Relaying in Wireless Sensor Networks with Interference Mitigation

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***Abstract*— We study the use of wireless relay channel in a one-hop sensor network with random packet arrival. Exploiting regular sensor nodes to serve in a wireless relay channel can increase the overall network capacity. However, due to asynchronous source transmission, the relays interfere with each other's transmission and reception. The fundamental trade-off between these two issues leads us to an optimization problem in which we find the optimum *relay zone radius* to maximize the overall sum rate of the network. We also propose a MAC protocol to choose the optimum number of sources allowed to transmit under this setting. The overall system capacity is proven to increase significantly under the proposed scheme, compared with cases where relay nodes are not exploited or where the relay zone radius is suboptimal.**

I. Introduction

Since long before the birth of new generation of sensors, mechanical sensors of varying capabilities were an integral part of automated systems. With the advent of new generation of sensors with higher sensing and communication capabilities the challenge lies in forming a complex information gathering network to maximize the quality of service. Capacity is a precious metric in this regard.

Relays can be exploited as a means to increase the capacity in a sensor network. The relay channel first introduced by van der Meulen in his PhD thesis leads to a communication scheme where instead of point to point communication between the source and destination, relays are exploited in a one-hop communication. The key capacity results for the case of a single relay were introduced by Cover and El Gamal in [1]. The capacity of a multi-user mobile system can be an interesting issue in this context. In an information theoretic point of view the literature is rich on the subject. However, the effect of wireless relay channel employment in a multiple source network, where the nodes have un-synchronized transmission, has not been studied to the best of our knowledge. While increasing the number of relays potentially increases the capacity, having more sources and relays leads to an increase in interference among nodes. Assuming that nodes can be employed as relays for other nodes during their idle periods, the fundamental question will be "To relay or not to relay." In this work we attempt to answer this question and find the best criteria on node decision to whether or not act as a relay in cooperating with its corresponding source.

The organization of this paper is as follows. Section II explains the network model. In section III we derive the information theoretic capacity results applied in our work. We discuss the use of multiple antennas at the destination in Section IV. Section V presents a discussion on MAC protocol and source selection procedure. The main optimization problem to find the optimum zones and make the relaying decision is solved in section VI. Section VII presents the simulation results. The concluding remarks are given in Section VIII.

II. Network Model

A collection of N nodes $X_1, X_2, ..., X_N$ placed randomly, uniformly and independently in the disk of unit area is considered. The transmission is assumed to be half-duplex. When node X_i transmits with power P_i, node X_j receives the transmission with power $\frac{P_i}{r_{ij}^{\alpha}}$, where r_{ij} is the distance between nodes i and j. The sink is located at height x, $0 < x < \infty$, above the center of the disk, and it is assumed that sink is a high processing power node which is equipped with multiple-antennas. The sink positioning in a height x above the field in our model resembles the SENMA model introduced by Tong *et al* in [2]. It is assumed that packet arrival has a Poisson distribution with rate λ. The transmission is slotted and we assume the length of each time slot T to be equal to the time needed to transmit a packet of length L. The signal path loss coefficients between the source and relays are represented as $h_m = \frac{1}{r_{s,m}^{\alpha}}$ and the coefficients between relays–sink as $g_m = \frac{1}{r_{m,d}^{\alpha}}$, where α is the path loss roll-off factor.

The relay channel model is depicted in Fig. 1. At time j, $1 \le j \le L$, where j denotes the jth bit in the packet, relay m observes a noisy version of the input $X[j]$. The bits $\{X[j]\}_{j=1}^{L}$ satisfy the power constraint $\frac{1}{L}\sum_{j=1}^{L} E[|X[j]|^2] \le P$. The received noisy version of the signal at relays can be expressed as,

$$Y_m[j] = h_m X[j] + I_m[j] + W_m[j], \tag{1}$$

where $I_m[j]$ is a sequence of independent and identically distributed circularly symmetric complex random variables and $|I_m[j]|^2$ is the interference power at relay node $R(m)$ at the jth sub-slot. $W_m[j]$ is a sequence of independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables of mean zero and variance $1/2$ ($E[|W_m[j]^2|] = 1$). This model resembles the channel model used by Gastpar *et al* [3].

