Beamforming and Power Control for Wireless Network Virtualization in Uplink MIMO Systems

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***Abstract*—We consider wireless network virtualization (WNV) in an uplink multiple-input multiple-output system, where multiple service providers (SPs) operate in virtually isolated networks managed by an infrastructure provider (InP) that owns the communication equipment. Service isolation is achieved at the physical layer by exploiting a large number of antennas at the base stations. We formulate this WNV as a non-convex optimization problem for the InP, jointly considering the uplink receive beamforming at the BS and the transmit power of the SPs' subscribing user devices. We decompose the problem into two subproblems and derive closed-form solutions to both. We then adopt an alternating optimization approach to combine the closed-form solutions to solve the original problem. Our simulation results show that the proposed method provides strong service isolation among the SPs while retaining efficiency similar to or better than centralized beamforming without virtualization, and it substantially outperforms traditional WNV with strict resource separation.**

I. Introduction

Implementation of new telecom infrastructure by communication service providers (SPs) continues to present significant barriers to market entry due to high initial capital expenses and deployment costs. To address this issue, wireless network virtualization (WNV) has been proposed as a framework for sharing the physical resources in a network among multiple SPs. A typical WNV system consists of the SPs and a separate entity that is called the infrastructure provider (InP). The InP manages the network's physical resources and splits them into virtual slices. These virtual slices are leased upon request to the SPs that, in turn, utilize them to provide services to their subscribing users. An SP demands services from the InP without needing knowledge of the existence of any other SPs. Although multiple SPs share the same infrastructure, none of them is expected to consider inter-SP interference in their design for the demands. Thus, it is the job of the InP to provide *service isolation*, i.e., to satisfy the demand of each SP without affecting the other SPs.

Although virtualization has been well studied for wired networks [1], WNV is more complicated, with the need to share both the hardware and the radio spectrum, and with new challenges arising in guaranteeing service isolation under wireless interference [2]. To achieve service isolation among the SPs in a wireless network, most existing works propose strict separation of the physical resources, an approach rooted in the traditional solution for wired network virtualization. This strict separation could be in the form of dividing the time, frequency spectrum, resource blocks, or the number of antennas among different SPs [3]–[8]. However, this strict separation limits the design space of virtualization since it does not explore the spatial dimension. It has been shown that strict separation can lead to inefficient resource utilization and severe loss of system throughput [9], [10].

The authors in [9] were the first to separate the SPs using multiple-input multiple-output (MIMO) signal processing techniques while they share all physical resources of a base station (BS). They minimized the InP's transmission power while providing a prescribed level of service isolation. The work in [10] also used beamforming to provide service isolation among the users of different SPs, while aiming to minimize the expected deviation between the InP's supply and the SPs' demands. Building on the idea in [10], the authors in [11] formulated an online WNV problem that minimizes the time-averaged expected deviation under long-term and short-term transmit power constraints at the BS. All these papers considered the virtualization of the wireless *downlink*. However, the problem of *uplink* WNV is equally important. The beamforming solution techniques developed for downlink WNV cannot be applied to the uplink. In particular, in uplink WNV, we need to additionally manage the transmit powers of the users of all SPs, to effectively reduce their interference with each other.

In this paper, we focus our attention on uplink WNV. We provide the required virtualization and service isolation by exploiting the spatial structure in MIMO communications. We jointly design the uplink receive beamforming vectors at the BS and the transmit power of user devices, for the InP to supply the signals demanded by the SPs while suppressing the inter-SP interference. The contributions of this paper are summarized below:

- We formulate the above uplink MIMO WNV as a joint beamforming and power control optimization problem, to minimize the deviation between the SPs' demanded received signals and the actual received signals supplied by the InP. Our formulation allows all SPs to simultaneously enjoy the full physical resources available at the InP while providing them with the required service isolation.
- Observing that the formulated joint optimization problem

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is biconvex, we solve it by first decomposing it into two subproblems. We derive closed-form solutions to both subproblems. Then, a solution to the joint optimization problem is obtained via alternating optimization, which guarantees convergence to a partial optimum. The proposed approach applies to both single-cell and multi-cell WNV.
- Our simulation results indicate that the proposed virtualization solution is no less efficient than centralized beamforming without virtualization, while providing strong service isolation among the SPs. It also substantially outperforms the traditional virtualization scheme of strictly separating the frequency bands among the SPs.

The rest of this paper is organized as follows. In Section II, the WNV system model is presented, and the joint beamforming and power control optimization problem is formulated. We then detail our solution in Sections III and IV, for single-cell and multi-cell systems, respectively. The proposed solution is evaluated via simulation and compared with other methods in Section V. Finally, Section VI concludes the paper.

II. System Model and Problem Formulation

We study uplink communication in WNV with M SPs. For clarity of exposition, initially we consider the scenario where they share a single BS with N antennas. In Section IV we will extend our solution to the multi-cell scenario. The BS is governed by an InP that performs the virtualization of the system, i.e., slicing the system into M virtual networks, each for an SP. We assume that all other parts of the network, including the core network and computational resources, are already virtualized and can be utilized by the InP and the SPs.

