Capacity Limits of Full-Duplex Cellular Network

Kaiming Shen, Reza Khosravi Farsani, and Wei Yu
Electrical and Computer Engineering Department, University of Toronto, ON M5S 3G4, Canada
Email: {kshen, rkfarsani, weiyu}@comm.utoronto.ca

***Abstract*—This paper explores the information theoretical capacity limits of uplink-downlink transmissions in a wireless cellular network with full-duplex (FD) base station (BS) and half-duplex user terminals. We recognize the cross-channel interference between the terminals as the main capacity bottleneck, and propose novel strategies that use BS as a relay to facilitate interference cancellation. We model the FD cellular system as a two-user interference channel with an extra cross-link feedback from the uplink receiver to the downlink transmitter, and show that the feedback allows a larger achievable rate region than the conventional non-feedback schemes. This paper further provides a converse and shows that the proposed scheme achieves the capacity of the full-duplex cellular network to within a constant additive gap. Finally, this paper considers a new scenario in which the uplink terminal has additional information to transmit to the downlink terminal directly. Relaying by the BS is shown to play a crucial role in maximizing the achievable rate in this case.**

I. Introduction

Traditional wireless cellular systems separate uplink and downlink signals in either the time-division duplex (TDD) or the frequency-division duplex (FDD) mode—because in a conventional analog front-end, the echo due to transmitting in one direction can overwhelm the receiver in the other direction. However, recent progress in analog and digital echo cancellation [1]–[3] has opened up the possibility of realizing full bi-directional communication in a full-duplex (FD) mode.

This paper considers an FD cellular network, in which the base station (BS) is capable of transmitting and receiving signals in the FD mode [4], but the user terminals still operate in half-duplex. In such a system, although uplink and downlink transmissions can occupy the same time-frequency resource block simultaneously from the BS's perspective, the cross-channel interference from the uplink terminal to the downlink terminal would present as a major source of interference. It is in fact the overall performance bottleneck [5], especially when the uplink and the downlink terminals are in close proximity to each other. One of the objectives of this paper is to show that such cross-channel interference can potentially be cancelled *with aid from the BS*—the BS can act as a relay, as it already needs to decode the uplink message, therefore can help the downlink terminal cancel cross interference.

As one of the main theoretical results of this paper, we model the FD cellular network as a two-user interference channel with a cross-feedback from the receiver of the uplink user to the transmitter of the downlink user, and characterize its capacity region to within a constant additive gap. The FD system can be modeled as a channel with feedback, because the FD BS is both the uplink receiver and the downlink transmitter, so the downlink transmit signal can be a function of the uplink received signal (with one unit delay). We remark that this channel model is different from the existing feedback interference channel model in the literature [6], where each transmitter obtains feedback from its intended receiver. We also mention here works [7], [8] that aim to optimize spectrum sharing to alleviate cross-channel interference for the FD cellular network and works [9], [10] on the multicell case.

This paper further considers the case where the uplink terminal has independent information to transmit directly to the downlink terminal. As a second main result of this paper, we show that such device-to-device (D2D) transmission can be significantly enhanced by the FD BS acting as a relay. In this regard, the channel model studied here is closely related to a relay channel model studied in [11], but the schemes proposed in this paper provides a larger rate region.

Notation: We use $[1:N]$ to denote the set $\{1,2,\ldots,N\}$, use $\mathsf{C}(x)$ to denote the function $\log_2(1+x)$ where $x\geq 0$, and use the superscripted bold font to denote a vector, e.g., $\mathbf{X}^N=(X_1,X_2,\ldots,X_N)$.

II. Cross Interference Cancellation in FD Cellular Systems

A. System Model

We first consider an FD cellular system without D2D transmission, as depicted in Fig. 1, with an input-output model:

$$Y_2[n]=g_{21}X_1[n]+g_{22}X_2[n]+Z_2[n], \qquad (1)$$

$$Y_3[n]=g_{31}X_1[n]+g_{32}X_2[n]+Z_3[n], \qquad (2)$$

in which the uplink user (node 1) wishes to communicate message $m_1\in[1:2^{NR_1}]$ to the BS (node 2), while the BS node 2 wishes to communicate message $m_2\in[1:2^{NR_2}]$ to the downlink user (node 3) over a block of length N; R_1 and R_2 are referred to as the uplink rate and the downlink rate, respectively. Here, $g_{ji}\in\mathbb{C}$ is the channel from node i to node j. At the nth channel use, $n\in[1:N]$, we use $X_i[n]\in\mathbb{C}$ to denote the signal transmitted from node i, $Y_i[n]\in\mathbb{C}$ to denote the signal received at node j, and $Z_j[n]\sim\mathcal{CN}(0,\sigma^2)$ to denote the i.i.d. Gaussian background noise at node j.