In wireless sensor networks, physical constraints restrict the relay from simultaneous transmission and reception. There-

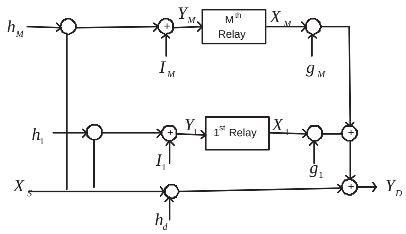

Fig. 1. Relay channel with M relays.

fore, we define two states for each relay as *receive* and *transmit* states. During each sub-slot of the receive state the signal received at the relay is expressed in (1) and the received signal at the destination is,

$$Y_d[j] = h_d X[j] + W_d[j], \tag{2}$$

where $h_d = \frac{1}{(\sqrt{r_{s,d}^2 + x^2})^\alpha}$. During the relay transmission state we have the following expressions

$$\begin{aligned} Y_m[j] &= 0 \\ Y_d[j] &= h_d X[i] + \sum_{i=1}^{M} g_m X_m[j] \end{aligned} \tag{3}$$

We next derive the achievable capacity for this model.

III. Capacity results for Multiple-Relay network

Each transmission frame is divided in to two equal length slots. It is assumed that the relay is in its receive state over the first transmission slot and in transmit state over the second slot (see Fig. 2) and each relaying zone has M relays. In the model of Fig 1, X_i is the input signals from the relays and Y_i is the output signals received at the relays. Let $R \subset S = \{1, \ldots, M\}$ and $\mathbf{X}_R = \{X_i | i \in R\}$, $\mathbf{Y}_R = \{Y_i | i \in R\}$. Also, R^c is defined as the complement of R in S. Using the cut-set bounds the following theorem gives an upper-bound on the capacity:

Theorem 1: The capacity of the multiple-relay channel depicted in Fig. 1, where the relays are in receive mode for the first time slot and in the transmit mode for the second slot is upper-bounded by,

$$C \leq \frac{1}{2} \max_{p(\mathbf{X})} \min \left(\begin{array}{c} I(X_s; Y_d, \mathbf{Y}_R | \mathbf{X}_R) + I(X_s; Y_d), \\ I(X_s; Y_d) + I(X_s, \mathbf{X}_R; Y_d | \mathbf{X}_{R^c}) \end{array} \right), \tag{4}$$

for all possible joint pdfs $p(X_s, X_1, \ldots, X_R)$.

Proof: This theorem is a direct result of applying Theorem 14.10.1 in [4] to the channel model at hand to find the broadcast and multiple-access achievable rates. The minimum of the two rates gives the upper bound on the achievable rate. ∎

We now apply Theorem 1 to the *decode-and-forward* strategy implemented in this work. In the decode-and-forward strategy the source transmits its message during the first phase, when the relay is in its receive state. The relay decodes this message and sends it to the destination in its *transmit* state.

Corollary 1: The decode and forward capacity expression is upper-bounded by,

$$R \leq \frac{1}{2} \min \left(\begin{array}{c} I(X_s; \mathbf{Y}_R | \mathbf{X}_R), \\ I(X_s; Y_d) + I(X_s, \mathbf{X}_R; Y_d | \mathbf{X}_{R^c}) \end{array} \right). \tag{5}$$

IV. Exploiting multiple antennas in the sink for interference removal

In this section the effect of multiple-antennas will be considered in the sink for interference removal. The capacity region for MIMO is expressed as [5],

$$\begin{aligned} R_i &\leq \log(1 + \frac{P||\mathbf{h}_i||^2}{N_0}) \\ \sum_{i=1}^{N_s} R_i &\leq \log\det(I_{n_r} + \frac{P}{N_0} \sum_{i=1}^{N_s} \mathbf{h}_i \mathbf{h}_i^T), \end{aligned} \tag{6}$$

where $\mathbf{h}_i$ is the individual source destination channel vector, n_r is the number of antennas in the destination and N_s is the number of transmitters. The first term is the individual rate constraint for each source and the second term is the maximum achievable sum-rate. If $\mathbf{h}_i \mathbf{h}_k^T = \mathbf{0}_{n_r}$ $\forall i \neq k$, in other words if $\mathbf{h}_i = [0, \ldots, \underbrace{r_{i,d}^{-\alpha}}_{i}, \ldots, 0]^T$, the sum-rate expression can be written as,

$$\sum_{i=1}^{N_s} R_i \leq \sum_{i=1}^{N_s} \log(1 + \frac{P||\mathbf{h}_i||^2}{N_0}) \tag{7}$$

Thus, in the case that source-destination channel vectors are orthogonal the destination can decode the message sent by each source, removing the interference caused by other sources. This assumption makes sense if the senders are geographically separated and we use directional antennas at the destination ($n_r \geq N_s$).

