ECE 466- Computer Networks II

Problem Set #4

1. Consider a sequence of arrivals as shown in Figure 1, which represent arrivals of video frames generated by a video codec.

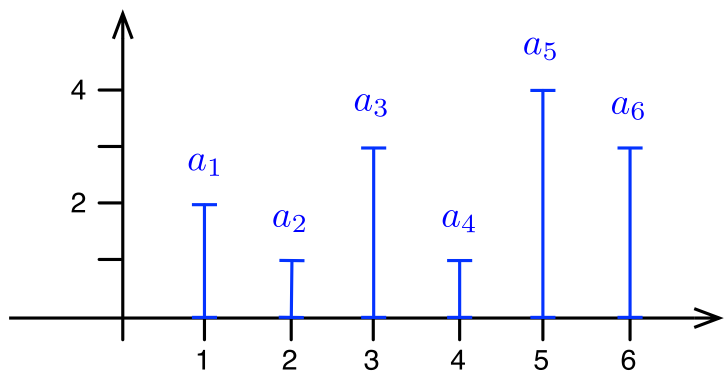

Figure 1:

   (a) Provide the values of the cumulative arrival function A and the minimal envelope $\mathcal{E}_A$ for a discrete-time domain, for times $t = 0, 1, 2, 3, 4, 5, 6$.

   (b) Sketch the cumulative arrival function A and the minimal envelope $\mathcal{E}_A$ for a continuous-time domain for the time interval $[0, 6]$.

Solution:

   (a) The arrival function is given by $A(n) = \sum_{i=1}^{n} a_n$, with $A(n) = 0$ for $n \leq 0$. Since the arrival sequence is finite with six elements, the discrete time version of the minimal envelope takes the form

$$\mathcal{E}_A(n) = \sup_{i=0,1,\ldots} \{A(n+i) - A(i)\} = \max_{i=0,1,\ldots,6-n} \{A(n+i) - A(i)\} \; .$$

   Thus, $\mathcal{E}_A(0) = 0$, $\mathcal{E}_A(1)$ is equal to the largest frame, $\mathcal{E}_A(2)$ is equal to the largest sum of a sequence of two consecutive frames, and so on. We obtain

$$
\begin{aligned}
\mathcal{E}_A(1) &= \max\{a_1\ ,\ a_2\ ,\ a_3\ ,\ a_4\ ,\ a_5\ , a_6\} &&= 4\ ,\\
\mathcal{E}_A(2) &= \max\{a_1+a_2\ ,\ a_2+a_3\ ,\ a_3+a_4\ ,\ a_4+a_5\ ,\ a_5+a_6\} &&= 7\ ,\\
\mathcal{E}_A(3) &= \max\{a_1+a_2+a_3\ ,\ a_2+a_3+a_4\ ,\ a_3+a_4+a_5\ ,\ a_4+a_5+a_6\} &&= 8\ ,\\
\mathcal{E}_A(4) &= \max\{a_1+a_2+a_3+a_4\ ,\ a_2+a_3+a_4+a_5\ ,\ a_3+a_4+a_5+a_6\} &&= 11\ ,\\
\mathcal{E}_A(5) &= \max\{a_1+a_2+a_3+a_4+a_5\ ,\ a_2+a_3+a_4+a_5+a_6\} &&= 13\ ,\\
\mathcal{E}_A(6) &= \max\{a_1+a_2+a_3+a_4+a_5+a_6\} &&= 14\ .
\end{aligned}
$$

(b) An explanation of the solution is in Chapter 2.

