ECE - Computer Networks II

Problem Set #1

This problems set contains exercises with the convolution operator '$\otimes$', defined as follows.

Definition: For two functions f and g the (min-plus) convolution is defined by:

$$f \otimes g(t) = \inf_{0 \le s \le t} \{f(s) + g(t-s)\}$$

Problem 1. Show that for two non-decreasing functions that satisfy $f(t) = g(t) = 0$ for $t \le 0$, the following holds:

$$f \otimes g(t) = \inf_{0 \le s \le t} \{f(s) + g(t-s)\} = \inf_{s \in \mathbb{R}} \{f(s) + g(t-s)\} \ .$$

Note: A function f for which $f(t) = 0$ if $t < 0$ is called *causal*.

Problem 1. Given the functions

$$f(t) = \begin{cases} 0 & \text{if } t \le 0 \\ t+3 & \text{if } t > 0 \end{cases}$$

$$g(t) = \begin{cases} 0 & \text{if } t \le 0 \\ 2t+1 & \text{if } t > 0 \end{cases}$$

Compute the *min-plus convolution* $f \otimes g$.

Problem 3.

Given the functions

$$A(t) = (\sigma + \rho t) I_{t>0}$$
$$S(t) = (C \cdot t) I_{t>0}$$

where σ, ρ and C are non-negative constants and $C > \rho$.

(a) Sketch the function $A \otimes S$.

(b) Use Reich's backlog equation to show that the queue length at any time t is bounded by σ.