ECE 466 - Computer Networks II

Problem Set #3

1. Consider the network with two buffered links with (with rates C_1 and C_2) and a delay element (with delay W) as shown in the figure.

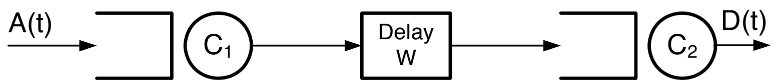


Figure 1:

(a) Express the service curve of the entire network in terms of C_1, C_2, and W.

(b) Determine the value of the network service curve at time $t = 10$ with parameters $C_1 = 20$ kbps, $C_2 = 30$ kbps, and $W = 5$ ms.

Solution:

(a) The service curve is a concatenation of constant rate servers and delay elements. We have

$$\begin{aligned} S_1(t) &= C_1 t \cdot I_{t>0} \\ S_2(t) &= \delta_W(t) \\ S_3(t) &= C_2 t \cdot I_{t>0} \end{aligned}$$

We compute

$$\begin{aligned} S_{net}(t) &= S_1 \otimes S_2 \otimes S_3(t) \\ &= S_1 \otimes S_3 \otimes S_2(t) \\ &= \begin{cases} \min(C_1, C_2)(t - W) & \text{if } t > W \\ 0 & \text{if } t \leq W \end{cases} \end{aligned}$$

The first step writes out the network service curve. The second step rearranges the service curves. The last step uses that the convolution of constant rate servers is the minimum of the rates, and the fact that $f \otimes \delta_d(t) = f(t-d)$.

We use, as usual, the short form:

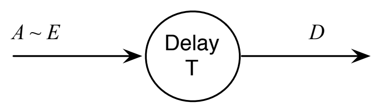
$$S_{net}(t) = \min(C_1, C_2)(t - W)_+$$

(b) Using the solution for (a) we get:

$$S_{net}(10) = \min(20, 30)(10 - 5) = 100 \; bits$$

2. Consider the arrivals to a delay element. The arrivals $A(t)$ are given by $A(t) = \sigma + \rho \cdot t$ $(t > 0)$. Consider two versions of the delay element:

- *Version 1:* The delay of each arrival is exactly T time units $(T > 0)$.
- *Version 2:* The delay is variable with a maximum delay of T_{max} time units.



(a) Characterize the service curves for both versions of the delay element.

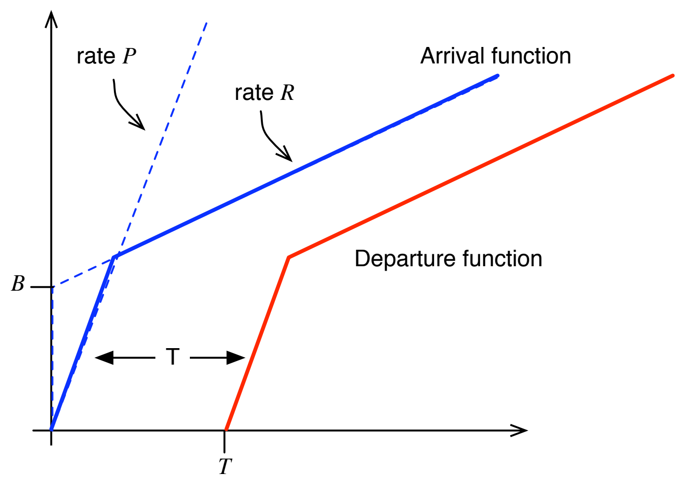
(b) For the first version, provide a sketch of an arrival function A and the corresponding departure function D at the delay element, and provide an expression for the departure function. Describe the departures that occur in any time interval of length $\tau > 0$.

(c) For the second version, show that the departures in a time interval of length $\tau > 0$ satisfy cannot exceed $\sigma + \rho \cdot T_{max} + \rho \cdot \tau$.

Solution:

(a) The delay element in *Version 1* describes an exact service curve $S(t) = \delta_{T_{max}}(t)$. In *Version 2*, it describes a lower service curve $\underline{S}(t) = \delta_{T_{max}}(t)$.

(b) See Figure. The figure shows that $D(t) = A(t - T)$.



