Power Allocation for Underlay Device-to-Device Communication over Multiple Channels

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Abstract*—In underlay device-to-device (D2D) communication, a D2D pair reuses the cellular spectrum, causing interference to exiting cellular users. The achieved D2D rate and the added interference to cellular users need to be jointly considered for optimal resource and power allocations. Unlike most existing work which only consider the simplified scenario of assigning each D2D pair a single channel or resource block (RB), we consider multiple RBs from different cellular users can be assigned to each D2D pair. We formulate the problem of optimal power allocation over multiple RBs at the D2D transmitter to maximize the sum-rate of D2D and cellular users, under the D2D transmitter power constraint and minimum signal-to-noise-and-interference ratio (SINR) requirement at each reused RB for all affected cellular users in all cells. To further lower the required overhead in a practical setting, we consider a second optimization problem for power allocation solution to maximize the D2D rate under the same constraints as the sum-rate maximization problem. We obtain the asymptotic power solution for the sum-rate maximization and the semi-closed-form optimal power solution for the D2D rate maximization. Our proposed optimization solutions are applicable to both uplink and downlink cellular spectrum sharing as well as to mutli-cell with multiple D2D pairs scenarios. Our simulation studies demonstrates the effectiveness of the two proposed methods for both uplink and downlink resource sharing, and further shed light into how the maximum rate is impacted by the system parameters such as available D2D transmit power, number of D2D pairs, minimum SINR requirements, and the cell size.

***Index Terms*—Device-to-Device communication, resource allocation, power allocation.**

I. Introduction

The rapid growth of proximity-based local services and applications has led to the development of device-to-device (D2D) communication as a new feature for the next-generation cellular networks, such as Long Term Evolution (LTE)-Advanced and the planned fifth generation (5G) evolution. D2D communication enables direct communication between two nearby devices, or even among a set of nearby devices, in the licensed cellular bandwidth without having the payload traverse through an evolved Node B (eNB) or the back-haul network. It is envisioned to provide many technical benefits such as improved resource utilization, spectral efficiency, and energy efficiency of the cellular network [2]–[4]. In addition, due to its local communication nature, D2D communication can be supported with less cost as compared with that for conventional downlink/uplink cellular communication. A fast growing number of context-aware services and applications, such as social-network applications, require D2D communication to support location discovery and communication with neighboring devices. Also, D2D communication has been proposed for the LTE-based public safety networks, in the event of cellular coverage failure, to meet more stringent reliability and security requirements [5]. Additionally, D2D communication is necessary when the cellular transmission is not accessible [4].

To establish D2D communication, there are different challenges that need to be addressed carefully (a survey on this topic can be found in [6]). Finding nearby users (peer discovery) and mode selection (between cellular and D2D modes) are necessary steps to setup a D2D session which may lead to the need to design new procedures. However, one fundamental issue for D2D communication is how to effectively share spectrum resources between D2D and cellular users, as sharing cellular recourses may cause intra-cell and inter-cell interference. Spectrum sharing for D2D communication can be either overlaid or underlaid [6]. For the former, orthogonal resources are allocated for cellular and D2D communications. While this approach can mitigate intra-cell interference, it may lead to inefficient use of spectrum resources, resulting in overall rate loss. In contrast, for underlay D2D communication, D2D users reuse spectrum assigned to cellular users. This approach has attracted more attention due to its potential to achieve higher spectrum efficiency [6]. For a D2D underlaying cellular network, to achieve potential data rate improvement with D2D communication, careful interference control as well as efficient resource and power allocations are the main design challenges. In this work, we consider underlaying D2D in a cellular system and focus on resource and power allocation for maximizing data rate of D2D and cellular users.