Note that X_1 and X_2 transmit in the same time-frequency band, i.e., the system operates in the full duplex mode and X_1 and X_2 interfere with each other. But since X_2 and Y_2 are co-located, the decoding of m_1 at $\mathbf{Y}_2^N$ can take $\mathbf{X}_2^N$ into account. In other words, at the end of N channel uses, node 2 can recover m_1 based on both $(\mathbf{Y}_2^N,\mathbf{X}_2^N)$, while node 3 recovers m_2 from $\mathbf{Y}_3^N$, i.e.,

$$\hat{m}_1=\mathcal{D}_1(\mathbf{Y}_2^N,\mathbf{X}_2^N)\ \text{ and }\ \hat{m}_2=\mathcal{D}_2(\mathbf{Y}_3^N) \qquad (3)$$

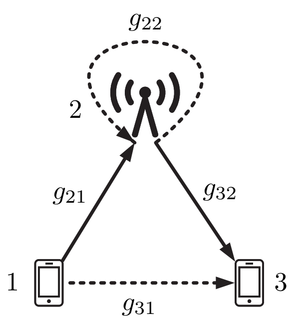

Fig. 1. A wireless FD cellular system consists of an uplink terminal (node 1), a BS (node 2), and a downlink terminal (node 3). The BS is capable of transmitting and receiving signals in FD mode. The cross interference from the uplink terminal to the downlink terminal is the dominant interference.

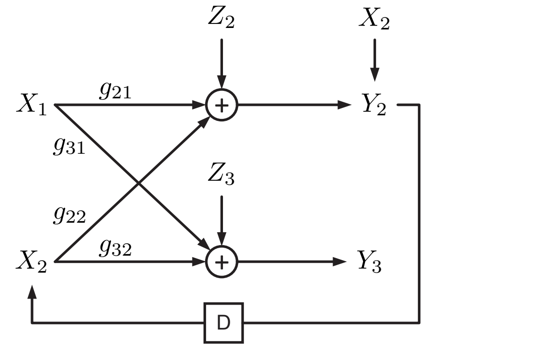

Fig. 2. FD cellular system as an interference channel with cross feedback. The block D represents one unit delay.

under deterministic functions $\mathcal{D}_1$ and $\mathcal{D}_2$. Hence, the FD capability at the BS allows its *self-interference* to be cancelled. Assuming complete self-interference cancellation, the FD channel described above is actually equivalent to a Z-interference channel with $g_{22} = 0$.

However, as the user terminals are half-duplex and distributed in space, the *cross-interference* from the uplink transmitter to the downlink receiver (namely g_{31}) still exists—such cross-interference is often the dominant noise in the network. The main idea of this paper is to explore the possibility of cancelling the cross-interference by taking advantage of another consequence of the fact that X_2 and Y_2 are co-located. Specifically, we recognize that the *encoding* of m_2 at X_2 can take (past) Y_2 into account, i.e., at the beginning of each channel use n, node 2 can form the transmit signal $X_2[n]$ based on both $(m_2, \mathbf{Y}_2^{n-1})$, while node 1 forms X_{1n} based on m_1, i.e.,

$$X_1[n] = \mathcal{E}_1(m_1) \text{ and } X_2[n] = \mathcal{E}_2(m_2, \mathbf{Y}_2^{n-1}) \tag{4}$$

using deterministic functions $\mathcal{E}_1$ and $\mathcal{E}_2$. Since Y_2 contains X_1 as a component and X_1 is the main interference to X_2, the encoding of X_2 as function of Y_2 can therefore possibly facilitate interference cancellation at its receiver Y_3.

One of the main objectives of this paper is to quantify the benefit of cross interference cancellation with the possible help from the BS. Toward this end, this paper models the above FD cellular channel model as an interference channel with side information X_2 at Y_2 and with cross feedback from Y_2 to X_2, as shown in Fig. 2. We impose an average power constraint P_i for $i \in \{1, 2\}$ on the transmitted signals, i.e., $\sum_{n=1}^{N} |X_i[n]|^2 \leq NP_i$. The rate pair (R_1, R_2) is said to be achievable if there exist functions $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{D}_1, \mathcal{D}_2)$ such that $\Pr\{(\hat{m}_1, \hat{m}_2) \neq (m_1, m_2)\}$ tends to zero as $N \to \infty$. In the remaining of this section, we derive an achievable rate region for this channel model and show that it is within a constant gap to an outer bound.

