On the Insensitivity of User Distribution in Multicell Networks under General Mobility and Session Patterns

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***Abstract*—The location of active users is an important factor in the performance analysis of mobile multicell networks, but it is difficult to quantify due to the wide variety of user mobility and session patterns. In particular, the channel holding times in each cell may be arbitrarily distributed and dependent on those in other cells. In this work, we study the stationary distribution of users by modeling the system as a multi-route queueing network with Poisson inputs. We consider arbitrary routing and arbitrary joint probability distributions for the channel holding times in each route. Using a decomposition-composition approach, we show that the user distribution (1) is insensitive to the user movement patterns, (2) is insensitive to general and dependent distributed channel holding times, (3) depends only on the average arrival rate and average channel holding time at each cell, and (4) is completely characterized by an open network with $M/M/\infty$ queues. This result is validated by experiments with the Dartmouth user mobility traces.**

I. Introduction

In designing ever more efficient and capable mobile access networks, the accurate modeling of how user mobility and session connectivity patterns affect network performance is of paramount interest. However, compared with wired networks, the analytical modeling of mobile networks is burdened with many additional technical challenges. Some of the most difficult factors are the following: (1) the movement of users may be individually arbitrary, without following any common mobility pattern [2]; (2) the session durations may have a general probability distribution, supporting diverse data and multimedia applications [3]; (3) the channel holding times at different cells are correlated, dependent on the speed or trajectory of different users [4].

In this paper, we study the distribution of active users in a multicell network, which has important utilization in network management and planning. Prior studies have led to several analytical models to estimate the user distribution with various degrees of detail and generality to facilitate tractable analysis [5]–[8]. In terms of user movement, [5], [6], and [7] assume that users move from one cell to another probabilistically and memorylessly, while [8] focuses on scattered single cells, so that user movement among multiple cells is not discussed. None of them consider the arbitrary user movement patterns. In terms of channel holding times, [5] uses the sum of hyper-exponentials or the Coxian distribution to approximate arbitrary distributions; [8] assumes generally distributed channel holding times but concerns only a single cell; and [6] and [7] consider generally but independently distributed channel holding times in different cells. None of the above works consider the dependence among channel holding times.

Different with these works, we consider general mobility and session patterns, only requiring that the new session arrivals form a Poisson process, which is well supported by experimental data [7], [9], [10]. We model the user mobility with a general system with multiple routes, each representing one type of users with a specific movement pattern. A general probability distribution is used to represent the session durations. As a consequence, the channel holding times at different cell sites are no longer independent.

Through a decomposition-composition approach, we derive a closed-form expression for the joint stationary distribution for the number of users in all cells. We observe five important conclusions on user distribution: *first*, it is insensitive to how the users move through the system; *second*, it is insensitive to the general distribution of channel holding times; *third*, it is intensive to the correlation among the channel holding times; *fourth*, it depends only on the average arrival rate and average channel holding time at each cell; and *fifth*, it is perfectly captured by an open Jackson network with $M/M/\infty$ queues.

Note that the authors of [7] have also observed a surprising match between analysis and real-life user mobility traces from the Dartmouth study [11], even though their analysis assumes simple $M/G/\infty$ mobility and session models without considering arbitrary user movement patterns or dependent channel holding times. No analytical explanation is given in [7] for this observation. In contrast, our work provides theoretical support for it, since we show that the user distribution is also insensitive to arbitrary user movement patterns and dependent channel holding times.

II. System Model

Consider a cellular network with C cells. There are L independent *routes*, each defined as a finite ordered sequence of cells. The jth stage on the lth route corresponds to the jth cell in the sequence, which is denoted as $c(l, j)$. Let N_l be the number of stages on the lth route. Each user of the lth route starts a new session in cell $c(l, 1)$; then it moves along the route through cells $c(l, 1), c(l, 2) \dots c(l, N_l)$, as long as the session remains active. The user is considered to have departed the network when its session terminates or when it exits cell $c(l, N_l)$. We allow an arbitrary number of arbitrary routes to cover all possible movement patterns.