Using the result of Corollary 1 for the capacity of a single source destination pair under the relay model and applying the MIMO capacity results from [6] and [7] the overall network capacity is bounded as,

$$C \leq \min \left(\begin{array}{c} \frac{1}{2} \sum_{i=1}^{N_s} \log(1 + \sum_{l=1}^{M} \frac{P||h_l^i||^2}{I_l}), \\ \frac{1}{2} \log\det(I_{n_r} + \frac{P}{N_0} \sum_{i=1}^{N_s} (\mathbf{h}_d^{i\,H} \mathbf{h}_d^i) + \\ \frac{1}{2} \log\det(I_{n_r} + \frac{P}{N_0} \sum_{i=1}^{N_s} \mathbf{g}^{i\,H} \mathbf{g}^i) \end{array} \right). \tag{8}$$

The first term corresponds to the broadcast cut capacity of the network and the second term to the multiple-access cut capacity (flow of information from the relays within a relaying zone to destination). The matrix $\mathbf{g}^i = \mathbf{h}_d^i + \sum_{l=1}^{M} \mathbf{g}_l^i$ corresponds to the sum of the relay-sink channel vectors in the ith relaying zone, plus the source-sink channel vector. Therefore, the size of the optimum relaying zones will be the result of a trade-off between two key factors. The first factor is the interference increase at the relays (during their reception) caused by the use of more relays while the second factor is the potential capacity increase resulting from using more relays. The overall network capacity is the sum of achievable rates of each relaying zone C_i where,

$$C = \sum_{i=1}^{N_s} C_i. \tag{9}$$

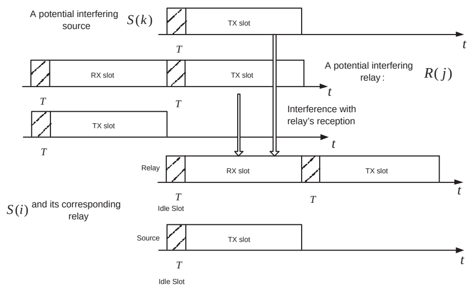

Fig. 2. Timing for a source and its corresponding relay in $\mathbb{REL}_i$ as well as possible interferer source and relays.

V. MAC LAYER AND SCHEDULING

Each node in the system at the beginning of each time slot can serve either as a source, relay or it can be turned off and take no action. The timing of this MAC scheme has been depicted in Fig. 2. After each transmission slot the node enters an idle state. During this shorter slot if a packet is generated at the node it will be transmitted to the destination. Otherwise, the node is considered to be available as a potential relay at the beginning of the next *transmission slot*. The relay can either forward the other nodes data in the relay mode or remain silent. This decision will be made based on the relay's distance from the sources at the beginning of each transmission slot. An *optimum relaying-zone distance* will be obtained that makes it possible for the relays to make this decision.

A. Source Selection Procedure

Each potential relay node can decode the data received from only one source during a specific transmission slot. Therefore, we introduce the idea of disjoint relaying zones. At the beginning of each slot all the sources enter the idle period of duration βT. In order to guarantee that each relay is used by only one source, the scheduling scheme has to decide whether a source is allowed to send or not. A potential relay $R(k)$ $1 \leq k \leq n$ is in source $S(i)$'s relaying zone if

$$d(R(k), S(i)) \leq d(R(k), S(j)) \quad \forall j \in \{1, \ldots, N_p\}, \tag{10}$$

where N_p is the maximum number of relaying zones of radius r_{rel}. The ith relaying zone will be called $\mathbb{REL}_i$ from now on. If a source $S(i)$ lies in $\mathbb{REL}_i$ during $S(i)$'s transmission the MAC layer will not allow another node $S(m) \in \mathbb{REL}_i$ to transmit. The results from circle packing theory [8] give the maximum number of packings N_p (equal to the number of sources N_s), which we use for our analytical results. Due to page limitation the analysis details are removed.