B. Achievable Rate Region

The FD cellular network is a two-user Z interference channel with cross-feedback from the interference-free receiver to the other transmitter. This paper proposes a combination of coding strategy for the interference channel, using the Han-Kobayash scheme of common-private message splitting at X_1, and a relaying strategy that helps the cancellation of interference produced by the common information at Y_3. Our achievability scheme further recognizes that the common information does not need to be decoded at Y_2.

Theorem 1: The rate pair (R_1, R_2) is achievable for the Gaussian FD cellular system if it is in the convex hull of

$$R_1 \leq \mathsf{C}\left(\frac{(\beta+\gamma)|g_{21}|^2 P_1}{\sigma^2}\right) \tag{5a}$$

$$R_2 \leq \mathsf{C}\left(\frac{(1-\eta)|g_{32}|^2 P_2}{\sigma^2 + \gamma|g_{31}|^2 P_1}\right) \tag{5b}$$

$$\begin{aligned} R_1 + R_2 \leq\ & \mathsf{C}\left(\frac{|g_{32}|^2 P_2 + (\alpha+\beta)|g_{31}|^2 P_1 + J\sqrt{\alpha\eta}}{\sigma^2 + \gamma|g_{31}|^2 P_1}\right) \\ & + \mathsf{C}\left(\frac{\gamma|g_{21}|^2 P_1}{\sigma^2}\right) \end{aligned} \tag{5c}$$

for $\alpha + \beta + \gamma \leq 1$, $\alpha, \beta, \gamma \geq 0$, and $0 \leq \eta \leq 1$, where $J = 2|g_{31} g_{32}|\sqrt{P_1 P_2}$.

Proof: We provide a proof sketch here. The idea is to split the X_1's uplink message $m_1 \in [1 : 2^{NR_1}]$ into a common-private message pair $(m_{10}, m_{11}) \in [1 : 2^{NR_{10}}] \times [1 : 2^{NR_{11}}]$ with $R_{10} + R_{11} = R_1$; the common message m_{10} is intended for both Y_2 and Y_3, whereas the private message m_{11} is intended for Y_2 alone. The uplink message at X_1 consists of only private message m_2 intended for Y_3. After the common message m_{10} is decoded at Y_2, it is fed back to X_2, which then uses a relay strategy (in a block-Markov fashion) to facilitate the simultaneous nonunique decoding of (m_{10}, m_2) at Y_3.

The relaying strategy involves the partitioning of m_{10} into 2^{NR_B} random bins (with $R_B \leq R_{10}$). We use the function $\ell = \mathcal{B}(m_{10})$ to denote the bin index ℓ of each m_{10}. The block-Markov encoding-and-decoding procedure is performed across a total of T blocks; we use $t \in [1 : T]$ to index the block. The codebooks are generated independently according to i.i.d. Gaussian distribution $\mathcal{CN}(0, 1)$ as follows:

- Private message $\{\mathbf{w}_1^N(m_{11})\}$, $m_{11} \in [1 : 2^{NR_{11}}]$,
- Private message $\{\mathbf{w}_2^N(m_2)\}$, $m_2 \in [1 : 2^{NR_2}]$,

- Common message $\{\tilde{\mathbf{v}}^N(m_{10})\}$, $m_{10} \in [1:2^{NR_{10}}]$,
- Relay message $\{\mathbf{u}^N(\ell)\}$, $\ell \in [1:2^{NR_B}]$.

In block t, given $(\ell^{t-1}, m_{10}^t, m_{11}^t)$, node 1 transmits

$$\mathbf{x}_1^N(t) = \mathbf{v}^N(t) + \sqrt{\gamma P_1}\mathbf{w}_1^N(m_{11}^t) \tag{6}$$

where

$$\mathbf{v}^N(t) = \sqrt{\alpha P_1}\mathbf{u}^N(\ell^{t-1}) + \sqrt{\beta P_1}\tilde{\mathbf{v}}^N(m_{10}^t). \tag{7}$$

