FPLinQ: A Cooperative Spectrum Sharing Strategy for Device-to-Device Communications

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***Abstract*—Interference management is a fundamental problem for the device-to-device (D2D) network, in which transmitter and receiver pairs may be arbitrarily located geographically with full frequency reuse, so active links may severely interfere with each other. This paper devises a new optimization strategy called FPLinQ that coordinates link scheduling decisions together with power control among the interfering links throughout the network. Scheduling and power optimization for the interference channel are challenging combinatorial and nonconvex optimization problems. This paper proposes a fractional programming (FP) approach that derives a problem reformulation whereby the optimization variables are determined analytically in each iterative step. As compared to the existing works of FlashLinQ, ITLinQ and ITLinQ+, a merit of the proposed strategy is that it does not require tuning of design parameters. FPLinQ shows significant performance advantage as compared to the benchmarks in maximizing system throughput in a typical D2D network.**

I. Introduction

Transmit power optimization for the interference channel is a fundamental problem, for which no efficient global optimal algorithm is yet available. The optimization problem is especially challenging when a large number of mutually interfering links are present; its essential difficulty boils down to deciding which links should be active at any given time, i.e., how to schedule, and also at what power levels. This scheduling problem is important especially in the emerging device-to-device (D2D) communications paradigm, where arbitrary peer-to-peer transmissions can take place.

This paper provides a novel approach to this classic problem. The problem formulation is that of maximizing a weighted sum rate of D2D links in a network, where the weights account for fairness and the D2D links are selectively activated in order to alleviate interference. This is a difficult combinatorial and nonconvex optimization problem, as the scheduling decision of each D2D link depends strongly on the transmission states of nearby links. This paper proposes a *fractional programming based link scheduling* (FPLinQ) strategy to solve this scheduling problem. The central idea is to reformulate the original combinatorial problem in an equivalent fractional programming (FP) form wherein the link schedules can be determined analytically with the assistance of some auxiliary variables in each iteration. The proposed FPLinQ has provable convergence, with variable updates all in closed form, so no tuning of design parameters is needed. It is further shown in the paper that FPLinQ can be naturally extended to integrate power control with scheduling.

Interference-aware scheduling for the D2D network has attracted considerable research interests over the past years. Because of the difficulty in solving the combinatorial and nonconvex optimization problem globally and efficiently, efforts in the past typically involve greedy or other heuristics. This paper is motivated by recent series of works that propose algorithms named FlashLinQ [1], ITLinQ [2], and ITLinQ+ [3], that address the D2D scheduling problem from an information theoretical perspective. The algorithms in [1]–[3] are reviewed in details in Section III. In contrast to these earlier works, this paper shows that an optimization based approach can in general do much better. This paper uses FlashLinQ, ITLinQ and ITLinQ+ as benchmarks and illustrates that direct and clever network optimization can significantly outperform these previous state-of-the-art methods in terms of overall network performance.

II. Problem Formulation

Consider a set of unicast D2D wireless links $\mathcal{D}$, with single-antenna transmitter and receiver pairs indexed by $i \in \mathcal{D}$. Let h_{ij} be the channel from the jth transmitter to the ith receiver; let p_i be the fixed transmit power level of the ith transmitter when its D2D link is activated; let σ^2 be the additive white Gaussian noise (AWGN) power level. Introduce an indicator variable x_i for each link i, which equals to 1 if the link is activated and 0 otherwise. The data rate of link i can be expressed as

$$R_i(\mathbf{x}) = \log\left(1 + \frac{|h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}, j\neq i} |h_{ij}|^2 p_j x_j + \sigma^2}\right). \quad (1)$$

The network objective is to maximize some utility function of the *long-term average* rates, $U(\bar{R}_i(\mathbf{x}))$, where the averaging is over many scheduling slots. In each scheduling slot, a carefully selected *subset* of links are activated to transmit at the same time. This scheduling problem can be formulated as a maximization of weighted *instantaneous* rates:

$$\underset{\mathbf{x}}{\text{maximize}} \quad \sum_{i\in\mathcal{D}} w_i R_i(\mathbf{x}) \quad (2a)$$

$$\text{subject to} \quad x_i \in \{0,1\},\ \forall i \quad (2b)$$

where the weights w_i are chosen for priority or fairness.

