# Beamforming Design for Uplink MU-MIMO Wireless Network Virtualization

Ahmed F. Almehdhar⋆, Ben Liang⋆, Min Dong‡, Gary Boudreau§, Yahia Ahmed§
⋆Department of Electrical and Computer Engineering, University of Toronto, Canada
‡Department of Electrical, Computer and Software Engineering, Ontario Tech University, Canada
§Ericsson Canada Inc., Canada

***Abstract*—We provide wireless network virtualization (WNV) for multiple service providers (SPs) that are assumed to virtually serve multi-antenna users via a base station (BS) with multiple antennas. The virtualization of this multi-user multiple-input multiple-output (MU-MIMO) system is managed by an infrastructure provider (InP) that owns the communication equipment. By exploiting the antennas at both the BS and the user devices, we jointly design the uplink receive beamforming at the BS and the transmit beamforming at the user devices, achieving service isolation at the physical layer. This WNV is formulated as a non-convex optimization problem in the transmit and receive beamforming vectors. We derive separately optimal closed-form and semi-closed form solutions for the beamforming vectors, and use these solutions in alternating iterations to solve the original problem. Our simulation results show that the proposed solution enables effective network virtualization, to support the independent operation of multiple SPs, while retaining efficiency no less than non-virtualized operation, and it substantially outperforms traditional WNV based on strict resource separation.**

## I. Introduction

Wireless network virtualization (WNV) is an important framework to enable the sharing of resources of a base station (BS) among multiple service providers (SPs), which encourages competition by reducing the market entry costs of new SPs. A typical WNV system consists of the SPs and a separate entity that is called the infrastructure provider (InP). The InP manages the network's physical resources and splits them into virtual slices. These virtual slices are leased upon request to the SPs that, in turn, utilize them to provide services to their subscribing users. An SP demands services from the InP, on behalf of it own users, without needing knowledge of the existence of any other SPs. Although multiple SPs share the same infrastructure, none of them is expected to consider inter-SP interference in their design for the demands. Thus, it is the job of the InP to provide *service isolation*, i.e., to satisfy the demand of each SP without affecting the other SPs.

Although virtualization has been well studied for wired networks [1], WNV is more complicated, with the need to share both the hardware and the radio spectrum, and with new challenges arising in guaranteeing service isolation under wireless interference [2]. To achieve service isolation among the SPs in a wireless network, most existing works propose strict separation of the physical resources, an approach rooted in the traditional solution for wired network virtualization. This strict separation could be in the form of dividing the time, frequency spectrum, resource blocks, or the number of antennas among different SPs [3]–[8]. However, this strict separation limits the design space of virtualization since it does not explore the spatial dimension. It has been shown that strict separation can lead to inefficient resource utilization and severe loss of system throughput. Other works proposed non-strict resource separation between SPs, where resources are shared among SPs, and interference caused to each other needs to be considered and managed. This interference was ignored in [9] and [10], while power-domain non-orthogonal multiple access (NOMA) was used in [11]–[13] to handle the interference between users.

Separating SPs in the downlink using multiple-input multiple-output (MIMO) processing techniques while sharing all physical resources of a BS was first proposed in [14]. Following this, transmit beamforming was utilized in [15] to provide downlink service isolation among the different SPs by minimizing the expected deviation between the InP's supply and the SPs' demands. The uplink WNV problem was first studied in [16] and [17] to achieve service isolation by minimizing the expected deviation. It was shown that the beamforming solutions developed for downlink WNV in [14] and [15] cannot be applied to the uplink. Furthermore, achieving service isolation in the uplink requires managing the transmit powers of users to effectively reduce interference among them. However, these studies have assumed single-antenna user devices. Given the current standards for next-generation communications, it is expected that user devices will be equipped with multiple antennas and will be able to leverage beamforming techniques as well.

In this paper, we consider a more general uplink WNV scenario where both the BS receiver and the user devices are equipped with multiple antennas. We jointly design the uplink receive beamforming for the BS and the transmit beamforming for the user devices to maintain SPs service isolation through an alternating-optimization approach. We propose a closed-form solution to the former subproblem, and a semi-closed-form solution to the latter. This enables the InP to effectively deliver the desired data transmission demanded by the SPs while minimizing the inter-SP interference, thereby providing the required service isolation.

The rest of this paper is organized as follows. In Section II, the WNV system model is presented, and the joint transmit-receive beamforming optimization problem for uplink MU-MIMO WNV is formulated. We then present our proposed solution in Section III. The proposed solution is evaluated via simulation and compared with other methods in Section IV. Finally, Section V concludes the paper.