For each route, we assume the arrivals of *new* sessions to form a Poisson process. Let λ_{l0} be the new session arrival rate at the lth route. The session duration of a user on the lth route is modeled as an arbitrarily distributed random variable T_l. After a new session arrival, let τ_{l1} denote the residual

This work has been supported in part by grants from Bell Canada and the Natural Sciences and Engineering Research Council (NSERC) of Canada. An extended technical report is available at [1].

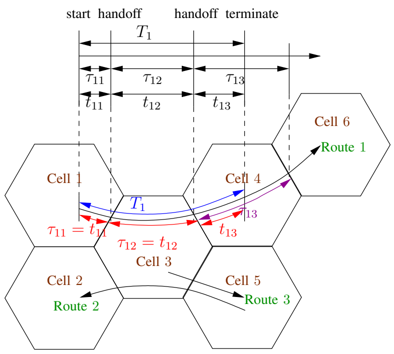


Fig. 1. System model.

cell dwell time of the user in the 1st stage on the lth route, which is arbitrarily distributed. Let τ_{lj}, $2 \leq j \leq N_l$, denote the cell dwell time of the user in the jth stage on the lth route, which are also arbitrarily distributed. Then, the channel holding time of the jth stage on the lth route, t_{lj}, if it exists, can be represented as follows:

$$\begin{cases} t_{l1} = \min\{T_l, \tau_{l1}\}, \\ t_{lj} = \min\{T_l - \sum_{i=1}^{j-1} \tau_{li}, \tau_{lj}\}, \text{ if } T_l > \sum_{i=1}^{j-1} \tau_{li}, \quad 2 \leq j \leq N_l. \end{cases}$$

Fig. 1 shows an example network with 3 routes. Route 1 starts from cell 1 and passes cell 3, 4 and 6 (i.e., $c(1,1) = 1$, $c(1,2) = 3$, $c(1,3) = 4$ and $c(1,4) = 6$). A user starts a session in cell 1, and the session is terminated in cell 4. The corresponding $T_1, \tau_{11}, \tau_{12}, \tau_{13}, t_{11}, t_{12}$, and t_{13} are labeled in the figure.

Let x_{lj}, $1 \leq l \leq L$, $1 \leq j \leq N_l$, denote the number of active users in the jth stage on the lth route; let y_n, $1 \leq n \leq C$, denote the number of active users in the nth cell. Let $\mathbf{x} = [\{x_{lj} : 1 \leq l \leq L, 1 \leq j \leq N_l\}]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_C]^T$. We aim to derive $\pi(\mathbf{x})$ and $\pi_1(\mathbf{y})$, the joint stationary distributions for $\mathbf{x}$ and $\mathbf{y}$, respectively.

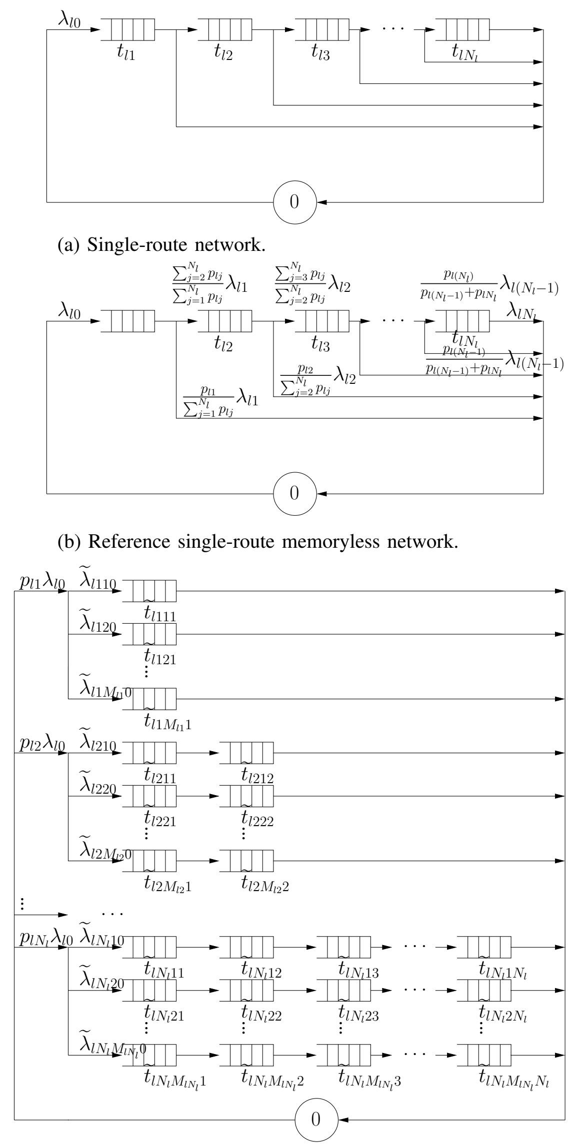
III. User Distribution in Single-Route Network

