Fair Bandwidth Allocation

ECE 466

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What is fair?
Max-Min Fair Allocation

$N$ flows, Link with rate $C$

- $r_i$: requested rate of flow $i$
- $a_i$: allocated rate of flow $i$
- $f$: fair share

1. If $\sum_j r_j \leq C$, then $a_i = r_i$ for each flows $i$.
2. If $\sum_j r_j > C$, then $a_i = \min\{r_i, f\}$ for $i$, where $f$ is selected such that $\sum_j a_i = C$.

Goal: Find fair share $f$ such that $C = \sum_j \min\{r_j, f\}$. 
$N$ flows, Link with rate $C$

$O = \{j \mid r_j > f\}$ Set of overloaded flows

$U = \{j \mid r_j \leq f\}$ Set of underloaded flows

$$C = \sum_{j \in U} r_j + |O| \cdot f,$$

$$f = \frac{C - \sum_{j \in U} r_j}{|O|}.$$
Algorithm: Fair Share Calculation

Input: \( N \) flows with request \( r_i \geq 0 \) for flow \( i \), link capacity \( C \)

Output: Fair share \( f \)

\[
f_0 \leftarrow 0 \\
n \leftarrow 0 \\
\text{repeat} \\
\quad n \leftarrow n + 1 \\
\quad U_n \leftarrow \{j \mid r_j \leq f_{n-1}\} \\
\quad O_n \leftarrow \{j \mid r_j > f_{n-1}\} \\
\quad f_n \leftarrow \frac{C - \sum_{i \in U_n} r_i}{|O_n|} \\
\quad \text{until } f_n = f_{n-1} \\
\text{return } f \leftarrow f_n
\]
Waterfilling \((C = 8)\)

\[ r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4 \]
Waterfilling \( (C = 8) \)

\[ r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4 \]

Diagram showing the waterfilling process with buckets labeled 1 to 4.
Waterfilling ($C = 8$)

$r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4$
Weighted Max-Min Fair Allocation

\( N \) flows, Link with rate \( C \)

- \( r_i \) requested rate of flow \( i \)
- \( a_i \) allocated rate of flow \( i \)
- \( \phi_i \) weight of flow \( i \)
- \( f \) fair share

1. If \( \sum_j r_j \leq C \), then \( a_i = r_i \) for each flows \( i \).
2. If \( \sum_j r_j > C \), then \( a_i = \min\{r_i, \phi_i f\} \) for flow \( i \), where \( f \) is selected such that \( \sum_j a_i = C \).

**Goal:** Find fair share \( f \) such that

\[
C = \sum_j \min\{r_j, \phi_j f\}.
\]
Weighted Max-Min Fair Allocation

$N$ flows, Link with rate $C$

$O = \{ j \mid r_j > \phi_j f \}$ Set of overloaded flows

$U = \{ j \mid r_j \leq \phi_j f \}$ Set of underloaded flows

\[
C = \sum_{j \in U} r_j + \sum_{j \in O} \phi_j \cdot f,
\]

\[
f = \frac{C - \sum_{j \in U} r_j}{\sum_{j \in O} \phi_j}.
\]
\( \phi_1 = \phi_2 = 1 \) and \( \phi_3 = \phi_4 = 2 \)
\( r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4 \)
\( \phi_1 = \phi_2 = 1 \) and \( \phi_3 = \phi_4 = 2 \)
\( r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4 \)
\( \phi_1 = \phi_2 = 1 \) and \( \phi_3 = \phi_4 = 2 \)

\( r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4 \)