## II. System Model and Problem Formulation

We study uplink communication in the WNV environment with $M$ SPs. We consider the case of one cell where all SPs share a single BS that is equipped with $N_r$ antennas. The BS is governed by an InP that performs the virtualization of the system, i.e., slicing the system into $M$ virtual networks, each for an SP. We assume that all other parts of the network, including the core network and computational resources, are already virtualized and can be utilized by the InP and the SPs.

Without loss of generality, we focus on a specific multiple-access channel that is shared by all SPs. Suppose that SP $m$ serves $K_m$ users on this shared channel. Let $K = \sum_{m=1}^{M} K_m$ be the total number of users. Each user device is equipped with $N_t$ transmit antennas. Each SP formulates its own demands to be fulfilled by the InP, so that its subscribing users can achieve specific performance goals, such as maximizing the sum-rate or ensuring certain fairness. The SPs design their demands without knowledge of each other, leading to interference between the SPs. The InP holds the responsibility to fulfill the requested demands of all SPs while managing the wireless interference among them, thereby ensuring service isolation.

Specifically, without the knowledge of other SPs, each SP $m$ designs a set of transmit beamforming vectors $\mathbf{f}_{m,k} \in \mathbb{C}^{N_t},\ \forall k \in \{1, \cdots, K_m\}$ for its users to transmit data and a set of receive beamforming vectors $\mathbf{w}_{m,k} \in \mathbb{C}^{N_r},\ \forall k \in \{1, \cdots, K_m\}$ for the InP to decode the data of its users. In this work, we assume that each user is limited to transmit only one data stream. Let $\mathbf{x}_m = [x_{m,1}, x_{m,2}, \cdots, x_{m,K_m}]^T \in \mathbb{C}^{K_m}$ be the transmitted symbol vector of the users of SP $m$. Without loss of generality, we set $\mathbb{E}\Big\{\mathbf{x}_m\Big\} = \mathbf{0}$ and $\mathbb{E}\Big\{\mathbf{x}_m^H \mathbf{x}_m\Big\} = \mathbf{I}_{K_m}$. The desired received signal at the BS, from SP $m$'s perspective, is

$$\hat{\mathbf{y}}_m^{\text{desired}} = \sum_{k=1}^{K_m} \mathbf{H}_{m,k} \mathbf{f}_{m,k} x_{m,k} + \mathbf{n} \tag{1}$$

$$= \mathbf{H}_m \mathbf{F}_m \mathbf{x}_m + \mathbf{n}, \tag{2}$$

where $\mathbf{H}_{m,k} \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix for user $k$ of SP $m$, $\mathbf{H}_m = [\mathbf{H}_{m,1}, \mathbf{H}_{m,2}, \cdots, \mathbf{H}_{m,K_m}] \in \mathbb{C}^{N_r \times K_m N_t}$ is the channel matrix from the users of SP $m$ to the BS, $\mathbf{F}_m = \text{blkdiag}\{\mathbf{f}_{m,1}, \mathbf{f}_{m,2}, \cdots, \mathbf{f}_{m,K_m}\}$ is the transmit beamforming matrix for all users with SP $m$, and $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_r})$ is the BS receiver's additive white Gaussian noise. The transmit power of each user is determined by $\|\mathbf{f}_{m,k}\|_2^2$. Then, by the design of SP $m$, the desired post-processed received signal vector of all users subscribed to SP $m$ is given by

$$\hat{\mathbf{x}}_m^{\text{desired}} = \mathbf{W}_m \hat{\mathbf{y}}_m^{\text{desired}} = \mathbf{W}_m (\mathbf{H}_m \mathbf{F}_m \mathbf{x}_m + \mathbf{n}), \tag{3}$$

where $\mathbf{W}_m = [\mathbf{w}_{m,1}, \mathbf{w}_{m,2}, \cdots, \mathbf{w}_{m,K_m}]^T \in \mathbb{C}^{K_m \times N_r}$ is the receive beamforming matrix applied at the BS to decode the messages of the users of SP $m$. The desired post-processed received signal vector for users of all SPs can be written as

$$\hat{\mathbf{x}}^{\text{desired}} = \left[\hat{\mathbf{x}}_1^{\text{desired}^T}, \cdots, \hat{\mathbf{x}}_M^{\text{desired}^T}\right]^T = \mathbf{D}\mathbf{x} + \mathbf{W}\mathbf{n}, \tag{4}$$

where $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_M^T]^T \in \mathbb{C}^K$ is the transmitted symbol vector of all users, $\mathbf{D}$ is a block diagonal matrix representing the *virtualization demand* given by $\mathbf{D} = \text{blkdiag}\{\mathbf{W}_1\mathbf{H}_1\mathbf{F}_1, \mathbf{W}_2\mathbf{H}_2\mathbf{F}_2, \cdots, \mathbf{W}_M\mathbf{H}_M\mathbf{F}_M\}$, and $\mathbf{W} = \left[\mathbf{W}_1^T, \mathbf{W}_2^T, \cdots, \mathbf{W}_M^T\right]^T$ contains a stack of the receive beamforming matrices designed by all SPs.