Without loss of generality, we focus on any one multiple-access channel that is shared by all SPs. Suppose that SP m serves K_m users in this shared channel. Let $K = \sum_{m=1}^{M} K_m$ be the total number of users. Each SP assembles certain demands to be fulfilled by the InP, so that its subscribing users achieve some desired performance, e.g., maximum sum-rate or fairness. The SPs design their demands in ignorance of each other, and thus, it is the responsibility of the InP to supply the requested demands of all SPs while managing the wireless interference among them, i.e., providing service isolation.

More precisely, each SP m requests that the InP uses a set of beamforming vectors $\mathbf{w}_{m,i} \in \mathbb{C}^{N\times 1}$, $\forall i \in \{1, \cdots, K_m\}$ to decode the messages of its users, and that its users transmit their signals with powers $\mathbf{p}_m = [p_{m,1}, p_{m,2}, \cdots, p_{m,K_m}]^T \in \mathbb{R}^{K_m}$. Let $\mathbf{x}_m = [x_{m,1}, x_{m,2}, \cdots, x_{m,K_m}]^T \in \mathbb{C}^{K_m}$ be the transmitted symbol vector of the users of SP m. Without loss of generality, we set $\mathbb{E}\Big\{\mathbf{x}_m\Big\} = \mathbf{0}$ and $\mathbb{E}\Big\{\mathbf{x}_m^H\mathbf{x}_m\Big\} = \mathbf{I}_{K_m}$. The received signal at the BS, from SP m's perspective, is

$$\hat{\mathbf{y}}_m^{\text{desired}} = \mathbf{H}_m \text{diag}(\mathbf{q}_m)\mathbf{x}_m + \mathbf{n}, \tag{1}$$

where $\mathbf{H}_m = [\mathbf{h}_{m,1}, \mathbf{h}_{m,2}, \cdots, \mathbf{h}_{m,K_m}] \in \mathbb{C}^{N\times K_m}$ is the channel matrix from the users of SP m to the different antenna elements at the BS, $\mathbf{q}_m = \left[\sqrt{p_{m,1}}, \cdots, \sqrt{p_{m,K_m}}\right]^T \in \mathbb{R}^{K_m}$ is the amplitude of the transmitted signals by all of those users, and $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I}_N)$ is the additive white Gaussian noise at the antenna's side. Then, by the design of SP m, the decoded received signal vector of all users subscribed to SP m is given by

$$\hat{\mathbf{x}}_m^{\text{desired}} = \mathbf{W}_m\hat{\mathbf{y}}_m^{\text{desired}} = \mathbf{W}_m(\mathbf{H}_m \text{diag}(\mathbf{q}_m)\mathbf{x}_m + \mathbf{n}), \tag{2}$$

where $\mathbf{W}_m = [\mathbf{w}_{m,1}, \mathbf{w}_{m,2}, \cdots, \mathbf{w}_{m,K_m}]^T \in \mathbb{C}^{K_m\times N}$ is the beamforming matrix applied at the BS to decode the messages of the users of SP m. The desired decoded received signals for all users in all SPs in the cell can thus be written as

$$\begin{aligned}\hat{\mathbf{x}}^{\text{desired}} &= \left[\hat{\mathbf{x}}_1^{\text{desired}^T}, \hat{\mathbf{x}}_2^{\text{desired}^T}, \cdots, \hat{\mathbf{x}}_M^{\text{desired}^T}\right]^T \\ &= \mathbf{D}\text{diag}(\mathbf{q})\mathbf{x} + \mathbf{W}\mathbf{n},\end{aligned} \tag{3}$$

where $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_M^T]^T \in \mathbb{C}^K$ is the transmitted symbol vector of all users, $\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T, \cdots, \mathbf{q}_M^T]^T \in \mathbb{R}^K$ denotes their transmitted signal amplitudes, $\mathbf{D}$ is a block diagonal matrix representing the virtualization demand and is given by $\mathbf{D} = \text{blkdiag}\{\mathbf{W}_1\mathbf{H}_1, \mathbf{W}_2\mathbf{H}_2, \cdots, \mathbf{W}_M\mathbf{H}_M\}$, and $\mathbf{W} = \left[\mathbf{W}_1^T, \mathbf{W}_2^T, \cdots, \mathbf{W}_M^T\right]^T$ is a stacking of the beamforming matrices designed by all SPs.

The desired signal in (3), designed by the SPs, is non-realistic and cannot be directly achieved since it does not account for the interference between SPs. This inter-SP interference occurs due to the fact that all SPs use the same time-frequency resources, while they are being oblivious to one another. It is, however, the InP that makes the actual beamforming design and chooses the user transmit powers to achieve its goal of satisfying the demands while providing service isolation among different SPs.