There are many methods proposed in the literature for interference control and resource allocation in underlay D2D systems. For example, power backoff approaches were studied in [7]–[9], and interference cancellation were proposed in [10]. Graph-based [11], [12] and game theoretic [13]–

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[17] approaches were also considered. None of them directly address the objective of sum-rate maximization. In contrast, power optimization methods were proposed in [18]–[22] to maximize the D2D rate, D2D-cellular sum rate, or power-rate efficiency. The authors of [18] gave an optimal power allocation solution for D2D users underlaying cellular users in downlink transmission without imposing any constraint on the D2D power. The authors of [19] provided performance bounds in the maximization of power efficiency under signal-to-noise ratio (SNR) constraints. In [20], the authors solved the sum-rate maximization problem under minimum SINR requirements and worst-case inter-cell interference limit in neighboring cells. In [21] and [22], sub-optimal power allocation solutions for D2D users in uplink transmission were proposed, which divide the original problem into several sub-problems to be solved separately. However, these methods only consider the overly simplified scenario where each D2D node accesses a *single* channel at a time. In reality, each user has access to *multiple* resource blocks (RBs) in an LTE network. The proposed methods in [18]–[22] cannot be directly applied to this multi-channel scenario, due to the difficulties arising from the non-convex objectives and the sum-power constraint over all channels. The authors of [23] considered the problem of resource allocation and mode selection for D2D users over multiple RBs in a single cell environment, in which a maximum transmit power limit on each RB is assumed, but there is no total power constraint considered for power allocation across multiple RBs.

In this paper, we consider a cellular system with underlay D2D communication, where the D2D users have access to multiple channels (*i.e.,* RBs). We first consider a new D2D pair arriving at the main cell of interest, and aim at optimizing D2D transmitter power allocation for cell sum-rate maximization and the D2D rate maximization. We focus on a practical scenario where the arrival of a new D2D pair does not alter the pre-existing spectrum and power allocation of other users. Besides the power constraint at the D2D transmitter, our optimization framework imposes minimum signal-to-interference-to-noise ratio (SINR) guarantees for cellular users over those RBs that the D2D pair reuses in both the main cell and neighboring cells. Our optimization framework is applicable to both uplink and downlink transmission spectrum sharing. With a given D2D resource assignment, we obtain an asymptotically optimal power allocation over each assigned RB at the D2D transmitter for cell sum-rate maximization, as well as the optimal power allocation in semi-closed-form for D2D rate maximization. The asymptotically optimal power allocation is obtained using a convex approximation of the non-convex sum-rate objective function. Our solutions are obtained by partitioning the interference scenario into high-interference and moderate-interference cases. We then show that our proposed power allocation solutions can be applied to a multi-cell network with multiple D2D pairs requesting resource in each cell. Our simulation studies in a multi-cell multi-D2D scenario for both uplink and downlink resource sharing provide insight into the effect of the transmit power of D2D and cellular users, minimum required SINR, the cell size, and the number of D2D pairs on the maximum achievable rate. The contributions of our proposed methods are as follows:

- We solve the resource allocation problem for a D2D pair without changing the existing resource allocation for other users. Therefore, the proposed D2D solutions are substantially simpler in terms of computational complexity than an optimal solution that considers joint resource allocation for the D2D pair and the other users in the cell. Yet, due to the localized and low-power nature of D2D communication, the performance of the other users is not substantially affected. Our power allocation solution provides the minimum SINR guarantees to all affected cellular users in the cell of interest and neighboring cells.
- Our proposed power allocation solution for the D2D and cellular sum-rate maximization is asymptotically optimal as SINR increases, while our proposed solution for the D2D rate maximization is optimal.
- Our proposed algorithms for power allocation have low computation complexity for implementation. The optimum power allocation for each assigned RB is obtained in semi-closed-form. The algorithms can be easily implemented either by at the eNB scheduler, or by each D2D pair in a distributed manner, due to its simplicity.
- Finally, we extend our proposed algorithms to the multi-cell network with multiple D2D pairs requesting resource per cell. Under the assumption of orthogonal RB assignments among D2D pairs, we show that our proposed algorithms can be applied to the power allocation of multiple D2D pairs in each cell.

A preliminary version of this work was presented in [1], in which only the sum-rate maximization problem for power allocation optimization was considered. In this journal version, we include the following new contributions: i) The system model is more general to allow the D2D pair to reuse RBs from multiple CUs, instead of only from a single CU. ii) We also consider the problem of optimizing power allocation for D2D-rate maximization. iii) We extend the proposed power allocation solutions to the multi-cell scenario and analyze the performance. iv) We provide extensive simulation comparisons between the two power allocation solutions for sum-rate and D2D-rate maximization problems. In addition, we provide the simulation study on the efficiency of our proposed solutions through convex approximation for the sum-rate maximization by comparing it with other known convexification methods in literature [24], [25].