The desired received signal in (4), by the design of the SPs, does not account for the interference among SPs. This inter-SP interference is due to the users of all SPs sharing the same time-frequency resources, while ignoring the existence of one another. Therefore, this desired signal is unrealistic and cannot be directly achieved. In WNV, the InP is responsible to use its physical resources to closely approximate this desired signal. Thus, the InP designs the actual transmit and receive beamforming vectors to achieve its goal of satisfying the demands while providing service isolation among different SPs.

The actual received signal vector of all users in the cell is given by

$$\hat{\mathbf{y}}^{\text{actual}} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{t}_k x_k + \mathbf{n} = \mathbf{H}\mathbf{T}\mathbf{x} + \mathbf{n}, \tag{5}$$

where $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix for user $k$, $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_M] \in \mathbb{C}^{N_r \times K N_t}$ is the channel matrix of all users to the BS, $\mathbf{t}_k$ is user $k$'s transmit beamforming vector, and $\mathbf{T} = \text{blkdiag}\{\mathbf{t}_1, \mathbf{t}_2, \cdots, \mathbf{t}_K\}$ is the transmit beamforming matrix for all users. Then, the actual post-processed received signal vector of all users is given by

$$\hat{\mathbf{x}}^{\text{actual}} = \mathbf{V}\hat{\mathbf{y}}^{\text{actual}} = \mathbf{V}\mathbf{H}\mathbf{T}\mathbf{x} + \mathbf{V}\mathbf{n}, \tag{6}$$

where $\mathbf{V} \in \mathbb{C}^{K \times N_r}$ is the receive beamforming matrix. Here, both $\mathbf{V}$ and $\mathbf{T}$ are designed and implemented by the InP for all users and all SPs. Note that the transmit power of user $k$, i.e., $\|\mathbf{t}_k\|_2^2$, may not be the same as its demanded power $\|\mathbf{f}_k\|_2^2$. This difference gives the InP additional degrees of freedom in its design to achieve its goal.

As an inherent characteristic of WNV, the InP aims to meet the demands requested by the different SPs, which may be based on some prior agreements between the InP and the SPs. The demands, as described in (4), are fully characterized by the transmit and receive beamforming vectors, i.e., $\mathbf{W}_m$ and $\mathbf{F}_m,\ \forall m$. Note that the expression in (4) represents perfect isolation among SPs without any interference. Thus, it is logical for the InP to aim at making $\hat{\mathbf{x}}^{\text{actual}}$ as close to $\hat{\mathbf{x}}^{\text{desired}}$ as possible. In this work, we consider the expected $l_2$-norm deviation between the virtual and actual received signals, which is given by

$$f(\mathbf{V}, \{\mathbf{t}_k\}_{k=1}^{K}) = \mathbb{E}\Big\{\Big\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\Big\|_2^2\Big\}, \tag{7}$$

where the expectation is taken over $\mathbf{x}$ and $\mathbf{n}$.

Thus, the InP need to solve the following optimization problem:

$$\min_{\mathbf{V},\{\mathbf{t}_k\}} \mathbb{E}\Big\{\left\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\right\|_2^2\Big\} \tag{8a}$$

$$\text{s.t. } \|\mathbf{t}_k\|_2^2 \leq p_{\max}, \forall k, \tag{8b}$$

where $p_{\max}$ is the maximum transmit power at each device. As seen above, the InP jointly optimizes the transmit and receive beamforming vectors to minimize the expected deviation. We remark that another practically meaningful variation of this constraint is to prevent the InP from assigning powers that are greater than the requested powers, given by $\|\mathbf{t}_k\|_2^2 \leq \|\mathbf{f}_k\|_2^2, \forall k$. The solution presented in Section III can be easily modified to accommodate this case.