The actual decoded received signal vector of all users in the cell is given by

$$\hat{\mathbf{x}}^{\text{actual}} = \mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}})\mathbf{x} + \mathbf{V}\mathbf{n}, \tag{4}$$

where $\mathbf{V} \in \mathbb{C}^{K\times N}$ is the beamforming matrix, designed and implemented by the InP, to all users and all SPs, $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_M] \in \mathbb{C}^{N\times K}$ is the overall channel matrix from all users to the BS, and $\bar{\mathbf{q}}$ is the signal amplitude vector set by the InP for all users. Note that $\bar{\mathbf{q}}$ in (4) could be the same as $\mathbf{q}$ in (3) but it generally is not, which provides the InP with more degrees of freedom in its design to achieve its goal.

As an inherent characteristic of WNV, the InP aims to supply the demands requested by the different SPs, which may be based on some prior agreements between the InP and the SPs. The demands, as described in (3), are fully characterized by the receive beamforming matrices and user transmit powers, i.e., $\mathbf{W}_m$ and $\mathbf{q}_m$ $\forall m$. Noting that the form in (3) represents perfect isolation between SPs, it is a logical choice for the InP to aim at making $\hat{\mathbf{x}}^{\text{actual}}$ as close to $\hat{\mathbf{x}}^{\text{desired}}$ as possible. In this work we consider the expected l_2-norm deviation between the virtual and actual received signals, which is given by

$$f(\mathbf{V}, \bar{\mathbf{q}}) = \mathbb{E}\Big\{\big\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\big\|_2^2\Big\}, \tag{5}$$

where the expectation is taken over $\mathbf{x}$ and $\mathbf{n}$.

Thus, it is the job of the InP to solve the following optimization problem:

$$\min_{\mathbf{V},\bar{\mathbf{q}}} \mathbb{E}\left\{\left\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\right\|_2^2\right\} \tag{6a}$$

$$\text{s.t. } 0 \preccurlyeq \bar{\mathbf{q}} \preccurlyeq \mathbf{q}_{max}. \tag{6b}$$

As seen above, the InP jointly optimizes the beamforming matrix and the transmit powers to minimize the expected deviation. The power constraint in (6b) gives the InP the permission to use any power value below the users' maximum available power $\mathbf{q}_{max}$. We remark that another practically meaningful variation of this constraint is to prevent the InP from assigning powers that are greater than the requested powers, which can be reflected by replacing constraint (6b) with $0 \preccurlyeq \bar{\mathbf{q}} \preccurlyeq \mathbf{q}$. The solution in Sections III and IV can be easily modified to facilitate this case as well.

III. Proposed Solution for Single-Cell WNV

The first step into tackling problem (6) is to simplify the deviation expression in the objective. We have

$$\begin{aligned} f(\mathbf{V},\bar{\mathbf{q}}) &= \mathbb{E}\left\{\left\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\right\|_2^2\right\} \\ &= \mathbb{E}\left\{\left\|\left(\mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right)\mathbf{x} + (\mathbf{V}-\mathbf{W})\,\mathbf{n}\right\|_2^2\right\} \\ &= \left\|\mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right\|_F^2 + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2, \end{aligned} \tag{7}$$

where the last line is obtained using the properties $\|\mathbf{x}\|_2^2 = \mathbf{x}^H\mathbf{x} = \text{tr}\left(\mathbf{x}^H\mathbf{x}\right)$, $\|\mathbf{A}\|_F^2 = \text{tr}\left(\mathbf{A}\mathbf{A}^H\right)$, and $\mathbb{E}\{\text{tr}(\cdot)\} = \text{tr}(\mathbb{E}\{\cdot\})$. With this, problem (6) can be written as

$$\min_{\mathbf{V},\bar{\mathbf{q}}} \left\|\mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right\|_F^2 + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2 \tag{8a}$$

$$\text{s.t. } 0 \preccurlyeq \bar{\mathbf{q}} \preccurlyeq \mathbf{q}_{max}. \tag{8b}$$

This is a non-convex optimization problem in two sets of decision variables, so it cannot be solved via regular convex optimization techniques. However, a careful look into the problem suggests that this problem is biconvex [12], i.e., it is convex in $\mathbf{V}$ and $\bar{\mathbf{q}}$ but not jointly in both.

Our approach to solving problem (8) is by decomposing it into a beamforming subproblem to optimize $\mathbf{V}$ and a power control subproblem to optimize $\bar{\mathbf{q}}$. In the following subsections, we provide a closed-form solution to each of them. Then, we use an alternating optimization approach to find a partial optimum of the original joint optimization problem.

1) Beamforming subproblem: Here, we treat $\bar{\mathbf{q}}$ in problem (8) as a constant and solve the beamforming subproblem by optimizing $\mathbf{V}$. We use $\bar{\mathbf{Q}} = \text{diag}(\bar{\mathbf{q}})$ and $\mathbf{Q} = \text{diag}(\mathbf{q})$ to simplify the notation. We find the optimal beamforming matrix $\mathbf{V}^\star$ by solving the beamforming subproblem given by

$$\min_{\mathbf{V}} \left\|\mathbf{V}\mathbf{H}\bar{\mathbf{Q}} - \mathbf{D}\mathbf{Q}\right\|_F^2 + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2. \tag{9}$$

This is an unconstrained convex program with a single minimum that can be located by finding the matrix $\mathbf{V}$ that makes the gradient of the objective function vanish.