B. Distributed Implementation of Circle Packing

In practice the nodes are randomly distributed. Therefore, the analytical results in finding the number of circle packings within a disk using hexagonal lattice are only upper-bounds on the exact number of possible packings. The maximum circle packing is an NP hard problem, which can be reduced to the maximal independent set problem [9]. We have implemented the parallel algorithm (in the sense that node addition to the independent set is done in a parallel manner rather than a sequential one) presented in [9] by Luby to solve the MIS problem.

VI. MAXIMAL CAPACITY RELAY-ZONE DESIGN AND INTERFERENCE ANALYSIS

We use the *decode and forward* scheme for the purpose of our analysis and simulation. The capacity C_i in (9) can be written as $C_i = f(r_{\text{opt}}, N_r, N_z, \mathbf{I})$ where N_z is the random variable equal to the number of nodes in zone $\mathbb{REL}_i$, N_r is the number of nodes acting as relays in the same zone, and $\mathbf{I}$ is the *random interference vector*. The goal is to design the relaying zones in order to maximize the network achievable sum-rate. Therefore, the optimization problem can be formulated as

$$r_{\text{opt}} = \arg\max_{r_{\text{rel}}} \sum_{i=1}^{N_s} \mathbb{E}_{N_r, N_z, \mathbf{I}}[C_i(r_{\text{rel}}, N_r, N_z, \mathbf{I})]. \tag{11}$$

If the destination is located in height x far enough from the disk center the source location will not affect the obtained r_{rel}. Therefore, the problem is relaxed as $r_{\text{opt}} = \arg\max_{r_{\text{rel}}} \mathbb{E}_{N_r, N_z, \mathbf{I}}[C_i(r_{\text{rel}}, N_r, N_z, \mathbf{I})]$. The conditional capacity $C_i(N_r, \mathbf{I}|N_r = M, \mathbf{I} = \mathbf{i})$ is expressed in (9) as a function of the number of relays. The number of nodes in each zone is a random variable N_z with binomial distribution $P(N_z = k) = \binom{N_z}{k} p^k (1-p)^{N_z - k}$, where $p = \pi r_{\text{rel}}^2$, since the nodes are uniformly distributed on the disk of unit area.

In Section V-A it was explained that the idle slot length equals βT, and based on the assumption of exponential inter arrival times the probability of being a potential relay equals $p_r = e^{-\lambda \beta T}$. The number of relays within each zone is also a random variable with binomial distribution which can be formulated as $P(N_r = l | N_z = N) = \binom{N}{l} p_r^l (1-p_r)^{N-l}$. The expected capacity of each relaying zone can be formulated as,

$$\begin{aligned} \mathbb{E}[C_i] &= \mathbb{E}_{N_r}[\mathbb{E}_{\mathbf{I}}[C_i | N_r = l]] \\ &= \sum_{j=0}^{N} \sum_{l=0}^{j} p(N_z = j) p(N_r = l | N_z = j) \mathbf{E}_{\mathbf{I}}[C_i(\mathbf{I} | N_r = l)] \end{aligned} \tag{12}$$

The only term in (12) that is a function of $\mathbf{I}$ is the conditional capacity $C_i(\mathbf{I}|N_r = l)$. Since the minimum of two concave functions is concave, we further apply Jensen's inequality to (8), (9) to approximate the expected values,

$$\mathbb{E}_{\mathbf{I}}[C_i(\mathbf{I}|N_r = l)] \leq \min \begin{pmatrix} \frac{1}{2}\log(1 + \sum_{j=1}^{l} \mathbb{E}[\frac{P||h_j^i||^2}{I_j}]), \\ \frac{1}{2}\log(1 + \frac{P}{N_0}\mathbb{E}[||\mathbf{h}_d^i||^2]) \\ +\frac{1}{2}\log(1 + \sum_{j=1}^{l} \frac{P}{N_0}\mathbb{E}[||\mathbf{g}_j^i||^2]) \end{pmatrix} \tag{13}$$

For a relay $l \in \mathbb{REL}_i$, the distribution of the distance $r_{s,l}$ from the corresponding source obeys $P[r_{l,s} < r | l \in \mathbb{REL}_i] = \frac{\pi r^2}{\pi r_{\text{opt}}^2}$. Since I_l is a function of $r_{s,l}$, the above expected values can be computed given the distribution of $r_{s,l}$, using numerical integration. The computational details are removed due to lack of space. We need to bear in mind that the number of interferer