## III. Proposed Solution for MU-MIMO WNV

The first step into tackling problem (8) is to simplify the deviation expression in the objective. We have

$$\begin{aligned} f(\mathbf{V},\{\mathbf{t}_k\}) &= \mathbb{E}\Big\{\Big\|(\mathbf{VHT}-\mathbf{D})\,\mathbf{x} + (\mathbf{V}-\mathbf{W})\,\mathbf{n}\Big\|_2^2\Big\} \\ &= \|\mathbf{VHT}-\mathbf{D}\|_F^2 + \sigma_n^2\,\|\mathbf{V}-\mathbf{W}\|_F^2\,, \end{aligned} \tag{9}$$

where the second equation is obtained using the properties $\|\mathbf{x}\|_2^2 = \mathbf{x}^H\mathbf{x} = \operatorname{tr}\Big(\mathbf{x}^H\mathbf{x}\Big)$, $\|\mathbf{A}\|_F^2 = \operatorname{tr}\Big(\mathbf{A}\mathbf{A}^H\Big)$, and $\mathbb{E}\{\operatorname{tr}(\cdot)\} = \operatorname{tr}(\mathbb{E}\{\cdot\})$. With this, problem (8) can be written as

$$\min_{\mathbf{V},\{\mathbf{t}_k\}} \|\mathbf{VHT}-\mathbf{D}\|_F^2 + \sigma_n^2\,\|\mathbf{V}-\mathbf{W}\|_F^2 \tag{10a}$$

$$\text{s.t. } \|\mathbf{t}_k\|_2^2 \leq p_{\max}, \forall k. \tag{10b}$$

This is a non-convex joint optimization problem with respect to (w.r.t.) $\mathbf{V}$, and $\{\mathbf{t}_k\}$, which cannot be solved via regular convex optimization techniques. Nonetheless, the problem is biconvex w.r.t. $\mathbf{V}$, and $\{\mathbf{t}_k\}$. Therefore, we can adopt alternating optimization to find a partial optimum.

In particular, our approach to solving problem (10) is by decoupling the two optimization variables, i.e., we decompose the joint problem into a receive beamforming subproblem to optimize $\mathbf{V}$ and a transmit beamforming subproblem to optimize $\mathbf{t}_k, \forall k$. In the following subsections, we provide a solution to each of them. Then, we use an alternating optimization approach to find a partial optimum of the original joint optimization problem.

### A. Receive Beamforming Design

Here, we treat $\{\mathbf{t}_k\}$ in problem (10) as constants and find the optimal receive beamforming matrix $\mathbf{V}^\star$ by solving

$$\min_{\mathbf{V}} \|\mathbf{VHT}-\mathbf{D}\|_F^2 + \sigma_n^2\,\|\mathbf{V}-\mathbf{W}\|_F^2\,. \tag{11}$$

This is an unconstrained convex optimization problem with a single minimum, for which we can derive its optimal solution analytically as shown below.

We start by writing the expression in (9) in the form of traces as

$$\begin{aligned} f(\mathbf{V},\{\mathbf{t}_k\}) &= \operatorname{tr}\Big(\mathbf{VHTT}^H\mathbf{H}^H\mathbf{V}^H\Big) - \operatorname{tr}\Big(\mathbf{DT}^H\mathbf{H}^H\mathbf{V}^H\Big) \\ &\quad - \operatorname{tr}\Big(\mathbf{VHTD}^H\Big) + \operatorname{tr}\Big(\mathbf{DD}^H\Big) + \sigma_n^2\operatorname{tr}\Big(\mathbf{VV}^H\Big) \\ &\quad - \sigma_n^2\operatorname{tr}\Big(\mathbf{WV}^H\Big) - \sigma_n^2\operatorname{tr}\Big(\mathbf{VW}^H\Big) + \sigma_n^2\operatorname{tr}\Big(\mathbf{WW}^H\Big). \end{aligned}$$

Let $\mathbf{V}^\dagger$ be the complex conjugate of $\mathbf{V}$. Differentiating $f(\mathbf{V},\{\mathbf{t}_k\})$ with respect to $\mathbf{V}^\dagger$ gives

$$\frac{\partial f(\mathbf{V},\{\mathbf{t}_k\})}{\partial \mathbf{V}^\dagger} = \mathbf{VHTT}^H\mathbf{H}^H - \mathbf{DT}^H\mathbf{H}^H + \sigma_n^2\mathbf{V} - \sigma_n^2\mathbf{W}.$$

Since $\left(\mathbf{HTT}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)$ is positive definite, we solve $\frac{\partial f(\mathbf{V},\{\mathbf{t}_k\})}{\partial \mathbf{V}^\dagger} = \mathbf{0}$ and obtain the following closed-form expression for $\mathbf{V}^\star$:

$$\mathbf{V}^\star = \left(\mathbf{DT}^H\mathbf{H}^H + \sigma_n^2\mathbf{W}\right)\left(\mathbf{HTT}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}. \tag{12}$$

