Homework #5

1. (*From ALG 8-1*) Let M_n denote the sequence of sample means from an iid random process X_n:

$$M_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

   a. Is M_n a Markov process?

   b. If the answer to part a is yes, find the following state transition pdf:

$$f_{M_n}(x|M_{n-1} = y) \; .$$

2. (*From ALG 8-5,9*) Let X_n be the Bernoulli iid process, let Y_n be given by

$$Y_n = X_n + X_{n-1} \; ,$$

   and define a vector process $\mathbf{Z}_n = (X_n, X_{n-1})$.

   a. Show that Y_n is not a Markov process.

   b. Show that $\mathbf{Z}_n$ is a Markov process.

   c. Find the state transition diagram for $\mathbf{Z}_n$.

   d. Find its one-step transition probability matrix P.

   e. Find P^2 and check your answers by computing the probability of going from state $(0, 1)$ to state $(0, 1)$ in two steps.

   f. Show that $P^n = P^2$ for all $n > 2$. Give an intuitive justification for why this is true for this random process.

   g. Find the steady state probabilities for the process.

3. (*From ALG 8-13*) A machine consists of two parts that fail and are repaired independently. A working part fails during any given day with probability a. A part that is not working is repaired by the next day with probability b. Let X_n be the number of working parts in day n.

   a. Show that X_n is a three-state Markov chain and give its one-step transition probability matrix P.

   b. Show that the steady state distribution is binomial with parameter $b/(a + b)$.

   c. What do you expect is steady state distribution for a machine that consists of n parts?

4. Papoulis $\&$ Pillai 15-1 (also find steady state distribution of the second MC)

5. (*From ALG 8-23*) Consider a random walk in the set $\{0, 1, \ldots, M\}$ with transition probabilities $p_{00} = q$, $p_{MM} = p$, and $p_{i,i-1} = q$, $p_{i,i+1} = p$ for $i = 0, 1, \ldots, M - 1$. Find the long-term proportion of time spent in each state, and the limit of ${p_{ii}}^{(n)}$ as $n \to \infty$.

6. Papoulis $\&$ Pillai 15-8 (Note the typo $p_{ij}p_{ik}p_{ki}$ - should be $p_{ij}p_{jk}p_{ki}$.)

7. Papoulis $\&$ Pillai 15-14 (You do not need to give exact answers. For further interest, program and observe the differences between these two genetic models using Matlab.)

8. Papoulis $\&$ Pillai 15-15. Also reconsider the gambler's ruin problem in Example 3-15 in terms of the absorption probability of a Markov chain.