We start by writing the Frobenius norms in the objective as traces to make the objective easier to differentiate:

$$\begin{aligned} f(\mathbf{V},\bar{\mathbf{q}}) = &\,\text{tr}\left(\mathbf{V}\mathbf{H}\bar{\mathbf{Q}}\bar{\mathbf{Q}}^H\mathbf{H}^H\mathbf{V}^H\right) - \text{tr}\left(\mathbf{D}\mathbf{Q}\bar{\mathbf{Q}}^H\mathbf{H}^H\mathbf{V}^H\right) \\ &- \text{tr}\left(\mathbf{V}\mathbf{H}\bar{\mathbf{Q}}\mathbf{Q}^H\mathbf{D}^H\right) + \text{tr}\left(\mathbf{D}\mathbf{Q}\mathbf{Q}^H\mathbf{D}^H\right) \\ &+ \sigma_n^2\,\text{tr}\left(\mathbf{V}\mathbf{V}^H\right) - \sigma_n^2\,\text{tr}\left(\mathbf{W}\mathbf{V}^H\right) - \sigma_n^2\,\text{tr}\left(\mathbf{V}\mathbf{W}^H\right) \\ &+ \sigma_n^2\,\text{tr}\left(\mathbf{W}\mathbf{W}^H\right). \end{aligned}$$

Let $\mathbf{V}^\dagger$ be the complex conjugate of V. Differentiating $f(\mathbf{V},\bar{\mathbf{q}})$ with respect to $\mathbf{V}^\dagger$ gives

$$\frac{\partial f(\mathbf{V},\bar{\mathbf{q}})}{\partial \mathbf{V}^\dagger} = \mathbf{V}\mathbf{H}\bar{\mathbf{Q}}\bar{\mathbf{Q}}^H\mathbf{H}^H - \mathbf{D}\mathbf{Q}\bar{\mathbf{Q}}^H\mathbf{H}^H + \sigma_n^2\mathbf{V} - \sigma_n^2\mathbf{W}$$

where we have used trace differentiation rules [13]. Since the matrix $\left(\mathbf{H}\bar{\mathbf{Q}}\bar{\mathbf{Q}}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)$ is positive definite, we solve $\frac{\partial f(\mathbf{V},\bar{\mathbf{q}})}{\partial \mathbf{V}^\dagger} = \mathbf{0}$ and obtain the following closed-form expression for $\mathbf{V}^\star$:

$$\mathbf{V}^\star = \left(\mathbf{D}\mathbf{Q}\bar{\mathbf{Q}}^H\mathbf{H}^H + \sigma_n^2\mathbf{W}\right)\left(\mathbf{H}\bar{\mathbf{Q}}\bar{\mathbf{Q}}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}. \tag{10}$$

2) Power control subproblem: In this part, we treat $\mathbf{V}$ in problem (8) as a constant and solve the power control subproblem to find the optimal $\bar{\mathbf{q}}^\star$. This subproblem is given by

$$\min_{\bar{\mathbf{q}}} \left\|\mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right\|_F^2 + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2 \tag{11a}$$

$$\text{s.t. } 0 \preccurlyeq \bar{\mathbf{q}} \preccurlyeq \mathbf{q}_{max}. \tag{11b}$$

This is a constrained convex optimization problem. Strong duality holds in this problem since Slater's condition is trivially satisfied.

We solve this problem by studying the KKT conditions [14]. To do so, we start by letting $\mathbf{A} = \mathbf{V}\mathbf{H}$. Now, the objective can be written as $f(\mathbf{V},\bar{\mathbf{q}}) = \left\|\mathbf{A}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right\|_F^2 + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2$. The Lagrangian of this problem is

$$\begin{aligned} \mathcal{L}(\bar{\mathbf{q}},\boldsymbol{\lambda},\boldsymbol{\mu}) &= \left\|\mathbf{A}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right\|_F^2 + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2 \\ &\qquad + \boldsymbol{\lambda}^T(\bar{\mathbf{q}} - \mathbf{q}_{max}) - \boldsymbol{\mu}^T\bar{\mathbf{q}} \\ &= \sum_{j=1}^{K}\bar{q}_j^2\mathbf{a}_j^H\mathbf{a}_j - \sum_{j=1}^{K} q_j\bar{q}_j\mathbf{d}_j^H\mathbf{a}_j - \sum_{j=1}^{K} q_j\bar{q}_j\mathbf{a}_j^H\mathbf{d}_j \\ &\qquad + C + \boldsymbol{\lambda}^T(\bar{\mathbf{q}} - \mathbf{q}_{max}) - \boldsymbol{\mu}^T\bar{\mathbf{q}}, \end{aligned} \tag{12}$$