This paper begins by considering the scheduling problem only: over the 0-1 variables x_i with p_i fixed. This is already a challenging combinatorial optimization because the optimal value of each x_i strongly depends on the choices of the other x_j's. A more general setting where the x_i's are relaxed to real numbers between 0 and 1 is considered later in the paper. In this case, the problem is still difficult to solve because the rate expression is non-convex in x_i.

III. FlashLinQ, ITLinQ, and ITLinQ+

The link scheduling problem (2) is a fundamental problem in communication network design. This section reviews the state-of-the-art approaches in the existing literature: FlashLinQ [1], ITLinQ [2], and ITLinQ+ [3]. To ease notation, we define signal-to-noise ratio (SNR), interference-to-noise ratio (INR), and signal-to-interference ratio (SIR) as:

$$\mathrm{SNR}_i = \frac{|h_{ii}|^2 p_i}{\sigma^2}, \;\; \mathrm{INR}_{ij} = \frac{|h_{ij}|^2 p_j}{\sigma^2}, \;\; \mathrm{SIR}_{ij} = \frac{|h_{ii}|^2 p_i}{|h_{ij}|^2 p_j}.$$

Because it is widely believed that optimizing all the x_i's at the same time is difficult (although our proposed method allows us to do so, as shown in the next section), a common strategy is to decide x_i sequentially, as stated in the algorithm below:

Algorithm 1 Sequential link selection

0) Initialize the set of scheduled links $\mathcal{S}$ to empty set $\emptyset$.
1) Sort all the links in a sequence $(i_1, i_2, \cdots, i_{|\mathcal{D}|})$.
for each link i in $(i_1, i_2, \cdots, i_{|\mathcal{D}|})$ **do**
  if link i does NOT "conflict" with any links in $\mathcal{S}$ **then**
    2) $x_i \leftarrow 1$ and $\mathcal{S} \leftarrow \mathcal{S} \cup \{i\}$
  else
    3) $x_i \leftarrow 0$
  end if
end for

FlashLinQ, ITLinQ and ITLinQ+ all adopt the above algorithmic framework. Their main difference lies in the criterion for deciding link conflict in the *if*-statement of the algorithm. Regarding Step 1 of the algorithm, a reasonable heuristic proposed in [3] is to sort all the links by a descending order of their rate weights.

A. FlashLinQ [1]

The FlashLinQ scheme in [1] applies a threshold θ to the link SIRs, i.e., link i does not conflict with links in $\mathcal{S}$ if

$$\mathrm{SIR}_{ji} \geq \theta, \; \forall j \in \mathcal{S} \;\; \text{and} \;\; \frac{|h_{ii}|^2 p_i}{\sum_{j\in\mathcal{S}} |h_{ij}|^2 p_j} \geq \theta. \tag{3}$$

The above criterion can be interpreted as that link i is scheduled if it does not cause too much interference to any already-activated links and also that itself does not suffer too much interference from the existing links. The performance of FlashLinQ is highly sensitive to the threshold θ; but choosing θ properly can be difficult in practice because of the complicated way network topology and channel magnitudes interact with each other. Further, using the same θ for all the links is usually suboptimal when the rate weight varies from link to link.

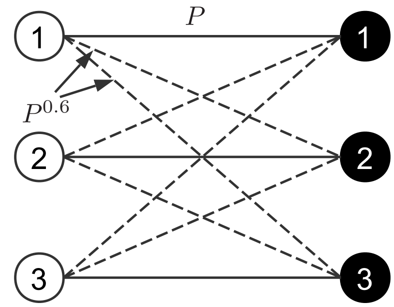

Fig. 1. Consider a 3-link Gaussian interference network with desired signal strength P, interfering signal strength $P^{0.6}$ and normalized noise. At most one link can be activated in order to satisfy the TIN condition, so the maximum sum GDoF under the TIN condition is 1. But, a higher sum GDoF of 1.2 can be achieved by simply activating all the links (although this does not meet the TIN condition). Thus, the TIN condition does not guarantee that a schedule is GDoF optimal.

