

# ECE - Computer Networks II

## Problem Set #2

**Problem 1.** Prove the following properties of the min-plus convolution for one-sided non-decreasing processes  $F$ ,  $G$ , and  $H$ :

- (a) *Associativity:*  $(F \otimes G) \otimes H = F \otimes (G \otimes H)$ .
- (b) *Commutativity:*  $F \otimes G = G \otimes F$ .
- (c) *Boundedness.*  $F \otimes G \leq F$ . In particular,  $F \otimes F \leq F$ .

**Problem 2.** Given the functions  $f$  and  $g$ , compute the convolution  $f \otimes g$ :

$$f(t) = \begin{cases} 0, & t \leq 1 \\ 2, & 1 < t \leq 2 \\ 2+t, & t > 2 \end{cases}, \quad g(t) = \begin{cases} 0, & t \leq 2 \\ 1+2t, & t > 2 \end{cases}.$$

**Problem 3.** Define the function  $f_n(t) = nI_{t>0}$  for a number  $n \in \mathbb{N}$ .

1. Show that the function is a traffic process, i.e.,  $f_n$  is one-sided and non-decreasing.
2. Show the result of the convolution  $f_3 \otimes f_2$ .