where $\mathbf{a}_j$ and $\mathbf{d}_j$ are respectively the j^{th} columns of $\mathbf{A}$ and $\mathbf{D}$, $C = \sum_{j=1}^{K} q_j^2\mathbf{d}_j^H\mathbf{d}_j + \sigma_n^2\left\|\mathbf{V}-\mathbf{W}\right\|_F^2$ is the sum of the terms that are independent of $\bar{\mathbf{q}}$, and $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the Lagrange multipliers associated with the constraints in (11b). Differentiating $\mathcal{L}(\bar{\mathbf{q}},\boldsymbol{\lambda},\boldsymbol{\mu})$ with respect to every $\bar{q}_j$ and setting it to zero gives the following stationarity condition:

$$2\bar{q}_j^\star\left\|\mathbf{a}_j\right\|_2^2 = 2\Re\left(q_j\mathbf{d}_j^H\mathbf{a}_j\right) - (\lambda_j^\star - \mu_j^\star) \quad \forall j. \tag{13}$$

The other KKT conditions for $(\bar{\mathbf{q}}^\star, \boldsymbol{\lambda}^\star, \boldsymbol{\mu}^\star)$ to form a globally optimal solution of problem (11) are

$$\bar{\mathbf{q}}^\star \preccurlyeq \mathbf{q}_{max}, \tag{14}$$

$$\bar{\mathbf{q}}^\star \succcurlyeq 0, \tag{15}$$

$$\boldsymbol{\lambda}^\star \succcurlyeq 0, \tag{16}$$

$$\boldsymbol{\mu}^\star \succcurlyeq 0, \tag{17}$$

$$\lambda_j^\star(\bar{q}_j^\star - q_{j_{max}}) = 0, \quad \forall j, \tag{18}$$

$$\mu_j^\star \bar{q}_j^\star = 0, \quad \forall j, \tag{19}$$

where equations (14)-(15) and (16)-(17) are, respectively, the conditions for primal and dual feasibility, and (18)-(19) are the complementary slackness conditions.

To solve for $\bar{\mathbf{q}}$, we look into the different possibilities of q_j, λ_j, and μ_j that satisfy the KKT conditions.

1) $\lambda_j^\star > 0$: From (18), $\bar{q}_j^\star = q_{j_{max}}$, and from (19), $\mu_j^\star = 0$. By (13) this implies that $\Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big) > q_{j_{max}} \|\mathbf{a}_j\|_2^2$.
2) $\lambda_j^\star = 0$: We have two cases for $\mu_j^\star$:
   i) $\mu_j^\star = 0$: By (13), $\bar{q}_j^\star$ is given by

$$\bar{q}_j^\star = \frac{1}{\|\mathbf{a}_j\|_2^2}\Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big)$$

   and from (14) and (15) we have $0 \leq \Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big) \leq q_{j_{max}} \|\mathbf{a}_j\|_2^2$.
   ii) $\mu_j^\star > 0$: From (19), $\bar{q}_j^\star = 0$. Then (13) implies that $\Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big) < 0$.

By combining these different cases, we have the following closed-form expression for the optimal $\bar{q}_j^\star$:

$$\bar{q}_j^\star = \begin{cases} \min\left\{\frac{1}{\|\mathbf{a}_j\|_2^2}\Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big), q_{j_{max}}\right\}, & \Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big) \geq 0, \\ 0, & \Re\big(q_j \mathbf{d}_j^H \mathbf{a}_j\big) < 0. \end{cases} \tag{20}$$

3) Solution to the joint optimization problem: Now that we have optimally solved problem (9) and problem (11), we make use of their closed-form solutions in (10) and (20), respectively. We employ an alternating optimization approach to find a solution to problem (8). The detailed steps are provided in Algorithm 1. Since our problem is biconvex, and we have found optimal solutions to the subproblems, each iteration of the algorithm results in a lower value of the objective function. Therefore, convergence is guaranteed by the monotone convergence theorem. Furthermore, at convergence, since $f(\mathbf{V}^\star, \bar{\mathbf{q}}^\star) \leq f(\mathbf{V}, \bar{\mathbf{q}}^\star)$ and $f(\mathbf{V}^\star, \bar{\mathbf{q}}^\star) \leq f(\mathbf{V}^\star, \bar{\mathbf{q}})$ for all feasible $\mathbf{V}$ and $\bar{\mathbf{q}}$, $(\mathbf{V}^\star, \bar{\mathbf{q}}^\star)$ is a partial optimum [12].