Node 2 decodes $\hat{m}_{10}^t \in [1:2^{NR_{10}}]$ and $\hat{m}_{11}^t \in [1:2^{NR_{11}}]$, which are possible whenever

$$R_{10} \le I(V;Y_2|U,X_2) = \mathsf{C}\left(\frac{\beta|g_{21}|^2P_1}{\sigma^2+\gamma|g_{21}|^2P_1}\right), \tag{8}$$

$$R_{11} \le I(X_1;Y_2|U,V,X_2) = \mathsf{C}\left(\frac{\gamma|g_{21}|^2P_1}{\sigma^2}\right). \tag{9}$$

The decoding of $\hat{m}_{10}^{t-1}$ at Y_2 in the previous block allows X_2 to compute $\hat{\ell}^{t-1} = \mathcal{B}(\hat{m}_{10}^{t-1})$ and to transmit

$$\mathbf{x}_2^N(t) = \sqrt{\eta P_2}\mathbf{u}^N(\hat{\ell}^{t-1}) + \sqrt{(1-\eta)P_2}\mathbf{w}_2^N(m_2^t). \tag{10}$$

Node 3 decodes the blocks in a backward direction, i.e., block $t-1$ prior to block t. In block t, it first decodes the relay message ℓ^{t-1} cooperatively transmitted by X_1 and X_2 at rate

$$R_B \le I(U;Y_3) \tag{11}$$

$$= \mathsf{C}\left(\frac{\left(|g_{31}|\sqrt{\alpha P_1}+|g_{32}|\sqrt{\eta P_2}\right)^2}{\sigma^2+(\beta+\gamma)|g_{31}|^2P_1+(1-\eta)|g_{32}|^2P_2}\right). \tag{12}$$

The decoded bin index $\hat{\hat{\ell}}^{t-1}$ is used later for decoding of block $t-1$. In block t, node 3 finds the unique $\hat{m}_2^t \in [1:2^{NR_2}]$ along with a (not necessarily unique) $\hat{\hat{m}}_{10}^t \in [1:2^{NR_{10}}]$ so that

$$\mathcal{B}(\hat{\hat{m}}_{10}^t) = \hat{\hat{\ell}}^t. \tag{13}$$

This simultaneous non-uniqueness decoding is successful if

$$R_2 \le I(X_2;Y_3|U,V) = \mathsf{C}\left(\frac{(1-\eta)|g_{32}|^2P_2}{\sigma^2+\gamma|g_{31}|^2P_1}\right), \tag{14}$$

and

$$R_2 + (R_{10}-R_B) \le I(V,X_2;Y_3|U) \tag{15}$$

$$= \mathsf{C}\left(\frac{(1-\eta)|g_{32}|^2P_2+\beta|g_{31}|^2P_1}{\sigma^2+\gamma|g_{31}|^2P_1}\right).$$

The set of inequalities (8), (9), (11), (14), and (15) along with the conditions $R_B \le R_{10}$, $R_{11} = R_1 - R_{10}$ and $R_B \ge 0, R_{10} \ge 0, R_{11} \ge 0$, give (5). ■

Remark 1: We remark that node 3 could have used simultaneous unique decoding to achieve the inner bound in Theorem 1, i.e., finding the unique $(\hat{m}_2^t, \hat{\hat{m}}_{10}^t)$ in the decoding process. In this case, to guarantee successful decoding, the condition

$$R_{10} - R_B \le I(V;Y_3|U,X_2) = \mathsf{C}\left(\frac{\beta|g_{31}|^2P_1}{\sigma^2+\gamma|g_{31}|^2P_1}\right) \tag{16}$$

is required in addition to (14) and (15). It is possible to prove however that this additional constraint does not reduce the achievable rate region.

Corollary 1: In the very strong interference regime defined as $|g_{31}|^2 \ge |g_{21}|^2(1+|g_{32}|^2)$, the achievable region (5) with $\alpha^\star = 0$, $\beta^\star = 1$, $\gamma^\star = 0$, and $\eta^\star = 0$ is the capacity region.

C. Outer Bound and Constant Gap Optimality

The following outer bound can be derived for the FD cellular network model, using a genie-aided technique. The outer bound allows a constant-gap optimality result for the proposed achievability scheme.