The paper is organized as follows. Section II presents the system model of the cellular network with underlay D2D communication and formulates the resource allocation optimization problem. Section III provides our proposed methods for solving the power allocation problems. Section IV presents the simulation results, followed by conclusion in Section V.

Notations: Throughout the paper, we use a, $\mathbf{a}$, and $\mathbf{A}$ to represent a scalar, a vector, and a matrix, respectively. The notation $\mathbf{a} \succcurlyeq 0$ means all entries of vector $\mathbf{a}$ are nonnegative. We define $\left[x\right]_a^b \triangleq \max\{a, \min\{x, b\}\}$. The main symbols used in this paper are summarized in Table I.

TABLE I: Notation Definition

N	cellular user in each cell
C	set of all available RBs in the cell
C_l	set of allocated RBs to the lth D2D user
S_j	set of neighboring cellular users accessing to RB j
$p^{\mathrm{D}}_{\mathrm{t},j}$	D2D transmitted power over RB j
$p^{\mathrm{C}}_{\mathrm{r},j}$	cellular user received power over RB j
$p^{(k)}_{\mathrm{r},j}$	neighboring cellular user received power over the RB j (for $k \in S_j$)
I_j	interference at D2D receiver over RB j
$p^{\mathrm{D}}_{\mathrm{t},j}\lvert h^{\mathrm{I}}_j\rvert^2$	interference to cellular user over the RB j from the new D2D pair
$p^{\mathrm{D}}_{\mathrm{t},j}\lvert h^{\mathrm{I},(k)}_j\rvert^2$	interference to neighboring cellular user k over the RB j (for $k \in S_j$) from the new D2D pair
h_j	channel coefficient between the D2D transmitter and receiver over RB j
I^0_j	interference to cellular user over RB j before the new D2D pair enter
$I^{0,(k)}_j$	interference to neighboring cellular user over RB j (for $k \in S_j$) before the new D2D pair enter
$P^{\mathrm{D}}_{\max}$	maximum power for the D2D transmitter
$\zeta^{\mathrm{intra}}_{j,\min}$	cellular user minimum required SINR over RB j
$\zeta^{(k)}_{j,\min}$	neighboring cellular user minimum required SINR over RB j (for $k \in S_j$)
σ^2	noise power over each RB

II. System Model and Problem Definition

A. System Model

We consider a cellular system with underlay D2D communication. A cell of interest consists of N cellular users and multiple D2D users, where the D2D users reuse the spectrum resources (*i.e.,* RBs) assigned to the cellular users. Each cellular user in the cell may be assigned multiple RBs, which is typical in practical cellular networks, such as LTE networks. The assigned RBs are orthogonal among the cellular users within the cell, and there is no intra-cell interference among them. We assume that an idle D2D pair registers at the cell of interest and requests access to RBs for D2D communication. Due to the localized and low-power transmission of D2D users, the resource planning (*i.e.,* RB assignment and power control) of existing cellular users and D2D pairs in the network is assumed unchanged. We assume the D2D pair can reuse multiple RBs assigned to different cellular users in the cell, and RB assignments among D2D pairs are orthogonal. Fig. 1 shows an example of a cellular network with underlay D2D users sharing uplink resources with cellular users.

Let C denote the set of all currently available RBs in the cell (not already reused by existing D2D pairs). We denote the new D2D pair as the lth D2D pair. It requests N_l RBs of the available RBs for reuse. Let C_l denote the set of RBs allocated to the lth D2D pair by the central coordinator (*e.g.,* eNB). Note that $|C_l| \le N_l$, due to resource availability and other quality of service constraints. Consider uplink resource sharing as shown in Fig. 1. For each RB $j \in C_l$, let h_j denote the D2D channel coefficient over RB j , and h^{I}_j the channel coefficient between the D2D transmitter and the evolved Node B (eNB) of this cell over RB j. Let $p^{\mathrm{D}}_{\mathrm{t},j}$ denote the transmit power at the D2D transmitter over RB j, and $p^{\mathrm{C}}_{\mathrm{r},j}$ the received power at the eNB from the cellular user who is assigned RB j[1]. For each RB $j \in C_l$, let S_j denote the set of all cellular users in the neighboring cells using RB j. For $k \in S_j$, let $h^{\mathrm{I},(k)}_j$ denote the channel coefficient between the D2D transmitter and the eNB of neighboring cellular user k over RB j. Let $p^{(k)}_{\mathrm{r},j}$ denote the received power from user k at its eNB over RB j.