B. ITLinQ [2] and ITLinQ+ [3]

ITLinQ is motivated by a recent result in the information theoretic study of interference channel [4] that identifies a sufficient (albeit not necessary) condition for the optimality of treating-interference-as-noise (TIN) in terms of generalized degree-of-freedom (GDoF). For the D2D scenario, this optimality condition for TIN can be written as

$$\mathrm{SNR}_i \geq \left(\max_{j\neq i} \mathrm{INR}_{ij}\right) \cdot \left(\max_{j\neq i} \mathrm{INR}_{ji}\right). \tag{4}$$

We refer to the above as the TIN condition.

The central idea of ITLinQ is to schedule a set of links that meet this TIN condition. Further, for distributed implementation purpose, [2] proposes to split (4) into

$$M\mathrm{SNR}_i^{\eta} \geq \max_{j\in\mathcal{S}} \mathrm{INR}_{ij} \tag{5}$$

and

$$M\mathrm{SNR}_i^{\eta} \geq \max_{j\in\mathcal{S}} \mathrm{INR}_{ji} \tag{6}$$

where M and η are the design parameters.

The ITLinQ+ scheme proposed in [3] follows the same approach, but splits the TIN condition differently:

$$\mathrm{SNR}_i^{\eta} \geq \max_{j\in\mathcal{S}} \left\{ \frac{\mathrm{INR}_{ij}}{(\min_{k\in\mathcal{S},k\neq j} \mathrm{INR}_{kj})^{\gamma}} \right\} \tag{7}$$

and

$$\mathrm{SNR}_i^{\eta} \geq \max_{j\in\mathcal{S}} \left\{ \frac{\mathrm{INR}_{ji}}{(\min_{k\in\mathcal{S},k\neq j} \mathrm{INR}_{jk})^{\gamma}} \right\} \tag{8}$$

where η and γ are introduced as the design parameters. We remark that the TIN condition requires power control, but ITLinQ and ITLinQ+ both assume full power for simplicity.

Like FlashLinQ, the performance of ITLinQ and ITLinQ+ is heavily dependent on the choice of design parameters, which can be difficult to choose optimally in practice. For example, [3] adopts two different sets of (η, γ) for ITLinQ+ for two different network models. However, it is unknown how the design parameters can be chosen so as to adapt ITLinQ and

ITLinQ+ to the particular network environment of interest.

It is important to point out that the theoretical basis of ITLinQ and ITLinQ+, namely the TIN condition, only helps decide whether for some particular schedule, treating interference as noise is the optimal coding strategy or not. It does not guarantee that if some schedule satisfies the TIN condition, then it must be the GDoF optimal schedule. For a given network topology, a schedule that does not satisfy the TIN condition can outperform one that does. This subtle point is illustrated with an example in Fig. 1.

IV. Scheduling via Fractional Programming

In contrast to the aforementioned works, this paper tackles the link scheduling problem (2) from an optimization perspective using tools from fractional programming.

A. Fractional Programming

Consider N pairs of non-negative functions $A_n(\mathbf{x})$ and strictly positive functions $B_n(\mathbf{x})$ of variable $\mathbf{x}$, for $n = 1, \cdots, N$. The sum-of-ratio problem is defined as

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & \sum_{n=1}^{N} \frac{A_n(\mathbf{x})}{B_n(\mathbf{x})} && \text{(9a)} \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X} && \text{(9b)} \end{aligned}$$

where $\mathcal{X}$ is some given constraint set.

The principal idea is to decouple the numerator and the denominator of each ratio terms, via a technique called *quadratic transform*, first introduced in [5], [6], as stated below:

Proposition 1. The sum-of-ratio problem (9) is equivalent to

$$\begin{aligned} \underset{\mathbf{x},\mathbf{y}}{\text{maximize}} \quad & \sum_{n=1}^{N} \left(2y_n\sqrt{A_n(\mathbf{x})} - y_n^2 B_n(\mathbf{x})\right) && \text{(10a)} \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X} && \text{(10b)} \end{aligned}$$

where $\mathbf{y} = (y_1, y_2, \cdots, y_N)$ is a set of auxiliary variables.

By introducing auxiliary variables y_i, this reformulation enables a more graceful numerical optimization of the numerators and the denominators of the ratios, and a more extensive exploration of the optimization landscape.