IV. Extension to Multi-Cell WNV

The proposed solution can be easily extended to multiple cells. Again we focus only on any one multiple-access channel that is shared by all SPs. We consider C cells where this channel is in use. Each cell has one BS, and the InP has control over all BSs. We further assume that BS c has N_c antennas and there are $K = \sum_{c=1}^{C} K_c = \sum_{c=1}^{C}\sum_{m=1}^{M} K_{c,m}$ users in the system, where K_c is the number of users in cell c, and $K_{c,m}$ of them are subscribing to SP m. We further assume

Algorithm 1 Proposed solution to (6)

Output: $\mathbf{V}^\star$, $\bar{\mathbf{q}}^\star$
Initialize: $\bar{\mathbf{q}}^{(0)}$, $i \leftarrow 0$
1: Compute $\mathbf{V}^{\star(0)}$ from $\bar{\mathbf{q}}^{(0)}$ using (10)
2: Compute $f^{(0)} = f(\mathbf{V}^{\star(0)}, \bar{\mathbf{q}}^{(0)})$ using (7)
3: **repeat**
4: $\quad i \leftarrow i+1$
5: $\quad$ Compute $\bar{\mathbf{q}}^{\star(i)}$ from $\mathbf{V}^{\star(i-1)}$ using (20)
6: $\quad$ Compute $\mathbf{V}^{\star(i)}$ from $\bar{\mathbf{q}}^{\star(i)}$ using (10)
7: $\quad$ Compute $f^{(i)} = f(\mathbf{V}^{\star(i)}, \bar{\mathbf{q}}^{\star(i)})$ using (7)
8: **until** convergence of $f^{(i)}$
9: Set $\mathbf{V}^\star \leftarrow \mathbf{V}^{\star(i)}$, $\bar{\mathbf{q}}^\star \leftarrow \bar{\mathbf{q}}^{\star(i)}$

that the SPs design their demands in a distributed fashion, i.e., each SP designs its demands in cell c without considering interference coming from its own users in other cells. This inter-cell interference is left for the InP to handle. Furthermore, from the prospective of the InP, we assume that the BSs operate in a fully cooperative manner.

The desired decoded received signal by SP m in cell c is

$$\hat{\mathbf{x}}_{c,m}^{\text{desired}} = \mathbf{W}_{c,m}\mathbf{H}_{cc,m}\text{diag}(\mathbf{q}_{c,m})\mathbf{x}_{c,m} + \mathbf{W}_{c,m}\mathbf{n}_c,$$

where $\mathbf{W}_{c,m}$ is the desired beamforming matrix designed by SP m to be used at BS c, $\mathbf{q}_{c,m}$ is the signal amplitude vector set by SP m for its users in cell c, $\mathbf{x}_{c,m}$ is the unit-power transmitted symbol vector of the users of SP m in cell c, $\mathbf{n}_c$ is the additive noise at the antennas of BS c, and $\mathbf{H}_{cc,m}$ is the channel matrix between users of SP m that are located in cell c to the BS in cell c. The desired decoded received signal vector from all SPs in cell c is given by

$$\hat{\mathbf{x}}_{c}^{\text{desired}} = \mathbf{D}_c\text{diag}(\mathbf{q}_c)\mathbf{x}_c + \mathbf{W}_c\mathbf{n}_c,$$

where $\mathbf{D}_c = \text{blkdiag}\{\mathbf{W}_{c,1}\mathbf{H}_{cc,1}, \cdots, \mathbf{W}_{c,M}\mathbf{H}_{cc,M}\}$, $\mathbf{q}_c = [\mathbf{q}_{c,1}^T, \cdots, \mathbf{q}_{c,M}^T]^T$, $\mathbf{x}_c = [\mathbf{x}_{c,1}^T, \cdots, \mathbf{x}_{c,M}^T]^T$, and $\mathbf{W}_c = [\mathbf{W}_{c,1}^T, \cdots, \mathbf{W}_{c,M}^T]^T$. By stacking them up, we can write the desired decoded received signal from all users in the system as

$$\hat{\mathbf{x}}^{\text{desired}} = \mathbf{D}\text{diag}(\mathbf{q})\mathbf{x} + \mathbf{W}\mathbf{n},$$

where $\mathbf{D} = \text{blkdiag}\{\mathbf{D}_1, \cdots, \mathbf{D}_C\}$, $\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T, \cdots, \mathbf{q}_C^T]^T$, $\mathbf{x} = [\mathbf{x}_1^T, \cdots, \mathbf{x}_C^T]^T$, $\mathbf{W} = \text{blkdiag}\{\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_C\}$, and $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_C^T]^T$.

The actual decoded received signal vector of all users in the system is given by

$$\hat{\mathbf{x}}^{\text{actual}} = \mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}})\mathbf{x} + \mathbf{V}\mathbf{n},$$

where $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \cdots, \mathbf{H}_C^T]^T \in \mathbb{C}^{\sum_c N_c \times K}$ is the overall channel matrix from all users to all BSs, $\mathbf{H}_c = [\mathbf{H}_{1c}, \mathbf{H}_{2c}, \cdots, \mathbf{H}_{Cc}] \in \mathbb{C}^{N_c \times K}$, and $\mathbf{H}_{lc}$ is the channel matrix between the users in cell l and the BS in cell c.