Theorem 2: If a rate pair (R_1, R_2) is achievable for the Gaussian FD cellular network, then it must be in the convex hull of

$$R_1 \le \mathsf{C}\left(\frac{(1-\rho^2)|g_{21}|^2P_1}{\sigma^2}\right) \tag{17a}$$

$$R_2 \le \mathsf{C}\left(\frac{(1-\rho^2)|g_{32}|^2P_2}{\sigma^2}\right) \tag{17b}$$

$$R_1 + R_2 \le \mathsf{C}\left(\frac{|g_{31}|^2P_1+|g_{32}|^2P_2+J\rho}{\sigma^2}\right) + \mathsf{C}\left(\frac{(1-\rho^2)|g_{21}|^2P_1}{1+(1-\rho^2)|g_{31}|^2P_1}\right) \tag{17c}$$

for some $\rho \in [0,1]$, where $J = 2|g_{31}g_{32}|\sqrt{P_1P_2}$.

Theorem 3: The achievable rate region of Theorem 1 is within 1 bit/s/Hz to the capacity region of the Gaussian FD cellular network.

Proof: Following the power splitting scheme of [12], we set $\alpha = 0$, $\gamma = \min\{1, \sigma^2/(|g_{31}|^2P_1)\}$, $\beta = 1-\gamma$, and $\eta = 0$ in Theorem 1. Contrasting the resulting inner bound (5) with the outer bound (17) gives a constant gap of 1 bit/s/Hz. ■

Theorem 4: In the strong interference regime, defined as $|g_{31}| \ge |g_{21}|$, the constant gap δ to the channel capacity can be reduced to $\frac{1}{2} + \frac{1}{2}\log_2(\frac{\sqrt{2}+1}{2}) \approx 0.6358$ bits/s/Hz.

Proof: Given $|g_{31}| \ge |g_{21}|$, the optimal power-splitting in the inner bound (5) is $\beta^\star = 1-\alpha$ and $\gamma^\star = 0$. Comparing the resulting inner bound to the outer bound (17), we find that

$$2\delta \le 1 + \mathsf{C}\left(\frac{|g_{31}|^2P_1+|g_{32}|^2P_2+2\rho|g_{31}g_{32}|\sqrt{P_1P_2}}{\sigma^2}\right) - \mathsf{C}\left(\frac{|g_{31}|^2P_1+|g_{32}|^2P_2+2\rho^2|g_{31}g_{32}|\sqrt{P_1P_2}}{\sigma^2}\right)$$

$$= 1 + \log_2\left(\frac{\lambda+\rho}{\lambda+\rho^2}\right) \tag{18}$$

where variable λ is defined as

$$\lambda = \frac{|g_{31}|^2P_1+|g_{32}|^2P_2+\sigma^2}{2|g_{31}g_{32}|\sqrt{P_1P_2}}. \tag{19}$$

Observe that $\lambda \ge 1$. Then, finding an upper bound on δ amounts to solving the problem:

$$\text{maximize} \quad \frac{\lambda+\rho}{\lambda+\rho^2} \tag{20a}$$

$$\text{subject to} \quad \lambda \ge 1,\ 0 \le \rho \le 1. \tag{20b}$$

The optimal solution of the above is $\lambda^\star = 1$, and $\rho^\star = \sqrt{2}-1$. Substituting $(\lambda^\star, \rho^\star)$ in (18) verifies the theorem. ■

$$R_1 \le \mathsf{C}\left(\frac{(b+c)|g_{21}|^2P_1}{\sigma^2 + d|g_{21}|^2P_1}\right) \tag{26a}$$

$$R_2 \le \mathsf{C}\left(\frac{f|g_{32}|^2P_2}{\sigma^2 + c|g_{31}|^2P_1}\right) \tag{26b}$$

$$R_3 \le \mathsf{C}\left(\frac{(b+d)|g_{31}|^2P_1}{\sigma^2 + c|g_{31}|^2P_1}\right) + \mathsf{C}\left(\frac{(|g_{31}|\sqrt{aP_1} + |g_{32}|\sqrt{eP_2})^2}{\sigma^2 + (b+c+d)|g_{31}|^2P_1 + f|g_{32}|^2P_2}\right) \tag{26c}$$

$$R_1 + R_3 \le \mathsf{C}\left(\frac{(b+c)|g_{21}|^2P_1}{\sigma^2 + d|g_{21}|^2P_1}\right) + \mathsf{C}\left(\frac{d|g_{31}|^2P_1}{\sigma^2 + c|g_{31}|^2P_1}\right) \tag{26d}$$

$$R_1 + R_3 \le \mathsf{C}\left(\frac{c|g_{21}|^2P_1}{\sigma^2 + d|g_{21}|^2P_1}\right) + \mathsf{C}\left(\frac{(b+d)|g_{31}|^2P_1}{\sigma^2 + c|g_{31}|^2P_1}\right) + \mathsf{C}\left(\frac{(|g_{31}|\sqrt{aP_1} + |g_{32}|\sqrt{eP_2})^2}{\sigma^2 + (b+c+d)|g_{31}|^2P_1 + f|g_{32}|^2P_2}\right) \tag{26e}$$