### B. Transmit Beamforming Design

Given $\mathbf{V}$ in problem (10), we now find the optimal transmit beamforming vectors $\mathbf{t}_k^\star, \forall k$. This subproblem is given by

$$\min_{\{\mathbf{t}_k\}} \|\mathbf{VHT}-\mathbf{D}\|_F^2 + \sigma_n^2\,\|\mathbf{V}-\mathbf{W}\|_F^2 \tag{13a}$$

$$\text{s.t. } \|\mathbf{t}_k\|_2^2 \leq p_{\max}, \forall k. \tag{13b}$$

This is a constrained convex optimization problem. Strong duality holds for this subproblem since Slater's condition is trivially satisfied, e.g. with $\mathbf{t}_k = \mathbf{0}$. Thus, we solve this problem by the KKT conditions. The Lagrangian of this problem is

$$\begin{aligned} \mathcal{L}(\{\mathbf{t}_k\},\boldsymbol{\lambda}) &= \|\mathbf{VHT}-\mathbf{D}\|_F^2 + \sigma_n^2\,\|\mathbf{V}-\mathbf{W}\|_F^2 \\ &\qquad + \sum_{k=1}^K \lambda_k(\|\mathbf{t}_k\|_2^2 - p_{\max}) \\ &= \sum_{k=1}^K \operatorname{tr}\Big(\mathbf{t}_k^H\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k\mathbf{t}_k\Big) - \operatorname{tr}\Big(\mathbf{t}_k^H\mathbf{H}_k^H\mathbf{V}^H\mathbf{d}_k\Big) \\ &\quad - \operatorname{tr}\Big(\mathbf{d}_k^H\mathbf{V}\mathbf{H}_k\mathbf{t}_k\Big) + \lambda_k(\|\mathbf{t}_k\|_2^2 - p_{\max}) + C, \end{aligned} \tag{14}$$

where $\mathbf{d}_j$ is the $j^{\text{th}}$ column of $\mathbf{D}$, $C = \sum_{j=1}^K \mathbf{d}_j^H\mathbf{d}_j + \sigma_n^2\,\|\mathbf{V}-\mathbf{W}\|_F^2$ is the sum of the terms that are independent of $\mathbf{t}_k$, and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_K]^T$ is the vector of Lagrange multipliers associated with the constraints in (13b). Let $\mathbf{t}_k^\star$ and $\lambda_k^\star$ be the optimal transmit beamforming vector and Lagrange multiplier for user $k$. Differentiating $\mathcal{L}(\{\mathbf{t}_k^\star\},\boldsymbol{\lambda}^\star)$ w.r.t. every $\mathbf{t}_k^{\star\dagger}$ and setting it to zero gives the following stationarity condition for $\mathbf{t}_k^\star$:

$$\left(\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k + \lambda_k^\star\mathbf{I}\right)\mathbf{t}_k^\star = \mathbf{H}_k^H\mathbf{V}^H\mathbf{d}_k \quad \forall k. \tag{15}$$

The other KKT conditions for $(\{\mathbf{t}_k^\star\},\boldsymbol{\lambda}^\star)$ to be a globally optimal solution of problem (13) are

$$\|\mathbf{t}_k^\star\|_2^2 \leq p_{\max}, \qquad \forall k, \tag{16}$$

$$\lambda_k^\star \geq 0, \qquad \forall k, \tag{17}$$

$$\lambda_k^\star(\|\mathbf{t}_k^\star\|_2^2 - p_{\max}) = 0, \qquad \forall k, \tag{18}$$

where (16) and (17) are, respectively, the conditions for primal and dual feasibility, and (18) represents the complementary slackness conditions.

Since strong duality holds, solving the KKT conditions yields the optimal solution to problem (13). Below, we discuss the solution in two cases of the value of $\lambda_k^\star$ that satisfy the KKT conditions.

1) $\lambda_k^\star = 0$: This case means that at optimality, the user $k$ may not use the full transmit power $p_{\text{max}}$. From (15), $\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k\mathbf{t}_k^\star = \mathbf{H}_k^H\mathbf{V}^H\mathbf{d}_k$. Since $\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k$ can be rank deficient, $\mathbf{t}_k^\star$ is given by

$$\mathbf{t}_k^\star = \left(\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k\right)^\ddagger \mathbf{H}_k^H\mathbf{V}^H\mathbf{d}_k, \qquad \forall k, \tag{19}$$

where $(\cdot)^\ddagger$ is the Moore–Penrose pseudo inverse of a matrix. Calculating this inverse requires computing eigenvalue decomposition, and inverting the non-zero eigenvalues. In the following special cases, the solution in (19) can be written in simplified forms:

a) $N_t \leq \min(N_r, K)$: In this case, $\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k$ is full-rank, then $\mathbf{t}_k^\star$ can be written as

$$\mathbf{t}_k^\star = \left(\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k\right)^{-1} \mathbf{H}_k^H\mathbf{V}^H\mathbf{d}_k, \qquad \forall k. \tag{20}$$

b) $K \leq \min(N_r, N_t)$: Here, $\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k$ is rank deficient, and (19) can be written as

$$\mathbf{t}_k^\star = \mathbf{H}_k^H\mathbf{V}^H\left(\mathbf{V}\mathbf{H}_k\mathbf{H}_k^H\mathbf{V}^H\right)^{-1}\mathbf{d}_k, \qquad \forall k. \tag{21}$$