However, our scheduling problem (2) is not in a sum-of-ratio form but actually a sum-of-logarithm-ratio form. In order to apply Proposition 1, we need to "move" the ratios to the outside of the logarithm functions. Toward this end, we use a Lagrangian dual reformulation, first introduced in [5].

B. Lagrangian Reformulation

The goal here is to reformulate the original problem (2) as a sum-of-ratio problem with respect to the primal variable $\mathbf{x}$. First rewrite (2) as

$$\begin{aligned} \underset{\mathbf{x},\mathbf{z}}{\text{maximize}} \quad & \sum_{i\in\mathcal{D}} w_i \log(1+z_i) && \text{(11a)} \\ \text{subject to} \quad & x_i \in \{0,1\},\ \forall i && \text{(11b)} \\ & z_i \le \frac{|h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}, j\neq i} |h_{ij}|^2 p_j x_j + \sigma^2},\ \forall i. && \text{(11c)} \end{aligned}$$

The above optimization can be thought of as an outer optimization over $\mathbf{x}$ and an inner optimization over $\mathbf{z}$ with fixed $\mathbf{x}$. The inner optimization is as follows:

$$\begin{aligned} \underset{\mathbf{z}}{\text{maximize}} \quad & \sum_{i\in\mathcal{D}} w_i \log(1+z_i) && \text{(12a)} \\ \text{subject to} \quad & z_i \le \frac{|h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}, j\neq i} |h_{ij}|^2 p_j x_j + \sigma^2},\ \forall i && \text{(12b)} \end{aligned}$$

The solution to this inner optimization is obviously that z_i should satisfy (12b) with equality. But, let's express the problem in a different way. Note that (12) is a convex optimization in $\mathbf{z}$, so strong duality holds. Introduce the dual variable λ_i for the constraint (12b) and form the Lagrangian function

$$L(\mathbf{z}, \boldsymbol{\lambda}) = \sum_{i\in\mathcal{D}} w_i \log(1+z_i) - \sum_{i\in\mathcal{D}} \lambda_i \left(z_i - \frac{|h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}, j\neq i} |h_{ij}|^2 p_j x_j + \sigma^2}\right). \tag{13}$$

The optimization (12) is then equivalent to

$$\underset{\boldsymbol{\lambda}\succeq\mathbf{0}}{\text{minimize}} \quad \underset{\mathbf{z}}{\text{maximize}} \quad L(\mathbf{z}, \boldsymbol{\lambda}). \tag{14}$$

Because (12) has a trivial solution, the optimal $\boldsymbol{\lambda}$ can be found analytically. Let $(\mathbf{z}^*, \boldsymbol{\lambda}^*)$ be the saddle point of the above. It must satisfy the first-order condition $\partial L/\partial z_i = 0$:

$$\lambda_i^* = \frac{w_i}{1+z_i^*}. \tag{15}$$

But we already know that $z_i^* = \frac{|h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}, j\neq i} |h_{ij}|^2 p_j x_j + \sigma^2}$, so

$$\lambda_i^* = w_i - \frac{w_i |h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}} |h_{ij}|^2 p_j x_j + \sigma^2}. \tag{16}$$

Note that $\lambda_i^* \ge 0$ is automatically satisfied here. Using (16) in (14), problem (12) can then be reformulated as

$$\underset{\mathbf{z}}{\text{maximize}} \quad L(\mathbf{z}, \boldsymbol{\lambda}^*). \tag{17}$$

Now, combining with the outer maximization over $\mathbf{x}$ and after some algebra, we arrive at the following reformulation of (11):

Proposition 2. The original problem (2) is equivalent to

$$\begin{aligned} \underset{\mathbf{x},\mathbf{z}}{\text{maximize}} \quad & f_r(\mathbf{x}, \mathbf{z}) && \text{(18a)} \\ \text{subject to} \quad & x_i \in \{0,1\},\ \forall i && \text{(18b)} \end{aligned}$$

where the new objective function is

$$f_r(\mathbf{x}, \mathbf{z}) = \sum_{i\in\mathcal{D}} w_i \log(1+z_i) - \sum_{i\in\mathcal{D}} w_i z_i + \underbrace{\sum_{i\in\mathcal{D}} \frac{w_i(1+z_i)|h_{ii}|^2 p_i x_i}{\sum_{j\in\mathcal{D}} |h_{ij}|^2 p_j x_j + \sigma^2}}_{\text{sum of ratios}}. \tag{19}$$