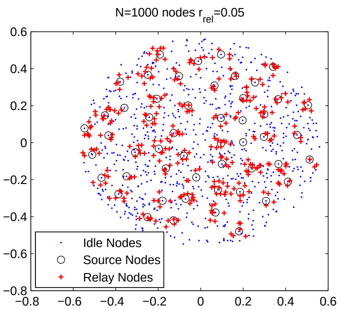

Fig. 3. Snapshot of the unit disk and relaying zones.

nodes is a random variable, since the interference at a relay node can be either caused by the other sources or other relays which are transmitting as depicted in Fig. 2. However, the scheduling scheme poses some restrictions on the location of interferers. Because of channel reservation for the source within a specific relaying zone, the interferers lie outside the relaying zone. Two sources are allowed to send simultaneously if and only if $\forall i, j \quad d(S(i), S(j)) \geq 2r_{\text{opt}}$. Correspondingly, $S(i)$ experiences interference from a relay $R(l)$ if and only if $\forall l \neq i \quad d(S(i), R(l)) \geq r_{\text{opt}}$, where $R(l) \in \mathbb{REL}_l$.

VII. Numerical Analysis and Simulation Results

Fig. 3 is a snapshot of source and relay location in an instance of our simulations. In the following example for numerical analysis, we assume a path loss rolling factor of $\alpha = 4$ in the flat network, due to partial cancelation by ground-reflected rays, and a path loss factor of $\alpha = 2$ in free space between the sensors and the sink [2]. In Fig. 4 the approximate upper bound on per node capacity for the case where the destination is located at height $x = 3$ is given for two different node numbers. The constant line represents the case where relaying is not employed. Consequently no MAC protocol is needed and all the sources can send a packet simultaneously, which will then be decoded correctly at the sink. It is clear from Fig. 4 that employing relays can significantly increase the capacity, even though the MAC protocol prevents some nodes from sending due to the selection of relaying zones. The optimal average number of relays in a relaying zone corresponding to radius r_{opt} can be easily computed as $\mathbb{E}[N_r] = \pi r_{\text{opt}}^2 N$. Fig. 5 provides an illustration of the optimum zone radius versus the number of nodes in the networks. As shown in the figure, increasing the arrival rate results in relaying zones with smaller radii which is expected.

VIII. Conclusions

In this work we consider a wireless sensor network with slot based transmission scheme and study how to optimally utilize the availability of nodes to serve as relays for other nodes during the time intervals in which they do not generate packets. Due to un-synchronized transmissions, the more relays we have for a specific source, the more interference will be introduced in the transmission of other sources. We have considered this trade-off and provided an analytical framework to optimize the size of relay-zone around each source node. Our numerical and simulation results show that the proposed scheme can lead to noticeable capacity increases.

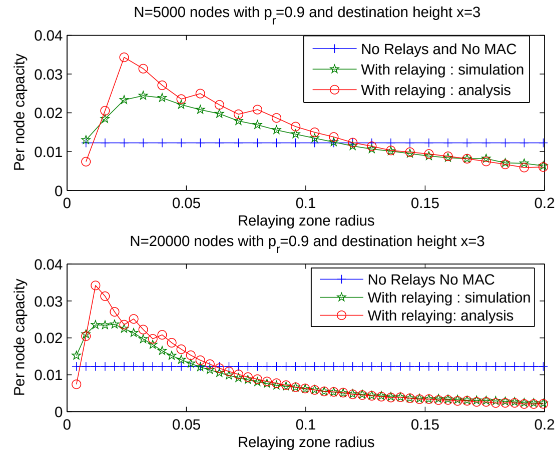

Fig. 4. Capacity increase using relay channel for destination height $x = 3$.

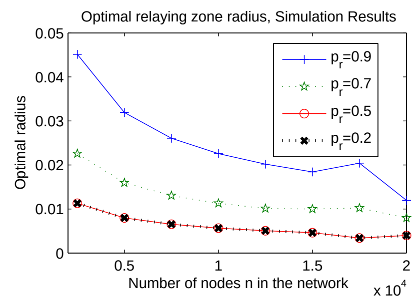

Fig. 5. Optimum relaying zone radius versus the number of nodes for different arrival rates.

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