2) $\lambda_k^\star > 0$: From (15), since $\left(\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k + \lambda_k\mathbf{I}\right)$ is positive definite, the optimal $\mathbf{t}_k^\star$ is

$$\mathbf{t}_k^\star = \left(\mathbf{H}_k^H\mathbf{V}^H\mathbf{V}\mathbf{H}_k + \lambda_k^\star\mathbf{I}\right)^{-1}\mathbf{V}^H\mathbf{H}_k^H\mathbf{d}_k. \tag{22}$$

To find $\lambda_k^\star$ in this case, we first note from (18) that at the optimality, the power constraint must hold with equality, i.e., $\|\mathbf{t}_k^\star\|_2^2 = p_{\text{max}}$. Then, we have the following propositions on the properties of $\lambda_k^\star$. We omit the proofs due to space limitation.

**Proposition 1.** *The power constraint function $\|\mathbf{t}_k^\star\|_2^2 - p_{max}$ is a monotonically decreasing function of $\lambda_k$.*

**Proposition 2.** *If $\lambda_k^\star > 0$, then it lies in the interval* $(0, \sqrt[4]{N_t}\,\|\mathbf{V}\mathbf{H}_k\|_F\,\|\mathbf{W}_{m_k}\mathbf{H}_k\|_F]$.

Using the above properties, we can find $\lambda_k^\star$ efficiently using bisection search for the $\|\mathbf{t}_k^\star\|_2^2$ to be equal to $p_{\text{max}}$ within the interval in Proposition 2.

### C. Solution to the Joint Optimization Problem

We have optimally solved problem (11) and problem (13), yielding closed-form and semi-closed form solutions in (12) and (19)-(22), respectively. We employ alternating optimization to find a solution to problem (10). Since we have two convex subproblems, this alternating optimization approach is guaranteed to converge to a partial optimal solution of the joint optimization in (10).

## IV. Simulation Results

We conduct simulation to study the performance of the proposed WNV method. We consider a cell coverage area of radius 500 m with a BS at the center. Unless otherwise specified, we set the number of SPs to $M = 4$ as default. Each SP $m$ has $K_m = \frac{K}{M}$ users. We model the channel from user $k$ to each BS as a Rayleigh fading channel given by $\mathbf{H}_k = \beta_k^{1/2}\mathbf{G}_k$. Here, $\beta_k$ is the large-scale fading coefficient that captures both pathloss and shadowing and is given as $10\log_{10}\beta_k = -31.54 - 33\log_{10}(d_k) + Z_k$, where $d_k$ is the distance from user $k$ to the BS, and $Z_k \sim \mathcal{CN}(0, \sigma_z^2)$ is the shadowing variable with $\sigma_z = 8$ dB. Matrix $\mathbf{G}_k \in \mathbb{C}^{N_r \times N_t}$ is a Gaussian random matrix of zero mean and unit variance representing the small-scale fading. The users utilize a bandwidth of $B = 1$ MHz for transmission with a power budget of $p_{\text{max}} = 27$ dBm for each user. The noise power spectral density is $N_0 = -174$ dBm/Hz, and we set the noise figure to $N_F = 2$ dB.

As an example of the virtualization requirements, we assume that the SPs set their demands with zero-forcing (ZF) receive beamforming and eigen-beamforming for their users' transmit beamforming vectors. That is, SP $m$'s demand is given by the receive beamforming matrix $\mathbf{W}_m = (\mathbf{F}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{F}_m)^{-1}\mathbf{F}_m^H\mathbf{H}_m^H$ and the transmit beamforming vector $\mathbf{f}_{m,k}$ being a scaled version of the right singular vector that corresponds to the largest singular value of $\mathbf{H}_k$, such that $\|\mathbf{f}_{m,k}\|_2^2 = p_{\text{max}}$. With this demand, a main performance metric is the average per-user rate normalized by the system bandwidth, which is given by $R = \frac{1}{B}\frac{1}{K}\sum_{k=1}^{K} B_k \log_2(1 + \text{SINR}_k)$, where $B_k$ is the bandwidth for user $k$, and $\text{SINR}_k = \frac{\left|\mathbf{v}_k^T\mathbf{H}_k\mathbf{t}_k\right|^2}{\sum_{j\in\mathcal{B}_k}\left|\mathbf{v}_k^T\mathbf{H}_j\mathbf{t}_j\right|^2 + \sigma_n^2\|\mathbf{v}_k\|_2^2}$, where $\mathbf{v}_k$ is the receive beamforming vector for user $k$, and $\mathcal{B}_k$ denotes the set of users that share the same frequency resource with user $k$. We initialize $\{\mathbf{t}_k\}^{(0)}$ with the eigen-beamforming vectors.

