ECE537H1 F

Random Processes

Fall 2025 Syllabus

Course Meetings

ECE537H1 F

Section	Day & Time	Delivery Mode & Location
LEC0101	Tuesday, 5:00 PM - 6:00 PM Wednesday, 3:00 PM - 5:00 PM	In Person: BA 2165 In Person: ES 4000
TUT0101	Tuesday, 3:00 PM - 5:00 PM	In Person: BA B026

Refer to ACORN for the most up-to-date information about the location of the course meetings.

Tutorials start in the second week.

Course Contacts

Instructor: Ben Liang
Email: liang@ece.utoronto.ca
Office Hours and Location: TBA

Course Overview

Introduction to the principles and properties of random processes, with applications to communications, control systems, and computer science. Random vectors, random convergence, random processes, specifying random processes, Poisson and Gaussian processes, stationarity, mean square derivatives and integrals, ergodicity, power spectrum, linear systems with stochastic input, mean square estimation, Markov chains, recurrence, absorption, limiting and steady-state distributions, time reversibility, and balance equations.

Course Learning Outcomes

- Explain and interpret the fundamental principles of random processes, convergence, and ergodicity.
- Apply the concepts of stationarity, mean-square continuity, derivatives, and integrals to model and evaluate stochastic signals.
- Utilize autocorrelation functions and power spectrum to represent random processes in the frequency domain.
- Analyze the behaviour of linear systems under stochastic inputs, including estimation in the mean-square sense.
- Construct and evaluate Markov chain models, including recurrence, absorption, limiting distributions, and steady-state distributions.

- Formulate and solve balance equations for discrete-time and continuous-time Markov chains, and interpret their implications in engineering applications.

Prerequisites: ECE286H1 and ECE355H1 or ECE302H1
Corequisites: ECE355H1 (can be taken at the same time as ECE537H1)
Exclusions: None
Recommended Preparation: None
Credit Value: 0.5
Graduate Attributes:

- 1B. Knowledge Base for Engineering: Demonstrate competence in natural sciences. [Developed] **Measured in Course**
- 1C. Knowledge Base for Engineering: Demonstrate competence in specialized engineering knowledge appropriate to the program. [Applied] **Measured in Course**
- 2B. Problem Analysis: Demonstrate the ability to formulate a solution plan (methodology) for an engineering problem. [Applied] **Measured in Course**
- 2C. Problem Analysis: Demonstrate the ability to formulate and interpret a model. [Applied] **Measured in Course**
- 2D. Problem Analysis: Demonstrate the ability to execute a solution process for an engineering problem. [Applied] **Measured in Course**

Course Materials

Required textbook:
A. Leon-Garcia, *Probability and Random Processes for Electrical Engineering*, 3rd Edition, Addison Wesley, 2008.

Recommended references:
A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th Edition, McGraw-Hill, 2002.
S. I. Resnick, *Adventures in Stochastic Processes*, Birkhauser, 1992.

Marking Scheme

Undergraduate and graduate students complete the same course work but are evaluated on separate scales.

Assessment	Percent	Details	Due Date
Midterm Exam	40%	Midterm exam will be held at 6 - 8 pm on Thursday, October 23.	2025-10-23
Homework	10%	Homework will be assigned weekly and will be graded solely on the basis of effort, not correctness. Solution to the	No Specific Date

Assessment	Percent	Details	Due Date
		homework problems will be discussed during tutorial.	
Final Exam	50%	The final examination is mandatory for all students. Assessed grades will not be provided to students with approved examination petitions; instead, a deferred examination will be scheduled.	Final Exam Period

Late Assessment Submissions Policy

10% per day of lateness.