$$R_2 + R_3 \le \mathsf{C}\left(\frac{(a+b+d)|g_{31}|^2P_1 + (e+f)|g_{32}|^2P_2 + J\sqrt{ae}}{\sigma^2 + c|g_{31}|^2P_1}\right) \tag{26f}$$

$$R_1 + R_2 + R_3 \le \mathsf{C}\left(\frac{(b+c)|g_{21}|^2P_1}{\sigma^2 + d|g_{21}|^2P_1}\right) + \mathsf{C}\left(\frac{d|g_{31}|^2P_1 + f|g_{32}|^2P_2}{\sigma^2 + c|g_{31}|^2P_1}\right) \tag{26g}$$

$$R_1 + R_2 + R_3 \le \mathsf{C}\left(\frac{c|g_{21}|^2P_1}{\sigma^2 + d|g_{21}|^2P_1}\right) + \mathsf{C}\left(\frac{(a+b+d)|g_{31}|^2P_1 + (e+f)|g_{32}|^2P_2 + J\sqrt{ae}}{\sigma^2 + c|g_{31}|^2P_1}\right) \tag{26h}$$

III. FD Cellular Network with D2D Link

We now consider an FD cellular network with additional independent D2D transmission from the uplink terminal to the downlink terminal. Specifically, node 1 wishes to send a message $m_3 \in [1 : 2^{NR_3}]$ to node 3 along with the uplink and downlink transmissions. The encoding functions are

$$X_{1n} = \mathcal{E}_1(n, m_1, m_3) \text{ and } X_{2n} = \mathcal{E}_2(n, m_2, \mathbf{Y}_2^{n-1}). \tag{21}$$

The decoding functions are

$$\hat{m}_1 = \mathcal{D}_1(\mathbf{Y}_2^N, \mathbf{X}_2^N),\ \hat{m}_2 = \mathcal{D}_2(\mathbf{Y}_3^N),\ \hat{m}_3 = \mathcal{D}_3(\mathbf{Y}_3^N). \tag{22}$$

The rate triple (R_1, R_2, R_3) is achievable if there exists a set of functions $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3)$ such that $\Pr\{(\hat{m}_1, \hat{m}_2, \hat{m}_3) \ne (m_1, m_2, m_3)\}$ tends to zero as $N \to \infty$.

We wish to take advantage of the possibility that the BS can act as a relay for the downlink user, by facilitating either interference cancellation or the D2D transmission. We split the uplink message m_1 and the D2D-link message m_3 into the common-private message pairs (m_{10}, m_{11}) and (m_{30}, m_{33}), respectively. Partition (m_{10}, m_{30}) into $2^{N(R_{B1}+R_{B3})}$ random bins with $R_{B1} < R_{10}$ and $R_{B3} < R_{30}$, where R_{10} is the rate of m_{10} and R_{30} is the rate of m_{30}. The bin index function is $b = \mathcal{B}(m_{10}, m_{30})$. We generate the following codebooks independently according to i.i.d. Gaussian distribution $\mathcal{CN}(0, 1)$:

- Private message $\{\mathbf{w}_1^N(m_{11})\}$, $m_{11} \in [1 : 2^{NR_{11}}]$,
- Private message $\{\mathbf{w}_2^N(m_2)\}$, $m_2 \in [1 : 2^{NR_2}]$,
- Private message $\{\mathbf{w}_3^N(m_{33})\}$, $m_{33} \in [1 : 2^{NR_{33}}]$,
- Common message $\{\tilde{\mathbf{v}}^N(m_{10}, m_{30})\}$, $(m_{10}, m_{30}) \in [1 : 2^{NR_{10}}] \times [1 : 2^{NR_{30}}]$,
- Relay message $\{\mathbf{u}^N(\ell)\}$, $\ell \in [1 : 2^{N(R_{B1}+R_{B2})}]$.