We compare the performance of our WNV approach with two other methods. 1) **Non-virtualized**: A fully non-virtualized approach, where the InP uses full channel bandwidth to simultaneously serve all users with ZF receive beamforming and eigen-beamforming for user transmit beamforming. 2) **FD-WNV**: An alternative WNV method, where service isolation is performed by allocating different frequency bands to different SPs and dividing the bandwidth $B$ equally among them. In FD-WNV, each SP uses ZF beamforming for the receiver and eigen-beamforming for the transmitters. In the following results, our WNV method is referred to as "Proposed".

Fig. 1 presents the normalized deviation between the InP's supply and the SPs' demands by the proposed method, defined as $\mathbb{E}\{\left\|\hat{\mathbf{x}}^{\text{actual}} - \hat{\mathbf{x}}^{\text{desired}}\right\|_2^2\}/\mathbb{E}\{\left\|\hat{\mathbf{x}}^{\text{desired}}\right\|_2^2\}$. This figure gives an indication on how well the proposed approach fulfills its main goal, i.e., service isolation. We observe that with a practical number of antennas at the transmitters and the receiver, the proposed method can keep the deviation small. Recall that the SPs' demands correspond to an idealized setting where there is no inter-SP interference, as if each SP owned a

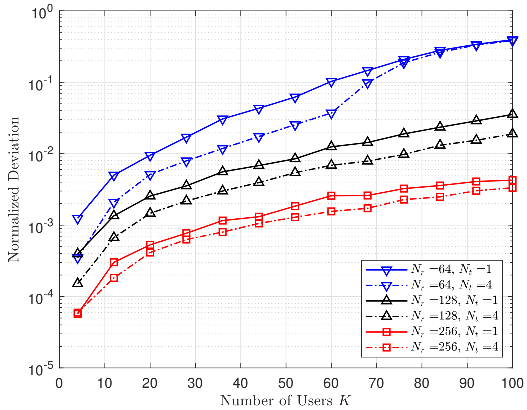


Fig. 1. Normalized deviation for various numbers of users and antennas.

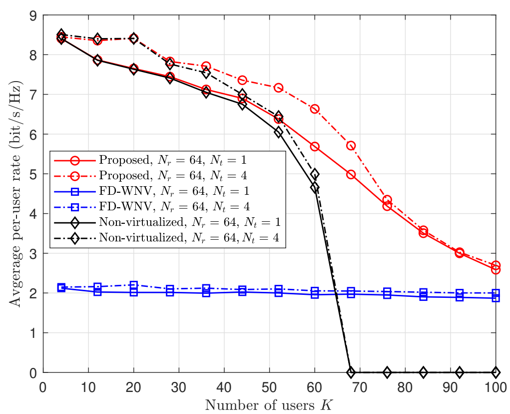


Fig. 2. Average per-user rate vs. $K$ and $N_t$.

separate copy of the network infrastructure. This suggests that, through proper transmit and receive beamforming design, there is an opportunity to substantially increase system efficiency by keeping the deviation from the SPs' demands small, i.e., implicitly managing inter-SP interference. This observation is further confirmed in terms of the average per-user rate in the results below.

Fig. 2 shows the average per-user rate of all three approaches versus the number of users $K$ for various numbers of transmit antennas $N_t$ and $N_r = 64$. As expected, we see a monotonically decreasing per-user rate in all systems. This figure shows a clear gap between the performance of FD-WNV and our proposed approach. Although the bandwidth separation in FD-WNV guarantees no inter-SP interference, the smaller bandwidth allocated to each SP causes a significant drop in the users' rates. Furthermore, the proposed method outperforms even the non-virtualized method over a wide range of $K$ values. This is clear in Fig. 2 when $K \in [40, 64]$. Note that, unlike the non-virtualized method, our method performs virtualization.