We first derive the stationary user distribution on a single route. We construct a reference single-route memoryless network, where all the channel holding times are independently and exponentially distributed. We prove insensitivity by showing an equivalence between the original network and the memoryless network in terms of user distribution.

A. Queueing Network Model for Single-Route Network

Consider exclusively the lth route in the network. Throughout this section, we will carry the route index l in most symbols, since they will be re-used in the analysis of multiple-route networks.

As shown in Fig. 2(a), we model the route as a tandem-liked queueing network, except with early exists. The node labeled with 0 represents the exogenous world. The jth queue, $1 \leq j \leq N_l$, represents the jth stage of the route, and units in this queue represent sessions in the jth stage. Each queue has infinite servers, since the sessions are served in parallel with no waiting. The channel holding time of a session in the jth



(a) Single-route network.

(b) Reference single-route memoryless network.

(c) Decoupled network.

Fig. 2. Single-route network and decomposition.

stage, t_{lj}, is equivalent to the service time of the jth queue. The handoff of a session from the jth stage to the $(j+1)$th stage is equivalent to a unit movement from the jth queue to the $(j+1)$th queue. The termination of a session is equivalent to the movement from a queue to node 0.

Let p_{lk} denote the probability that a session lasts for k stages. It is given by

$$p_{lk} = P\Big[\sum_{j=1}^{k-1} \tau_{lj} < T_l \leq \sum_{j=1}^{k} \tau_{lj}\Big], \text{ for } 2 \leq k \leq N_l - 1,$$

with $p_{l1} = P[T_l \leq \tau_{l1}]$ and $p_{lN_l} = P\Big[\sum_{j=1}^{N_l-1} \tau_{lj} < T_l\Big]$. Note that we have $\sum_{k=1}^{N_l} p_{lk} = 1$. Given a session in the kth stage, it enters the $(k+1)$th stage with probability $\frac{\sum_{j=k+1}^{N_l} p_{lj}}{\sum_{j=k}^{N_l} p_{lj}}$ and terminates with probability $\frac{p_{lk}}{\sum_{j=k}^{N_l} p_{lj}}$.

B. Reference Single-Route Memoryless Network

We define a reference *single-route memoryless network*, as a Jackson network with the same topology as the original single-route network, where each queue has infinitely many independent and exponential servers. An illustration is shown in Fig. 2(b). Let $\bar{t}_{lj}$ denote the average value of t_{lj}, given that the number of stages is larger than j on the lth route. By matching the mean service times in this memoryless network with those of the original network, we see that its external arrival rate is λ_{l0}, the service rate of the jth queue is $\lambda_{lj} = \frac{1}{\bar{t}_{lj}}$, the service rate from the kth queue to the $(k+1)$th queue is $\frac{\sum_{j=k+1}^{N_l} p_{lj}}{\sum_{j=k}^{N_l} p_{lj}}\lambda_{lk}$, and the service rate from the kth queue to node 0 is $\frac{p_{lk}}{\sum_{j=k}^{N_l} p_{lj}}\lambda_{lk}$.

Let w'_{lj} be the positive invariant measure of the jth queue that satisfies the routing balance equations of the single-route memoryless network, with the convention that at node 0, $w'_0 = 1$. It can be derived from the topology of Fig. 2(b) that

$$\begin{cases} w'_{l1} = \lambda_{l0}, \\ w'_{lj} = \lambda_{l0}(1 - \sum_{n=1}^{j-1} p_{ln}), \quad 2 \leq j \leq N_l. \end{cases} \tag{1}$$

Let $w_{lj} = \frac{w'_{lj}}{\lambda_{lj}}$. Then the stationary distribution of this network is [12]

$$\pi_0(\mathbf{x}) = \prod_{j=1}^{N_l} e^{-w_{lj}} w_{lj}^{x_{lj}} \frac{1}{x_{lj}!}. \tag{2}$$

C. Insensitivity of Single-Route Network

For the original single route network, we employ a decomposition-composition approach to derive its stationary user distribution.