As before, we propose to perform the block-Markov encoding-and-decoding procedure across T blocks. In each block $t \in [1 : T]$, node 1 transmits

$$\mathbf{x}_1^N(t) = \mathbf{v}^N(t) + \sqrt{cP_1}\mathbf{w}_1^N(m_{11}^t) + \sqrt{dP_1}\mathbf{w}_3^N(m_{33}^t) \tag{23}$$

where

$$\mathbf{v}^N(t) = \sqrt{aP_1}\mathbf{u}^N(\ell^{t-1}) + \sqrt{bP_1}\tilde{\mathbf{v}}^N(m_{10}^t, m_{30}^t). \tag{24}$$

The BS fully recovers (m_{10}, m_{30}, m_{11}) then uses the bin index of (m_{10}, m_{30}) to help node 3 decode the downlink signal. Specifically, in block t, after decoding $(\hat{m}_{10}^{t-1}, \hat{m}_{30}^{t-1}, \hat{m}_{11}^{t-1})$ and computing $\hat{\ell}^{t-1} = \mathcal{B}(\hat{m}_{10}^{t-1}, \hat{m}_{30}^{t-1})$ in the previous block $t-1$, node 2 transmits

$$\mathbf{x}_2^N(t) = \sqrt{eP_2}\mathbf{u}^N(\hat{\ell}^{t-1}) + \sqrt{fP_2}\mathbf{w}_2^N(m_2^t). \tag{25}$$

In a backward direction across the T blocks, node 3 fully recovers (m_{10}, m_{30}, m_{33}) with the help of the bin index ℓ. The resulting achievable rate region is stated below:

Theorem 5: The rate triple (R_1, R_2, R_3) is achievable for the Gaussian FD cellular network with D2D transmission, if it is in the convex hull of the region defined in (26) as displayed at the top of the page for $a + b + c + d \le 1$, $e + f \le 1$, and $a, b, c, d, e, f \ge 0$, where $J = 2|g_{31}g_{32}|\sqrt{P_1P_2}$.

Remark 2: The inner bound of Theorem 5 reduces to that of Theorem 1 when R_3 is set to zero.

Remark 3: In the case where the terminals are equipped with multiple antennas, it is possible to generalize the above scheme using Marton's coding for the broadcast channel.

We note here that a similar channel has been analyzed in [11], but without rate splitting and interference cancellation. The proposed scheme of this paper can provide a strictly larger achievable rate region than the decode-forward scheme of [11].

IV. Numerical Examples

We consider an FD cellular network in which the uplink/downlink user-to-BS distance is fixed at 300m, the user-

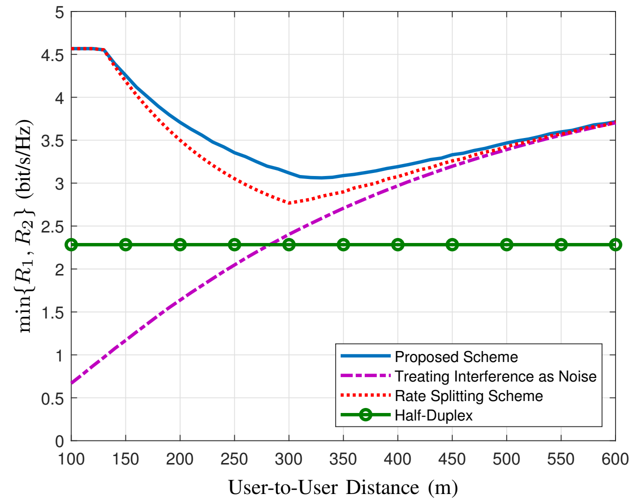

Fig. 3. $\min\{R_1, R_2\}$ in the FD cellular network without D2D link.

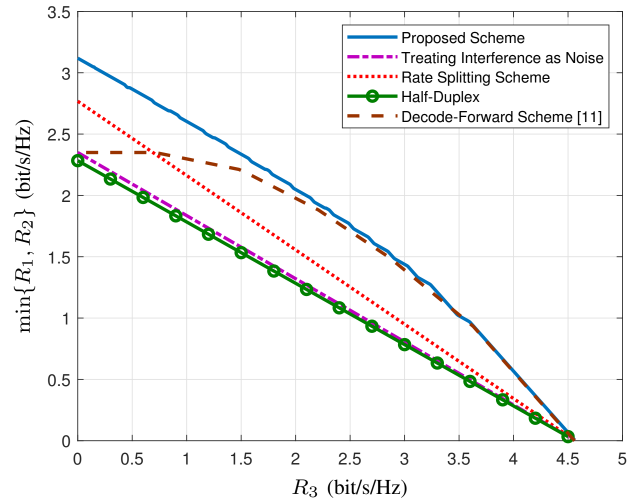

Fig. 4. R_3 vs. $\min\{R_1, R_2\}$ in the D2D case when users are 300m apart.