Course Schedule

Outline of topics:

Weeks 1 – 2: Probability definition, sigma algebra, probability axioms, combined experiments, random variables, joint moments, conditional expectation

Weeks 3 – 4: Random vectors, independence, transformation, Gaussian random vector, convergence of random sequences, Laws of Large Numbers, Central Limit Theorem

Weeks 5 – 6: Random processes, specifying random processes, discrete-time processes, Poisson process, shot noise, Gaussian process, stationarity, mean square continuity, derivatives and integrals

Weeks 7 – 9: Ergodicity, power spectrum, linear systems with stochastic input, mean square estimation, orthogonality principle, Wiener filtering

Weeks 10 – 11: Discrete-time Markov chains, limiting and steady-state distributions, irreducibility, recurrence, ergodicity, absorption probability, time to absorption

Weeks 11 – 12: Continuous-time Markov chains, steady-state distribution, birth-death processes, Markovian queues, time reversibility, balance equations

Planned Lecture Schedule:

Week of Sep 1	Course Introduction, Subtlety of Probability, Probability Definitions	Probability Space, Probability over Real Line	(review) Conditional Probability, Total Probability, Bayes Rule, Combined Experiments, Bernoulli Trials	
Week of Sep 8	(review) Random Variables, CDF, PDF, PMF, Poisson Theorem	(review) Function of RV, Expected Value, Moments,	(review) Two RVs, Independence, Correlation,	Tut 1

		Characteristic Function, Moment Generating Function, Complex RVs	Orthogonality, Conditional Density, Conditional Expectation	
Week of Sep 15	Random Vectors, Independence, Group/Pairwise/Linear Independence, Correlation Matrix	Transformation of Random Vectors	Gaussian Vectors	Tut 2
Week of Sep 22	Convergence of Random Sequence: Sure, Almost Sure, in Probability	Convergence of Random Sequence: in Distribution, in Mean Square Sense, Cauchy Criterion	Revisiting LLN and CLT	Tut 3
Week of Sep 29	Random Processes, Statistics of Random Processes, Multiple Random Processes	IID Processes, Sum Processes	Poisson Process	Tut 4
Week of Oct 6	Random Telegraph Process, Shot Noise, Gaussian Process, Wiener Process	Stationarity, Wide-Sense Stationarity	Continuity, Derivatives, Integrals	Tut 5
Week of Oct 13	Time Averages, Ergodicity	Power Spectral Density, Wiener-Khinchin Theorem	PSD of D-T Processes, Jointly WSS Processes	Tut 6
Week of Oct 20	LTI Systems with WSS Inputs	Brownian Motion	Midterm Review	Tut 7, Midterm Exam
Week of Oct 27	Fall Study Break			
Week of Nov 3	MS Estimation	Linear MS Estimation, Orthogonality Principle	Wiener Filtering	Tut 8
Week of Nov 10	Markov Chains, Motivating Example	D-T MC, Limiting and Steady-State Distributions	Irreducibility, Periodicity, Recurrence	Tut 9
Week of Nov 17	Null Recurrence, Ergodicity	Absorption Probability, Mean Time to Absorption	C-T MC, State Sojourn Time, Construction from D-T MC	Tut 10
Week of Nov 24	Transition Densities, Kolmogorov Equations, Limiting and Steady State Distributions	Birth-Death Processes, Markovian Queues	Global and Local Balance Equations	Tut 11
Week of Dec 1	No lecture			Tut 12

Policies & Statements

University Land Acknowledgement

I wish to acknowledge this land on which the University of Toronto operates. For thousands of years, it has been the traditional land of the Huron-Wendat, the Seneca, and the Mississaugas of the Credit. Today, this meeting place is still the home to many Indigenous people from across Turtle Island and we are grateful to have the opportunity to work on this land.

Learn more about Canada's relationship with Indigenous Peoples here.

Indigenous Students' Supports

If you are an Indigenous engineering student, you are invited to join a private Discord channel to meet other Indigenous students, professors, and staff, chat about scholarships, awards, work opportunities, Indigenous-related events, and receive mentorship. Email Professor Bazylak if you are interested.