Given that one session lasts for k stages, we denote the channel holding times as a k-dimensional random vector $\widehat{\mathbf{t}}_{\mathbf{lk}} = \{\widehat{t}_{lk1}, \dots \widehat{t}_{lkj}, \dots, \widehat{t}_{lkk}\}$, where $\widehat{t}_{lkj}$ is the channel holding time at the jth stage. We assume that $\widehat{\mathbf{t}}_{\mathbf{lk}}$ is an arbitrarily distributed discrete random vector with M_{lk} possible realizations[1]. For any i, $1 \leq i \leq M_{lk}$, we define a k-dimensional deterministic vector $\widetilde{\mathbf{t}}_{\mathbf{lki}} = [\widetilde{t}_{lki1}, \dots, \widetilde{t}_{lkij}, \dots, \widetilde{t}_{lkik}]^T$ corresponding to the ith realization of $\widehat{\mathbf{t}}_{\mathbf{lk}}$. Let q_{lki} be the probability of the ith realization given that the session lasts for k stages. Also, let $P_{lki} = p_{lk}q_{lki}$ denote the probability that a session lasts for k stages and it is in the ith realization.

By do so, we decompose the original network into a multiple-branch queuing network as shown in Fig. 2(c), which is referred to as the *decoupled network*. In this network, there are N_l main branches, where the kth main branch represents the event that a session lasts for k stages. The kth main branch contains M_{lk} sub-branches, where the ith sub-branch represents the realization where $\widehat{\mathbf{t}}_{\mathbf{lk}} = \widetilde{\mathbf{t}}_{\mathbf{lki}}$. Furthermore, the jth queue in the ith sub-branch of the kth main branch represents the jth stage of the ith realization of the sessions that last for k stages.

[1]For a vector of continuous channel holding times, we can use a sequence of discrete distributions with decreasing granularity to approach its distribution.

Hence, each queue of the decoupled network has infinite servers with *deterministic* service time, $\widetilde{t}_{lkij}$, for the jth stage of the ith sub-branch of the kth main branch. Furthermore, the arrival rate of the ith sub-branch of the kth main branch is $\widetilde{\lambda}_{lki0} = P_{lki}\lambda_{l0}$. Let $\widetilde{\mathbf{x}} = [\{\widetilde{x}_{lkij} : 1 \leq k \leq N_l, 1 \leq j \leq k, 1 \leq i \leq M_{lk}\}]^T$ be the vector of number of sessions in the jth stage of the ith sub-branch of the kth main branch. Denote by $\pi_D(\widetilde{\mathbf{x}})$ the stationary distribution of the decoupled network.

Note that the stationary distribution of a Jackson network with infinite servers at each queue is insensitive with respect to the distribution of the service times [13]. Therefore, $\pi_D(\widetilde{\mathbf{x}})$ remains unchanged if we create a reference Jackson network by replacing each queue in the decoupled network with a queue that has exponential service time with service rate $\widetilde{\lambda}_{lkij} = \frac{1}{\widetilde{t}_{lkij}}$. Let $\widetilde{w}'_{lkij}$ be the positive invariant measure of the jth stage of the ith sub-branch of the kth main branch of the memoryless version of the decoupled network, which satisfies the routing balance equations with the convention that at node 0, $w'_0 = 1$. Since each sub-branch is a chain network, we have

$$\widetilde{w}'_{lkij} = P_{lki}\lambda_{l0}. \tag{3}$$

Let $\widetilde{w}_{lkij} = \frac{\widetilde{w}'_{lkij}}{\widetilde{\lambda}_{lkij}}$. Then the stationary distribution of the decoupled network is

$$\pi_D(\widetilde{\mathbf{x}}) = \prod_{j=1}^{N_l} \prod_{k=j}^{N_l} \prod_{i=1}^{M_{lk}} e^{-\widetilde{w}_{lkij}} \widetilde{w}_{lkij}^{\widetilde{x}_{lkij}} \frac{1}{\widetilde{x}_{lkij}!}. \tag{4}$$

Next, we re-compose $\pi(\mathbf{x})$ by summing up $\pi_D(\widetilde{\mathbf{x}})$ satisfying $x_{lj} = \sum_{k=j}^{N_l} \sum_{i=1}^{M_{lk}} \widetilde{x}_{lkij}$, $\forall j$. To derive $\pi(\mathbf{x})$, we first introduce the following lemma.