The InP achieves virtualization by minimizing the expected deviation, given by

$$\mathbb{E}\Big\{\left\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\right\|_2^2\Big\} = \left\|\mathbf{V}\mathbf{H}\text{diag}(\bar{\mathbf{q}}) - \mathbf{D}\text{diag}(\mathbf{q})\right\|_F^2 + \sigma_n^2\left\|\mathbf{V} - \mathbf{W}\right\|_F^2.$$

This is identical in structure to the objective in problem (8) and thus can be solved using the same approach.

V. Simulation Results

We conduct simulation in Matlab to study the performance of the proposed WNV method. For the single-cell scenario, we consider a circular coverage area of radius 500 m with a BS at its center, and for the multi-cell scenario, we consider a square area of 2 km in width, covered by 4 identical square cells each with a BS at the center. Unless otherwise specified, we set the number of SPs to $M = 4$ as default. Each SP m has $K_m = \frac{K}{M}$ users, and in the multi-cell scenario they are evenly split among all cells, i.e., $K_{c,m} = \frac{K}{4M}$. The users in each cell are uniformly distributed in space. We model the channel from user k to each BS as a Rayleigh fading channel given by $\mathbf{h}_k = \beta_k^{1/2}\mathbf{g}_k$. Here, β_k is the large-scale fading coefficient that captures both pathloss and shadowing and is given as $10\log_{10}\beta_k = -31.54 - 33\log_{10}(d_k) + Z_k$, where d_k is the Euclidean distance from user k to the BS, and $Z_k \sim \mathcal{CN}(0, \sigma_z^2)$ is the shadowing at that user with $\sigma_z = 8$ dB; and $\mathbf{g}_k \sim \mathcal{CN}(0, \mathbf{I})$ denotes the small-scale fading. The users utilize a bandwidth of $B = 1$ MHz for transmission with a power budget of $p_{max} = q_{max}^2 = 27$ dBm for each user. We set the noise power spectral density to $N_0 = -174$ dBm/Hz, and the noise figure to $N_F = 2$ dB.

As an example of the virtualization requirements, we assume that the SPs set their demands with zero-forcing (ZF) beamforming and full power transmission. In single-cell WNV, SP m's demand is given by the beamforming matrix $\mathbf{W}_m = (\mathbf{H}_m^H\mathbf{H}_m)^{-1}\mathbf{H}_m^H$ and the signal amplitude vector $\mathbf{q}_m = \mathbf{q}_{max}$. This choice of power is known to maximize the sum-rate when ZF beamforming is used [15]. In multi-cell WNV, the SPs' demands are determined in the same way within each cell. With this demand, a main performance metric is the average per-user rate normalized by the system bandwidth, which is given by $R = \frac{1}{B}\frac{1}{K}\sum_{k=1}^{K} B_k \log_2(1 + \mathrm{SINR}_k)$, where B_k is the bandwidth for user k, and SINR_k is the SINR of user k given by $\mathrm{SINR}_k = \frac{|\mathbf{v}_k\mathbf{h}_k|^2 q_k^2}{\sum_{j\in\mathcal{B}_k}|\mathbf{v}_k\mathbf{h}_j|^2 q_j^2 + \sigma_n^2\|\mathbf{v}_k\|_2^2}$, where $\mathbf{v}_k$ is the receive beamforming vector for user k, and $\mathcal{B}_k$ is the set of users that share bandwidth with user k other than k itself. We initialize the power values in Algorithm 1 with full power, i.e., $\bar{\mathbf{q}}^{(0)} = \mathbf{q}_{max}$.

We compare the performance of our WNV approach with two other methods. 1) A fully centralized non-virtualized approach, referred to as "Non-virtualized", where the InP uses full channel bandwidth to simultaneously serve all users with ZF beamforming and full power. 2) An alternative WNV method, referred to as "FD-WNV", where service isolation is performed by allocating different frequency bands to different SPs and dividing the bandwidth B equally among them. In FD-WNV, each SP uses ZF beamforming and maximum user transmit power, with fully cooperative BSs in the multi-cell case. In the following results, our WNV method is referred to as "Proposed".

Fig. 1 presents the normalized deviation between the InP's supply and the SPs' demands in the proposed method, defined as $\mathbb{E}\{\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\|_2^2\}/\mathbb{E}\{\|\hat{\mathbf{x}}^{\text{desired}}\|_2^2\}$. This figure gives an indication on how well the proposed approach fulfills its main goal, i.e., service isolation. We observe that with a practical number of antennas, the proposed method can keep the deviation small. Recall that the SPs' demands correspond to an idealized setting where there is no inter-SP interference, as if each SP owned a separate copy of the network infrastructure. This suggests that, through proper beamforming design and user power control, there is an opportunity to substantially increase system efficiency by limiting the deviation from the SPs' demands. This observation is further confirmed in terms of the average per-user rate in the results below.

