Problem 1. Prove the following properties of the min-plus convolution for one-sided non-decreasing processes $F$, $G$, and $H$:

(a) Associativity: $(F \otimes G) \otimes H = F \otimes (G \otimes H)$.

(b) Commutativity: $F \otimes G = G \otimes F$.

(c) Boundedness. $F \otimes G \leq F$. In particular, $F \otimes F \leq F$.

Problem 2. Given the functions $f$ and $g$, compute the convolution $f \otimes g$:

$$f(t) = \begin{cases} 
0, & t \leq 1 \\
2, & 1 < t \leq 2 \\
2 + t, & t > 2 
\end{cases} \quad g(t) = \begin{cases} 
0, & t \leq 2 \\
1 + 2t, & t > 2 
\end{cases}$$

Problem 3. Define the function $f_n(t) = n I_{t>0}$ for a number $n \in \mathbb{N}$.

1. Show that the function is a traffic process, i.e., $f_n$ is one-sided and non-decreasing.

2. Show the result of the convolution $f_3 \otimes f_2$. 