Note that the last term is now in the sum-of-ratio form for which we can apply Proposition 1. The overall strategy is to optimize $\mathbf{x}$ and $\mathbf{z}$ in an iterative fashion. Note that when $\mathbf{x}$ is held fixed, the optimal $\mathbf{z}$ can be determined in closed form by solving $\partial f_r / \partial z_i = 0$, that is

$$z_i^* = \frac{|h_{ii}|^2 p_i x_i}{\sum_{j \in \mathcal{D}, j \neq i} |h_{ij}|^2 p_j x_j + \sigma^2}. \tag{20}$$

It only remains to optimize $\mathbf{x}$ for fixed $\mathbf{z}$ using FP.

C. Coordinated Link Scheduling

Applying Proposition 1 to the last term of f_r, we arrive at the following further reformulation:

$$f_q(\mathbf{x}, \mathbf{z}, \mathbf{y}) = \text{const}(\mathbf{z}) + \sum_{i \in \mathcal{D}} 2y_i \sqrt{w_i(1+z_i)|h_{ii}|^2 p_i x_i} - \sum_{i \in \mathcal{D}} y_i^2 \left(\sum_{j \in \mathcal{D}} |h_{ij}|^2 p_j x_j + \sigma^2 \right) \tag{21}$$

where const($\mathbf{z}$) refers to a constant term when $\mathbf{z}$ is fixed. The overall strategy is again to update $\mathbf{x}$, $\mathbf{z}$ and $\mathbf{y}$ iteratively. The update of $\mathbf{z}$ is already as shown in (20). When all the other variables are held fixed, the optimal $\mathbf{y}$ can be analytically determined by solving $\partial f_q / \partial y_i = 0$, i.e.,

$$y_i^* = \frac{\sqrt{w_i(1+z_i)|h_{ii}|^2 p_i x_i}}{\sum_{j \in \mathcal{D}} |h_{ij}|^2 p_j x_j + \sigma^2}. \tag{22}$$

A desirable property of f_q is that all the terms related to $\mathbf{x}$ can be grouped together according to i, the link index, i.e.,

$$f_q(\mathbf{x}, \mathbf{z}, \mathbf{y}) = \text{const}(\mathbf{z}, \mathbf{y}) + \sum_{i \in \mathcal{D}} Q_i(x_i, \mathbf{z}, \mathbf{y}) \tag{23}$$

where const($\mathbf{z}, \mathbf{y}$) refers to a constant term when both $\mathbf{z}$ and $\mathbf{y}$ are fixed; the per-link function Q_i is defined as

$$Q_i(x_i, \mathbf{z}, \mathbf{y}) = 2y_i \sqrt{w_i(1+z_i)|h_{ii}|^2 p_i x_i} - \sum_{j \in \mathcal{D}} y_j^2 |h_{ji}|^2 p_i x_i. \tag{24}$$

The optimal solution for $\mathbf{x}$ now becomes straightforward:

$$x_i^* = \begin{cases} 1, & \text{if } Q_i(1, \mathbf{z}, \mathbf{y}) > Q_i(0, \mathbf{z}, \mathbf{y}) \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

In fact, since Q_i equals 0 when x_i is 0, the activation of link i just depends on whether $Q_i(1, \mathbf{z}, \mathbf{y})$ is positive or not.

An iterative optimization can be readily devised by combining (20), (22) and (25). However, (25) makes a hard decision for turning off link i, which may not be the best practice, because once a link is turned off, it may never come back on. This is due to the fact that once x_i is set to zero, then y_i will also be zero; consequently Q_i will be zero in all future iterations and link i will never be re-activated again. To avoid the premature de-activation of links at the beginning of the iterations due to poor starting point, we propose to relax the integer variable $\mathbf{x}$ to be a real number between 0 and 1 throughout the iterations, i.e., set $\mathbf{x}$ in closed-form as

$$\tilde{x}_i = \min \left\{ 1, \left(\frac{y_i \sqrt{w_i(1+z_i)|h_{ii}|^2 p_i}}{\sum_{j \in \mathcal{D}} y_j^2 |h_{ji}|^2 p_i} \right)^2 \right\}, \tag{26}$$

then recover the integer solution by (25) after convergence. This approach is summarized in the following algorithm.