to-user distance is set to different values. Let the maximum transmit power spectrum density (PSD) be -47 dBm/Hz for both uplink and downlink; we set the PSD of the background noise be -169 dBm/Hz. The channel magnitude is modeled as $-128.1 - 37.6\log_{10}(L)$ in dB scale, where L is the distance (in km). The following baselines are evaluated:

- Half-duplex with separate uplink and downlink;
- Treating interference as noise with FD uplink/dowlink;
- Rate splitting scheme: The uplink message is split for interference cancellation, but without relaying by the BS.
- Decode-forward scheme (only for the D2D case): This corresponds to the scheme of [11]. It is a special case of our proposed scheme without rate splitting and interference cancellation.

The first three baselines are extended to the D2D case by time or frequency division multiplex of cellular and D2D traffic.

Fig. 3 compares the symmetric uplink-downlink rate $\min\{R_1, R_2\}$ in the no D2D case. It shows that the proposed BS-aided scheme is more effective than simple interference cancellation, gaining up to about 0.5 bits/s/Hz. Observe that the proposed scheme shows the most benefit when the user-to-user distance is not too close or too far. Fig. 4 shows the trade-off between the D2D rate and the symmetric uplink-downlink rate when the two users are 300m apart. The proposed scheme achieves a larger rate region than the baseline schemes.

V. Concluding Remarks

This paper shows that in a cellular network with full-duplex BS and half-duplex user terminals, the FD BS can play a crucial role in facilitating the cancellation of the cross-terminal interference and the relaying of the direction data transmission between the terminals. A crucial component of the relaying strategy used in this paper is binning: the BS uses the bin index of the decoded common information to help the downlink user. In practice, such a binning strategy can be implemented using incremental redundancy coding, similar to its use in hybrid automatic repeat request (HARQ) where the parity bits of the message are transmitted to help the eventual decoding.

The results of this paper can be readily extended to the case where the BSs and the terminals are equipped with multiple antennas, in which case a beamforming strategy can be used to transmit multiple data streams for each of the uplink, downlink, and D2D transmissions.

References

[1] J. I. Choi, M. Jain, K. Srinivasan, P. Levis, and S. Katti, "Achieving single channel, full duplex wireless communication," in *ACM Int. Conf. Mobile Comput. Netw. (MobiCom)*, Aug. 2010, pp. 375–386.

[2] E. Aryafar, M. Khojastepour, K. Sundaresan, S. Rangarajan, and M. Chiang, "MIDU: Enabling MIMO full duplex," in *ACM Int. Conf. Mobile Comput. Netw. (MobiCom)*, 2012, pp. 257–268.

[3] E. Everett, A. Sahai, and A. Sabharwal, "Passive self-interference suppression for full-duplex infrastructure nodes," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 680–694, Feb. 2014.

[4] A. Sahbarwal, P. Schniter, D. Guo, D. W. Bliss, S. Rangarajan, and R. Wichman, "In-band full-duplex wireless: Challenges and opportunities," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 9, pp. 1637–1652, Sept. 2014.

[5] X. Xie and X. Zhang, "Does full-duplex double the capacity of wireless networks?," in *IEEE INFOCOM*, Apr. 2014, pp. 253–261.

[6] C. Suh and D. N. C. Tse, "Feedback capacity of the Gaussian interference channel to within 2 bits," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2667–2685, May 2011.

[7] J. Maraševíc, J. Zhou, H. Krishnaswamy, Y. Zhong, and G. Zussman, "Resource allocation and rate gains in practical full-duplex systems," *IEEE/ACM Trans. Netw.*, vol. 25, no. 1, pp. 292–305, Feb. 2017.

[8] K. Shen and W. Yu, "Interference management in full-duplex wireless cellular networks via fractional programming," in *IEEE Veh. Technol. Conf. (VTC) Spring*, June 2018.

[9] M. A. Khojastepour, K. Sundaresan, and S. Rangarajan, "Scaling wireless full-duplex in multi-cell networks," in *IEEE INFOCOM*, May 2015, pp. 1751–1759.

[10] S. H. Chae, S.-W. Jeon, and S. H. Lim, "Fundamental limits of spectrum sharing full-duplex multicell networks," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 11, pp. 3048–3061, Nov. 2016.

[11] R. Tannious and A. Nosratinia, "Relay channel with private messages," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3777–3785, Oct. 2007.

[12] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Nov. 2008.