Lemma 1: Consider a stationary open Jackson network with N queues each with an infinite number of servers. Let x_j be the number of units in the jth queue and $\mathbf{x} = [x_1, \dots x_N]^T$. Suppose $\{\mathcal{J}_1, \mathcal{J}_2, \dots \mathcal{J}_M\}$ is a set of mutually exclusive subsets of $\{1, 2, \dots, N\}$. Let $z_i = \sum_{j \in \mathcal{J}_i} x_j$, $i = 1, 2, \dots, M$, denoting the sum of units in the queues inside $\mathcal{J}_i$. Then, the distribution of $\mathbf{z} = [z_1, \dots z_M]^T$ is

$$\pi(\mathbf{z}) = \prod_{i=1}^{M} e^{-v_i} v_i^{z_i} \frac{1}{z_i!}, \tag{5}$$

where $v_i = \sum_{j \in \mathcal{J}_i} w_j$, and w_j is the expected number of units in the jth queue.

Proof: For a Jackson network with infinite servers at each queue, the stationary queue lengths are independent Poisson random variables with mean w_j for the jth queue. Hence, z_i is Poisson with mean $v_i = \sum_{j \in \mathcal{J}_i} w_j$ for all i. Furthermore, since $\{\mathcal{J}_i\}$ are mutually exclusive, $\{z_i\}$ are independent. ■

Next, we note that the expected service time spent in the jth stage given that the jth stage exists, i.e., $j \leq k$ for the kth main branch, can be computed as

$$\bar{t}_{lj} = \frac{\sum_{k=j}^{N_l} \sum_{i=1}^{M_{lk}} P_{lki}\widetilde{t}_{lkij}}{\sum_{k=j}^{N_l} \sum_{i=1}^{M_{lk}} P_{lki}} = \frac{\sum_{k=j}^{N_l} \sum_{i=1}^{M_{lk}} P_{lki}\widetilde{t}_{lkij}}{1 - \sum_{n=1}^{j-1} p_{ln}}. \tag{6}$$

Combining this with (3), we have

$$\sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}} \widetilde{w}_{lkij} = \sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}} \frac{\lambda_{l0}P_{lki}}{\widetilde{\lambda}_{lkij}}$$
$$= \sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}} \lambda_{l0}P_{lki}\widetilde{t}_{lkij} = \lambda_{l0}(1-\sum_{n=1}^{j-1}p_{ln})\bar{t}_{lj}$$
$$= \frac{\lambda_{l0}}{\lambda_{lj}}(1-\sum_{n=1}^{j-1}p_{ln}) = w_{lj}. \quad (7)$$

Therefore, by Lemma 1, we have

$$\pi(\mathbf{x}) = \sum_{\widetilde{\mathbf{x}}:x_{lj}=\sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}}\widetilde{x}_{lkij},\forall j} \pi_D(\widetilde{\mathbf{x}})$$
$$= \prod_{j=1}^{N_l} e^{-w_{lj}} \frac{w_{lj}^{x_{lj}}}{x_{lj}!}, \quad (8)$$

which is restated as the following theorem:

Theorem 1: The single-route network has the same stationary distribution as that of the corresponding single-route memoryless network: $\pi(\mathbf{x}) = \pi_0(\mathbf{x})$.