Algorithm 2 FPLinQ for scheduling D2D links

0) Initialize all the variables to feasible values.
repeat
  1) Update $\mathbf{z}$ by (20);
  2) Update $\mathbf{y}$ by (22);
  3) Update $\tilde{\mathbf{x}}$ by (26);
until Convergence
4) Recover the integer $\mathbf{x}$ by (25).

Algorithm 2 is guaranteed to converge, because the weighted sum rate objective (with relaxed $\mathbf{x}$) is nondecreasing after each of the Steps 1 to 3a. A desirable feature of FPLinQ is that no tuning of parameters is needed. But, FPLinQ is also somewhat more difficult to implement in a distributed fashion than FlashLinQ, ITLinQ and ITLinQ+, because it requires the update of $\mathbf{y}$ at all the links in every iteration.

D. Joint Scheduling and Power Control

The proposed link scheduling algorithm can be further improved by recognizing that instead of setting variables x_i to $\{0, 1\}$, we can allow transmission at variable power levels. The setting of $\tilde{x}_i \in [0, 1]$ as in (26) in fact already gives a continuous power control mechanism. In the following, we define a slightly different problem in which x_i must be chosen from a discrete set

$$\mathcal{X} = \{\xi_1, \cdots, \xi_M\} \tag{27}$$

with $0 \leq \xi_m \leq 1$ for $m = 1, \cdots, M$. Replacing the binary set $\{0, 1\}$ with $\mathcal{X}$ in (2) gives rise to a joint scheduling and discrete power control problem.

The extension of the FPLinQ algorithm to this case is straightforward. The problem reformulation still involves the maximization of f_q over $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$. The updates of $\mathbf{y}$ and $\mathbf{z}$ remain the same as in (22) and (20), respectively. When it comes to optimizing $\mathbf{x}$, we derive the following solution from the form of f_q in (23):

$$x_i^* = \arg\max_{x_i \in \mathcal{X}} Q_i(x_i, \mathbf{z}, \mathbf{y}). \tag{28}$$

Therefore, the optimal x_i for each link i can be determined through a search over $\mathcal{X}$. We note that Q_i is a quadratic function of $\sqrt{x_i}$, thus the optimal quantized value for x_i is some ξ_m whose square root $\sqrt{\xi_m}$ is the closest to where Q_i is maximized. Since the relaxed solution $\tilde{x}_i$ in (26) is the one that maximizes Q_i, the optimal quantization can be stated as

$$x_i^* = \arg\min_{x_i \in \mathcal{X}} |\sqrt{x_i} - \sqrt{\tilde{x}_i}|, \tag{29}$$

which can be interpreted as projecting $\tilde{x}_i$ to the nearest point in $\mathcal{X}$ in a square-root scale. We can then incorporate power control into Algorithm 2 by replacing (25) with (29). Note that the algorithm suggests to round the relaxed x_i to $\mathcal{X}$ in a *square-root* scale, as opposed to the common heuristic of rounding the relaxed x_i to the nearest $\mathcal{X}$ directly.