Consider $D(t+\tau) - D(t)$ (for arbitrary $t \geq 0, \tau \leq t$), which are the departures in a time interval of length τ. We have

$$\begin{aligned} D(t+\tau) - D(t) &= A(t+\tau-T) - A(t-T) \\ &= (\sigma + \rho(t+\tau-T)) \cdot I_{t+\tau>T} - (\sigma + \rho(t+-T)) \cdot I_{t>T} \\ &= \begin{cases} 0 \ , & t \leq T-\tau \\ \sup\limits_{T-\tau<t\leq T} \{\sigma + \rho(t+\tau-T)\} = \sigma + \rho\tau \ , & T-\tau < t \leq T \\ \rho\tau \ , & t > T \end{cases} \end{aligned}$$

From the above we have that for any $\tau \leq t$ we have

$$D(t+\tau) - D(t) \leq \sigma + \rho\tau \ .$$

(c) Let D denote the departure function. Let us look at $D(t+\tau) - D(t)$ (for arbitrary $t > 0, \tau \leq t$), which are the departures in a time interval of length τ. At t, no data older than $t - T_{max}$ can be in the system. Hence

$$\begin{aligned} D(t+\tau) - D(t) &\leq A(t+\tau) - A(t-T_{max}) \\ &\leq \sigma + \rho(\tau + T_{max}) \\ &= (\sigma + \rho \cdot T_{max}) + \rho \cdot \tau \end{aligned}$$

3. Consider a non-work-conserving FCFS server that takes vacations every other T seconds, i.e., for T seconds, the server services traffic in FCFS order at rate C, and for the next T seconds, the server services no traffic, even if traffic is queued.

(a) What is the lower service curve of this server.

(b) Derive an expression for the maximum delay $d_{\max}$ for N (σ, ρ) flows multiplexed at this server. Justify your answer with a graphical illustration.

Solution:

(a) In the worst case, a packet arrives to find that the server has just begun an idle phase. Hence, the lower service curve is 0 for T seconds to indicate that no service can occur for intervals of up to length T. For larger intervals, the service curve has a stair-step shape as shown. Hence

$$S(t) = \begin{cases} \frac{n}{2}CT & t \in [nT, (n+1)T] \text{ and } n \text{ even} \\ \frac{n-1}{2}CT + C(t - NT) & t \in [nT, (n+1)T] \text{ and } n \text{ odd} \end{cases}$$

(b) For stability (finite delay), we require that $N\rho < C/2$. If the stability condition is satisfied, the maximum delay is given by the maximum horizontal distance between the arrival envelope and the service curve. This maximum necessarily occurs at an inflection point of one of the two piece-wise linear curves. In this case, it will occur at either $d'_{\max}$ or $d''_{\max}$.
Define $\eta \in \{1, 2, 3, \cdots\}$ such that $N\sigma$ lies just below ηCT, i.e., $(\eta - 1)CT < N\sigma \leq \eta CT$.[1]
Thus, $d'_{\max}$ is the solution to

$$S(d'_{\max}) = N\sigma$$

and $d''_{\max}$ is given by

$$d''_{\max} = (2\eta + 1)T - \frac{\eta CT - N\sigma}{N\rho}$$

Thus, $d_{\max} = \max(d'_{\max}, d''_{\max})$.

[1] For example, $\eta = 2$ in the figure.

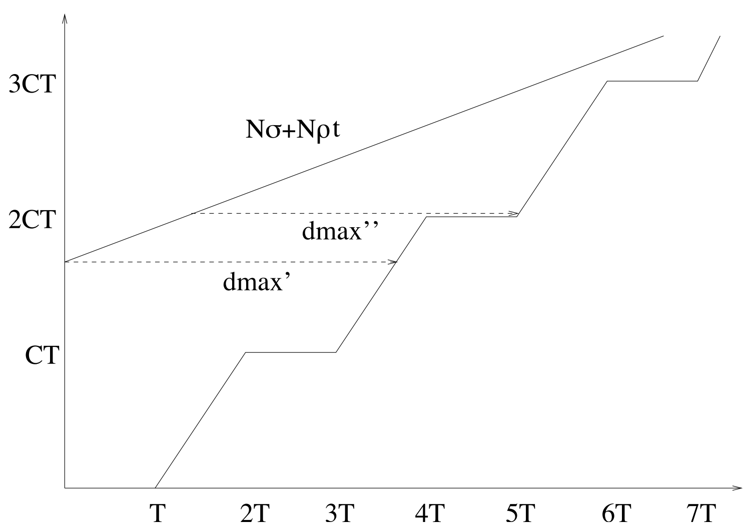


Figure 2: Aggregate arrival envelope and lower service curve.