IV. User Distribution in Multiple-Route Network

In this section, we briefly describe the general case with multiple L routes in the network. Since the L routes are independent, we model the multiple-route network as a paralleling of L single-route networks. Similar to Section III, we consider a reference multiple-route memoryless network, which is a paralleling of L corresponding single-route memoryless networks. Due to the independence of the routes, the stationary distributions of the multiple-route network and multiple-route memoryless network can be computed as the product of the stationary distributions of corresponding single-route networks and single-route memoryless networks respectively. Since each single-route network has the same stationary distribution with its corresponding single-route memoryless network, the multiple-route network has the same stationary distribution as that of the corresponding multiple-route memoryless network $\pi(\mathbf{x}) = \pi_0(\mathbf{x})$.

Let $\overline{\lambda}_n$ be the average total arrival rate to cell n, including both new and handoff arrivals. Let $\bar{\mathfrak{t}}_n$ be the average channel holding time in cell n, considering all routes and stages. Thus,

$$\overline{\lambda}_n = \sum_{l,j:c(l,j)=n}\sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}} \lambda_{l0}P_{lki}, \quad (9)$$

$$\bar{\mathfrak{t}}_n = \frac{\sum_{l,j:c(l,j)=n}\sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}}\lambda_{l0}P_{lki}\widetilde{t}_{lkij}}{\sum_{l,j:c(l,j)=n}\sum_{k=j}^{N_l}\sum_{i=1}^{M_{lk}}\lambda_{l0}P_{lki}}. \quad (10)$$

Then from (7) and Lemma 1, we can obtain

$$\pi_1(\mathbf{y}) = \prod_n e^{-\left(\overline{\lambda}_n\bar{\mathfrak{t}}_n\right)}\left(\overline{\lambda}_n\bar{\mathfrak{t}}_n\right)^{y_n}\frac{1}{y_n!}. \quad (11)$$

From (11), we can observe the stationary distribution (1) is insensitive with respect to movement patterns; (2) is insensitive with respect to the distribution of channel holding times, or the correlation among them; (3) only depends on the average arrival rates and average channel holding times in individual cells; (4) has the exact same form of an $M/M/\infty$ open Jackson network (the number of users in each cell is independent and Poisson).

V. Experimental Study

In this section, our analysis is validated via experimenting with real-world traces, the Dartmouth traces [11]. In our experiment, we use data from the academic area in the Dartmouth traces, with 152 APs and more than 5000 users, during a 17-week period (Nov. 1, 2003 to Feb. 28, 2004). We focus on the Simple Network Management Protocol (SNMP) logs, which are constructed every five minutes, when each AP polls all the users attached to it. By analyzing such SNMP logs, we can derive the average arrival rate, average channel holding time, and the stationary distribution by relative frequency.

A. Data Preprocessing

1) Data Extraction: We focus on data accumulated from 9 am to 5 pm on Monday to Friday. We also discard the data accumulated during the periods of holiday breaks, including Thanksgiving (Nov. 26, 2003 to Nov. 30, 2003) and Christmas and New Year (Dec. 17, 2003 to Jan. 4, 2004).

2) Trace Gap Padding: In the SNMP logs, a user may disappear from the SNMP report and soon reappear. This may be caused by the user departing and then returning to the network, or due to the missing of an SNMP report. Following the solution proposed in [7], we set a departure length threshold $T_d = 10$ minutes. Only if a user disappears and reappears within T_d, it is regarded as staying in the network and the missing SNMP logs are padded.

3) Open Users: A fraction of the users may stay in the system during almost all working hours. These users are regarded as closed users. Since our analytical model assumes an open network, the closed users are excluded in our experiment. If a user stays for greater than or equal to 7.5 hours during working hours on a valid day, it is regarded as a closed user. In our experiment, we observe that 9.91% of all users are closed users. An analytical model for accommodating closed users is provided in [7], which can also be applied to our work.

B. Trace Analysis

1) Poisson Arrivals: We test the arrival process of new session *at each AP* against the Poisson assumption in two steps. In the first step, we run an *independence test*, which indicates whether the number of arrivals in different time intervals are independent. In the second step, we run a *Poisson distribution test*, which indicates whether the number of arrivals are Poisson distributed in a fixed time interval. The details of the two-step test can be found in [1]. We observe that 124 of the 152 APs pass the two-step test. Those APs are referred to as *valid* APs, while the other 28 APs are referred to as *invalid* APs. In our experiments, we study the effects of both including and excluding the non-Poisson new sessions, which refer to sessions that are initiated at invalid APs. We emphasize that the Poisson test is for new arrivals only. Even for those APs that pass the Poisson test, the overall session arrival process includes both new arrivals and handoff arrivals and hence is non-Poisson.