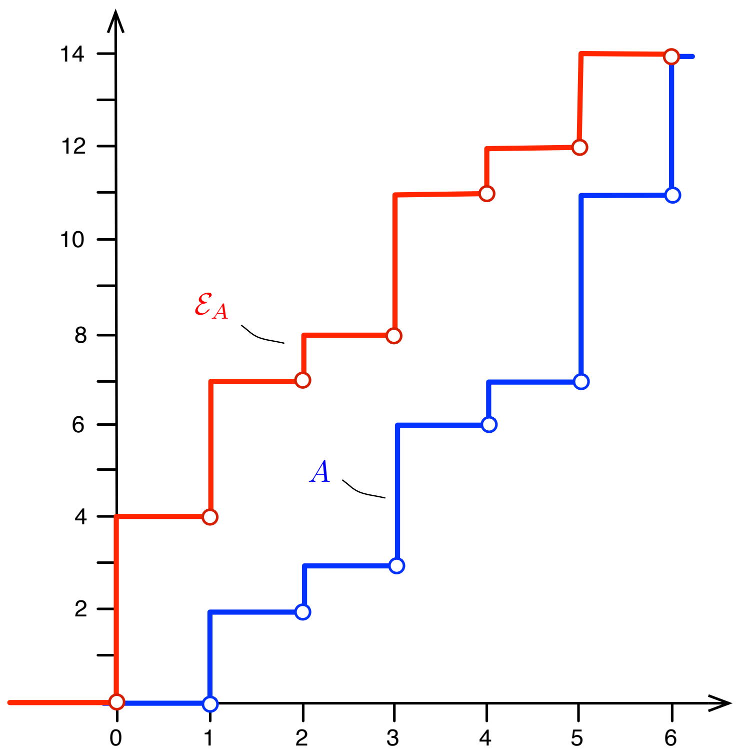

Figure 2:

2. Consider the following fluid-flow arrival function of a flow in continuous

time

$$A(t) = \begin{cases} 0\ , & t \leq 0 \\ t + k\ , & k \leq t \leq k+2 \\ 2(2t-k-3) & k+2 \leq t \leq k+3 \\ 2t\ , & k+3 \leq t \leq k+5 \end{cases} \qquad , k = 0, 5, 10, \ldots$$

Provide a sketch of the arrival function A and the minimal envelope $\mathcal{E}_A$.

Solution:

The arrival function A follows a pattern that cycles through transmission phases at rate 1 for two time units, rate 4 for one time unit, and rate 2 for two time units.

To construct $\mathcal{E}_A$ graphically, it is sufficient to look at the most amount of traffic that is created by A in time intervals of length $1, 2, \ldots, 10$.

Note that the result is a shifted version of A. That is,
$\mathcal{E}_A(t) = A(t-2) - 2$ for $t > 0$.

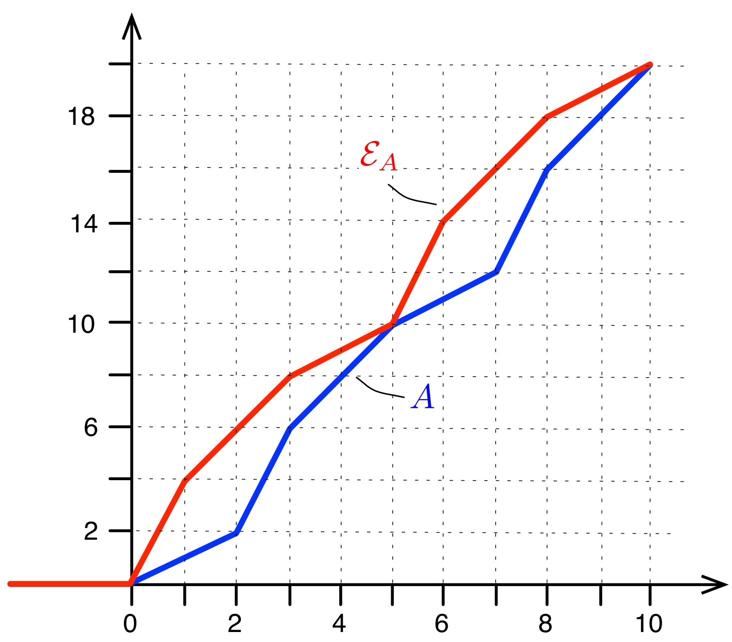

Figure 3:

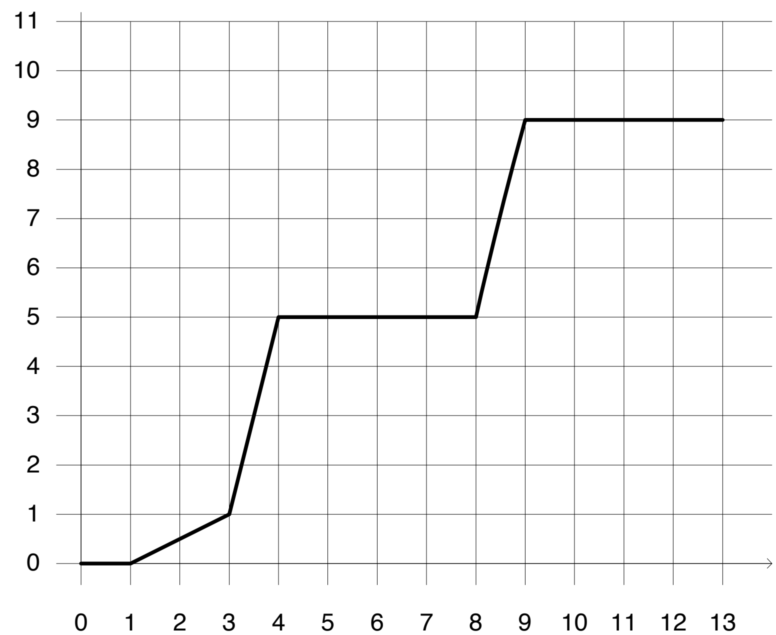

Figure 4:

3. Consider the graph of the evolution of an arrival sample path shown in Figure 4.

   (a) Suppose that the arrivals are regulated by a token bucket with parameters $\sigma = 2$ and $\rho = 1$. Indicate in the figure the cumulative amount of tokens entering the token bucket as a function $F(t)$, with initial value is $F(0) = 2$.
   (b) Indicate in the figure the maximum backlog and the maximum delay for arriving traffic.
   (c) Draw the minimal envelope for the arrivals shown in Figure 4.
   (d) Select values σ and ρ for the token bucket so that traffic is never backlogged in the buffer.

Solution:

   (a) See Figure 5.
   Content of tokens is the difference between the curve from "(a)" and the arrival function (as long as the curve from "(a)" is above the arrival curve.

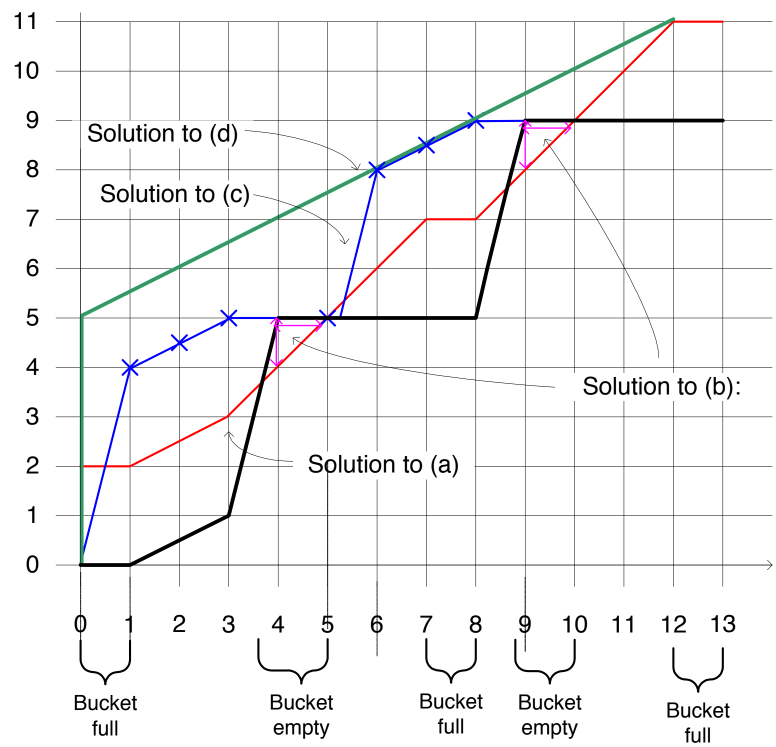

Figure 5:

(b) See Figure 5.
Max. delay and max. backlog occur at $t = 4$ and $t = 9$.

(c) See Figure 5.
The minimal envelope is shown as a dashed line.

(d) See Figure 5.
Note that there are many 'sensible' solutions to (d), as long as the curve is larger than the minimal envelope. A good choice is $\sigma = 3$ and $\rho = 1$ are a good choice.