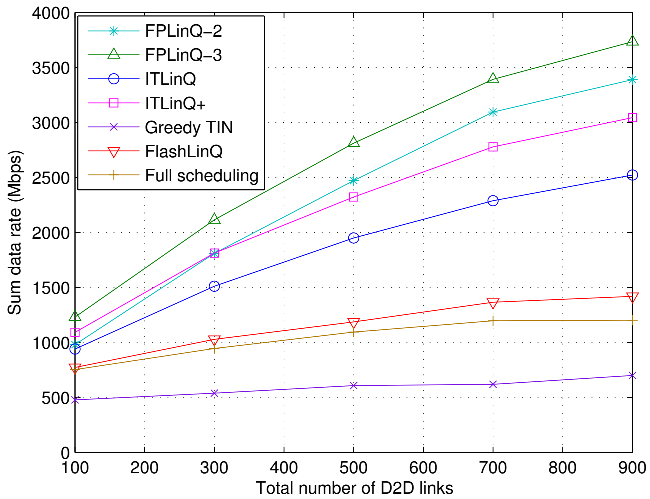

Fig. 2. Sum rate maximization in a D2D network

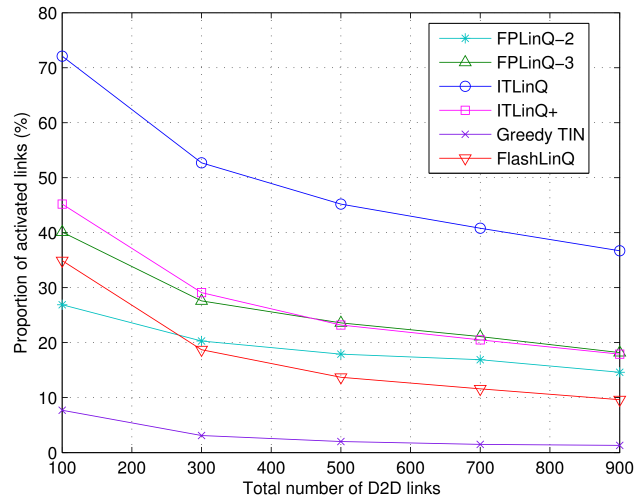

Fig. 3. Proportion of activated links in sum rate maximization

V. Numerical Results

To numerically evaluate FPLinQ, we simulate a 1km$\times$1km square area with randomly placed D2D links. The distance from any sender to its intended receiver is uniformly distributed between 2m to 65m as suggested in [2]. Following [1]–[3], we adopt the short-range outdoor channel model ITU-1411, with 5MHz bandwidth at carrier frequency of 2.4GHz. The antenna height of each device is 1.5m; antenna gain is 2.5dB; noise power spectrum density is -184dBm/Hz; noise figure is 7dB. As these parameter settings are almost the same as in [1]–[3], we use design parameters as recommended in these papers, i.e., $\theta = 9$dB for FlashLinQ, $\eta = 0.7$ and $M = 25$dB for ITLinQ, $\eta = 0.9$ and $\gamma = 0.1$ for ITLinQ+. We also introduce a greedy algorithm that activates links one by one for as long as the TIN condition is met, and a baseline of fully activating all the links. The proposed FPLinQ strategy is simulated with (i) $\mathcal{X} = \{0, 1\}$, referred to as FPLinQ-2, and (ii) $\mathcal{X} = \{0, 0.5, 1\}$, referred to as FPLinQ-3.

We first consider the sum rate maximization objective by setting all weights w_i to 1. Fig. 2 compares the sum rate achieved by the different methods. As expected from the conclusions of [2], [3], ITLinQ+ outperforms ITLinQ, and ITLinQ outperforms FlashLinQ. But FPLinQ-2 and FPLinQ-3 are seen to significantly outperform ITLinQ and ITLinQ+ when there are a large number of links (i.e., more than 300) densely deployed in the area. FPLinQ-3 has better performance than FPLinQ-2, illustrating the benefit of including just one additional power level $x_i = 0.5$ in $\mathcal{X}$. We also observe that greedily activating links according to the TIN condition is inferior to even the baseline of simply activating all links. This is because the TIN condition does not guarantee GDoF optimality for scheduling. Fig. 3 shows the proportion of activated links by these methods. The greedy TIN scheme is overly strict so that very few links get activated. This explains why ITLinQ and ITLinQ+, which are also based on TIN, need to relax the TIN condition through their respective parameter setting. Fig. 3 also illustrates that FPLinQ-3 activates more links than FPLinQ-2. This shows the advantage of allowing some of links to transmit at lower power, which enables more links to be activated, thereby attaining higher throughput. Finally, although not shown here, FPLinQ is also observed to significantly outperform the existing schemes in log-utility maximization with weights updated according to the proportional fairness objective.

VI. Conclusion

This paper proposes a novel optimization strategy called FPLinQ for coordinating spectrum sharing and power optimization in a D2D communications network. FPLinQ is based on reformulating the network utility maximization problem as a fractional program. It requires no parameter tuning and is shown to significantly outperform existing state-of-the-art methods.

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