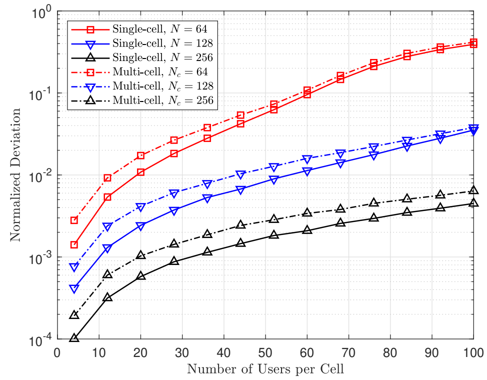

Fig. 1. Normalized deviation for various numbers of users and antennas.

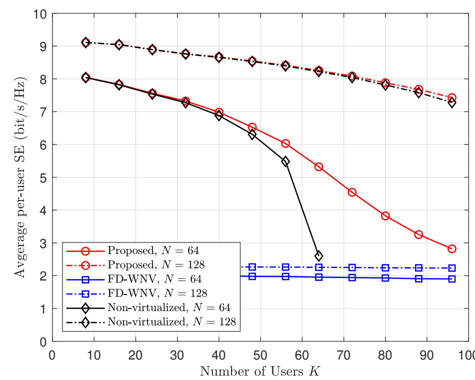

Fig. 2. Average per-user rate vs. K and N in single-cell case.

Fig. 2 shows the average per-user rate of all three approaches in the single-cell case versus the number of users K for various numbers of antennas N. As expected, we see a monotonically decreasing per-user rate in all systems. This figure shows a clear gap between the performance of FD-WNV and our proposed approach. Although the bandwidth separation in FD-WNV guarantees no inter-SP interference, the smaller bandwidth allocated to each SP causes a huge drop in the users' rates. Furthermore, the proposed method substantially outperforms even the non-virtualized method over a wide range of K values. This is clear in Fig. 2 when $N = 64$ and $K \in [40, 64]$. Note that, unlike the non-virtualized method, our method performs

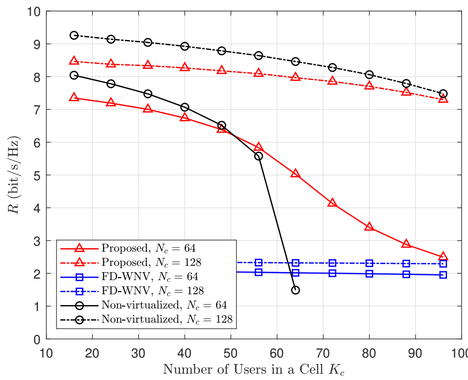

Fig. 3. Average per-user rate vs. K_c and N_c in multi-cell case.

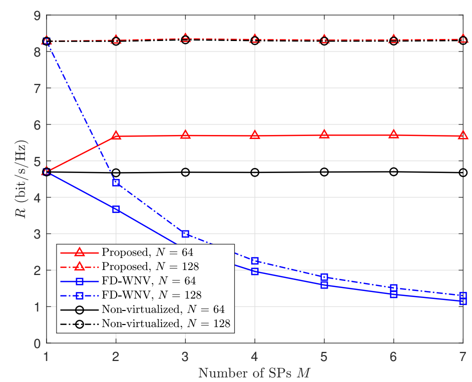

Fig. 4. Average per-user rate vs. M and N in single-cell case.

virtualization. It achieves average rates that are at least as high as those achieved by the non-virtualized method, even when $K \ll N$. Notice that the non-virtualized method is not defined when $K > N$ because of rank-deficient matrix inversion in the formula for ZF. The proposed method, however, does not have such limitation.

Fig. 3 shows the average per-user rate for the multi-cell case. Similar to Fig. 2, it is again observed that the proposed method substantially outperforms FD-WNV. Furthermore, for $K_c \ll N_c$, the proposed method achieves approximately 90% of the average rate of the non-virtualized method, while again it can substantially outperform the non-virtualized as K_c increases to near or above N_c.

In Fig. 4, we show the average per-user rate in the single-cell case as the number of SPs M varies from 1 to 7, while the total number of users remains at $K = 60$. This figure illustrates that the performance of the proposed approach does not deteriorate as more SPs subscribe to the InP's services. In contrast, we observe a drastic drop in the performance of FD-WNV. This is due to its strict separation of frequency bands between different SPs, which is a highly inefficient means to achieve service isolation. Similar observations are made in the multi-cell case, which are omitted due to the page limitation.

VI. Conclusions

This paper provides a new look into uplink wireless network virtualization that is not inherited from wired network virtualization. We propose to exploit the MIMO structure and provide service isolation by spatially separating SPs using beamforming. We have formulated a joint receive beamforming and power control optimization problem to minimize the expected deviation between the virtual demand and the actual supply. Closed-form solutions are derived for each set of optimization variables, and a solution to the joint optimization problem is obtained using alternating optimization. In our examples with 4 SPs and typical numbers of antennas and users, simulation results confirm that the proposed virtualization method provides 3 to 4 times the user data rate compared with traditional virtualization approaches that depend on strict resource separation among the SPs, and even higher performance gains for larger numbers of SPs. Furthermore, the proposed method can achieve data rates similar to or higher than centralized beamforming that does not provide virtualization.

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