2) Dependency of Channel Holding Times: We check the dependency of channel holding times in different stages. The entropies of the distributions of channel holding times at stages $1, 2, 3$ and 4 are 4.0657, 3.4172, 3.3942 and 2.9792, respectively, in bits. The entropy of their joint distribution is 10.2998 bits. Hence, the entropy gap is $4.0657 + 3.4172 + 3.3942 + 2.9792 - 10.2998 = 3.5565$ bits, much larger than 0. This shows that the channel holding times at different stages are dependent.

3) AP Locations and Distance Constraint: APs that are far away are likely to have little effect on each other, regardless of the mobility and session patterns. Therefore, to rigorously test the joint distribution of several APs, we are more interested in selecting APs located close to each other. We set a *distance constraint*, under which APs are located pairwisely less than 500 meters from each other. In the experiments, we will test for cases with this distance constraint.

C. KL Divergence and Entropy Gap for User Distributions

To compare the real and analytical joint distributions of multiple APs, we compute the Kullback-Leibler (KL) divergence H_{kl} between them. We also test the independence of the user distributions in different cells by computing the entropy gap H_{gap}, between the sum of the entropies of real marginal distributions and the entropy of the real joint distribution. The entropy of the real joint distribution H_{real} are also presented for reference.

Given n, the number of APs we aim to study, we randomly choose n different APs with the distance constraint. Then we compute H_{kl}, H_{gap}, and H_{real} with respect to these APs. By running this procedure 100 times, we obtain the sample mean and sample standard deviation of H_{kl}, H_{gap}, and H_{real}. In subsequent studies, we plot the sample mean versus n, along with bars showing one sample standard deviation.

Fig. 3 shows H_{kl}, H_{gap}, and H_{real} versus n under the conditions of either including or excluding non-Poisson sessions. Note that the plot points are slightly shifted to avoid overlaps. We observe that both H_{kl} and H_{gap} are much smaller than H_{real}, when we either exclude invalid sessions or exclude invalid APs, illustrating that the real distributions are close to the analytical distributions, and the real marginal distributions of single APs are approximately independent. When we do not exclude invalid sessions or invalid APs, H_{kl} and H_{gap} become larger, showing that the analytical distribution is influenced by the non-Poisson arrivals. However, since there is only a small fraction of non-Poisson arrival sessions, H_{kl} and H_{gap} remain much smaller than H_{real}. Thus, our analytical conclusions are validated by the real-world trace experiment.

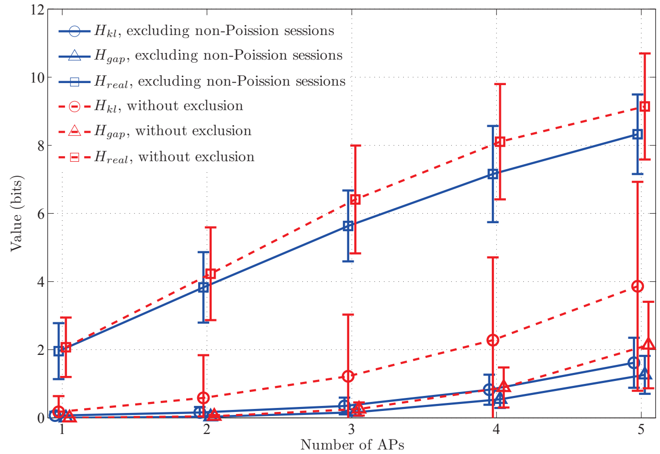


Fig. 3. H_{kl}, H_{gap} and H_{real} under the influence of non-Poisson arrivals.

VI. Conclusions

In this paper, we have studied the user distribution in multicell network by establishing a precise analytical model, considering arbitrary user movement and arbitrarily and dependently distributed channel holding times. We have derived the stationary distribution of the number of users in each cell, which is only related to the average arrival rate and the average channel holding time of each cell, and hance is insensitivity with respect to the general movement and session patterns. We use the Dartmouth trace to validate our analysis, which show that the analytical model is accurate.

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