## V. Conclusions

This paper addresses the virtualization of multiple SPs at the physical layer of an uplink wireless network by utilizing the spatial dimensions offered by multiple antennas at both the BS and the user devices. We have formulated a joint optimization problem of user transmit beamforming and BS receive beamforming to minimize the expected deviation between the virtual demand and the actual supply. We have employed an alternating optimization approach for the joint optimization and have developed closed-form and semi-closed-form solutions for the transmit and receive beamforming optimization subproblems. In our examples with 4 SPs and typical configurations of antennas and users, the proposed virtualization method achieves user data rates that are 3 to 4 times higher than those of traditional virtualization approaches that rely on strict resource separation among SPs. Furthermore, the proposed method achieves data rates that are comparable to or higher than conventional transmit-receive beamforming methods in the non-virtualized system.

## References


[1] X. Wang, P. Krishnamurthy, and D. Tipper, "Wireless network virtualization," in *Proc. IEEE Int. Conf. Comput., Netw. and Commun. (ICNC)*, 2013, pp. 818–822.

[2] M. Richart, J. Baliosian, J. Serrat, and J.-L. Gorricho, "Resource slicing in virtual wireless networks: A survey," *Trans. Netw. Serv. Manag.*, vol. 13, no. 3, pp. 462–476, 2016.

[3] M. I. Kamel, L. B. Le, and A. Girard, "LTE wireless network virtualization: Dynamic slicing via flexible scheduling," in *Proc. IEEE Veh. Technol. Conf. (VTC-Fall)*, 2014.

[4] V. Jumba, S. Parsaeefard, M. Derakhshani, and T. Le-Ngoc, "Resource provisioning in wireless virtualized networks via massive-MIMO," *IEEE Wirel. Commun. Lett.*, vol. 4, no. 3, pp. 237–240, 2015.

[5] S. Parsaeefard, R. Dawadi, M. Derakhshani, and T. Le-Ngoc, "Joint user-association and resource-allocation in virtualized wireless networks," *IEEE Access*, vol. 4, pp. 2738–2750, 2016.

[6] Z. Chang, Z. Han, and T. Ristaniemi, "Energy efficient optimization for wireless virtualized small cell networks with large-scale multiple antenna," *IEEE Trans. Commun.*, vol. 65, no. 4, pp. 1696–1707, 2017.

[7] K. Zhu and E. Hossain, "Virtualization of 5G cellular networks as a hierarchical combinatorial auction," *IEEE Trans. Mob. Comput.*, vol. 15, no. 10, pp. 2640–2654, 2016.

[8] G. Liu, F. R. Yu, H. Ji, and V. C. M. Leung, "Distributed resource allocation in virtualized full-duplex relaying networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 10, pp. 8444–8460, 2016.

[9] S. Parsaeefard, R. Dawadi, M. Derakhshani, T. Le-Ngoc, and M. Baghani, "Dynamic resource allocation for virtualized wireless networks in massive-MIMO-aided and fronthaul-limited C-RAN," *IEEE Trans. Veh. Technol.*, vol. 66, no. 10, pp. 9512–9520, 2017.

[10] Y. Liu, M. Derakhshani, S. Parsaeefard, S. Lambotharan, and K.-K. Wong, "Antenna allocation and pricing in virtualized massive MIMO networks via stackelberg game," *IEEE Trans. Commun.*, vol. 66, no. 11, pp. 5220–5234, 2018.

[11] D. Tweed, S. Parsaeefard, M. Derakhshani, and T. Le-Ngoc, "Dynamic resource allocation for MC-NOMA VWNs with imperfect SIC," in *Proc. IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, 2017.

[12] D. Tweed and T. Le-Ngoc, "Dynamic resource allocation for uplink MIMO NOMA VWN with imperfect SIC," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2018.

[13] C. Liu, Y. Tao, and S. Xing, "Interference management in NOMA-enabled virtualized wireless networks," *Wireless Networks*, vol. 28, pp. 1457 – 1474, 2022.

[14] M. Soltanizadeh, B. Liang, G. Boudreau, and S. H. Seyedmehdi, "Power minimization in wireless network virtualization with massive MIMO," in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, 2018.

[15] J. Wang, M. Dong, B. Liang, G. Boudreau, and H. Abou-zeid, "Distributed coordinated precoding for MIMO cellular network virtualization," *IEEE Trans. Wirel. Commun.*, vol. 21, no. 1, pp. 106–120, 2022.

[16] A. F. Almehdhar, B. Liang, M. Dong, Y. Ahmed, and G. Boudreau, "Beamforming and power control for wireless network virtualization in uplink MIMO systems," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2024.

[17] A. F. Almehdhar, M. Dong, B. Liang, Y. Ahmed, and G. Boudreau, "Wireless network virtualization in uplink coordinated multi-cell mimo systems," in *Proc. IEEE Conf. Comput. Commu. (INFOCOM)*, 2025.