Indigenous students at U of T are also invited to visit Nations House's (FNH) Indigenous Student Services for culturally relevant programs and services. If you want more information on how to apply for Indigenous specific funding opportunities, cultural programs, traditional medicines, academic support, monthly social events or receive the weekly newsletter, go to the FNH website, email or follow FNH on social media: Facebook, Instagram, or TikTok. A full event calendar is on the CLNX platform. Check CLNX often to see what new events are added!

Wellness and Mental Health Support

Your personal wellness and mental health are important. The University of Toronto and the Faculty of Applied Science & Engineering offer a wide range of free and confidential services that can support your well-being.

As a U of T Engineering student, you have a Departmental Undergraduate Advisor or a Departmental Graduate Administrator who can support you by advising on personal matters that impact your academics. Other resources that you may find helpful are listed on the U of T Engineering Mental Health & Wellness webpage, and a small selection are also included here:

- U of T Engineering's Student & Community Wellness Coordinator
- Health & Wellness and the On-Location Engineering Wellness Counsellor
- Health & Wellness Peer Support Program
- Accessibility Services & the On-Location Advisor
- Graduate Engineering Council of Students' Mental Wellness Commission
- SKULE™ Mental Wellness
- U of T Engineering's Learning Strategist and Centre for Learning Strategy Support
- Registrar's Office and Scholarships & Financial Aid Office & Advisor

We encourage you to access these resources as soon as you feel you need support; no issue is too small. You may reach out to the counsellors at U of T Telus Health Student Support for 24/7 free and confidential counselling support.

If you find yourself feeling distressed and in need of more immediate support visit uoft.me/feelingdistressed or U of T Engineering's Urgent Support – Talk to Someone Right Now.

Accommodations

The University of Toronto supports accommodations for students with diverse learning needs, which may be associated with mental health conditions, learning disabilities, autism spectrum, ADHD, mobility impairments, functional/fine motor impairments, concussion or head injury, visual impairments, chronic health conditions, addictions, D/deaf, deafened or hard of hearing, communication disorders and/or temporary disabilities, such as fractures and severe sprains, or recovery from an operation.

If you have a learning need requiring an accommodation the University of Toronto recommends that students register with Accessibility Services as soon as possible.

We know that many students may be hesitant to reach out to Accessibility Services for accommodations. The process of accommodation is private; we will not share details of your needs or condition with any instructor.

If you feel hesitant to register with us, we encourage you to reach out for further information and resources on how we can support. It may feel difficult to ask for help, but it can make all the difference during your time here.

Phone: 416-978-8060
Email: accessibility.services@utoronto.ca

Equity, Diversity and Inclusion

Looking for community? Feeling isolated? Not being understood or heard?

You are not alone. You can talk to anyone in the Faculty that you feel comfortable approaching, anytime – professors, instructors, teaching assistants, first-year or upper years academic advisors, student leaders or the Assistant Dean of Diversity, Inclusion and Professionalism.

You belong here. In this class, the participation and perspectives of everyone is invited and encouraged. The broad range of identities and the intersections of those identities are valued and create an inclusive team environment that will help you achieve academic success. You can read the evidence for this approach here.

You have rights. The University Code of Student Conduct and the Ontario Human Rights Code protect you against all forms of harassment or discrimination, including but not limited to acts of racism, sexism, Islamophobia, antisemitism, homophobia, transphobia, ableism, classism and ageism. Engineering denounces unprofessionalism or intolerance in language, actions or interactions, in person or online, on- or off-campus. Engineering takes these concerns extremely seriously and you can confidentially disclose directly to the Assistant Dean for help here.

Resource List:

- Engineering Equity, Diversity & Inclusion Groups, Initiatives & Student Resources
- Engineering Positive Space Resources
- Request a religious-based accommodation here
- Email Marisa Sterling, P.Eng, the Assistant Dean, Diversity, Inclusion & Professionalism here
- Make a confidential disclosure of harassment, discrimination or unprofessionalism here or email engineering@utoronto.ca or call 416.946.3986
- Email the Engineering Society Equity & Inclusivity Director here
- U of T Equity Offices & First Nations House Resources