The interference scenarios for uplink resource sharing are shown in Fig. 1. In the cell of interest, a cellular user suffers intra-cell interference from D2D users reusing its assigned RBs, and vise versa. In addition, they suffer inter-cell interference due to frequency reuse at neighboring cells. For each RB $j \in C_l$, let I_j denote the interference power at the D2D receiver over RB j (including both inter-cell and intra-cell interference). Let I^C_j and $I^{C,(k)}_j$ denote the inter-cell interference over RB j at the eNB in the cell of interest and at the eNB of neighboring cellular user k, respectively, in the absence of the lth D2D pair.

Based on the above, for each RB $j \in C_l$, the received SINR at the D2D receiver, for the cellular user using RB j in the cell of interest, and for each cellular user $k \in S_j$ in a neighboring cell are respectively given by

$$\mathrm{SINR}^{\mathrm{D}}_j = \frac{p^{\mathrm{D}}_{\mathrm{t},j}|h_j|^2}{I_j + \sigma^2} \tag{1}$$

$$\mathrm{SINR}^{\mathrm{C}}_j = \frac{p^{\mathrm{C}}_{\mathrm{r},j}}{p^{\mathrm{D}}_{\mathrm{t},j}|h^{\mathrm{I}}_j|^2 + I^C_j + \sigma^2} \tag{2}$$

$$\mathrm{SINR}^{\mathrm{C},(k)}_j = \frac{p^{(k)}_{\mathrm{r},j}}{p^{\mathrm{D}}_{\mathrm{t},j}|h^{\mathrm{I},(k)}_j|^2 + I^{C,(k)}_j + \sigma^2}, \ k \in S_j. \tag{3}$$

Note that, although so far we have focused on modeling D2D uplink resource sharing, our system model can be straightforwardly applied to the downlink scenario. In that case, the same notations can be used, with the eNBs being replaced by the corresponding cellular users, and vise versa. The same problem formulation and analysis would follow. For brevity, we skip the description of the downlink system model.

B. Problem Formulation

We study how to schedule the new D2D pair for resource sharing with existing cellular users. For spectrum resource reuse by the D2D pair, we impose the minimum SINR

[1]We assume the transmit power of each cellular user is known. To simplify the expression, it is suffices to only consider the received power at the eNB from each cellular user.

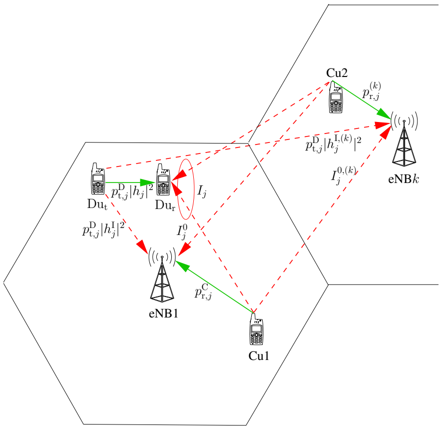


Fig. 1: A cellular network with underlay D2D users sharing uplink resources with cellular users. $\mathrm{Du_t}$ and $\mathrm{Du_r}$ denote transmit and receive nodes of a D2D pair, respectively. Cu1 and Cu2 denote cellular users. Solid and dashed lines denote desired and interfering signals, respectively.

constraints for the cellular users in the cell of interest and for the neighboring cellular user k as follows

$$\mathrm{SINR}_j^{\mathrm{C}} \geq \zeta_{j,\min}^{\mathrm{intra}}, \ j \in C_l \tag{4}$$

$$\mathrm{SINR}_j^{\mathrm{C},(k)} \geq \zeta_{j,\min}^{(k)}, \ j \in C_l, \ k \in S_j. \tag{5}$$

where $\zeta_{j,\min}^{\mathrm{intra}}$ and $\zeta_{j,\min}^{(k)}$ are the respective minimum SINR requirements. These two constraints set minimum quality of service guarantee to the cellular users for D2D resource sharing.

The transmit power at the D2D transmitter is constrained by the maximum transmit power as follows

$$\sum_{j \in C_l} p_{\mathrm{t},j}^{\mathrm{D}} \leq P_{\max}^{\mathrm{D}}. \tag{6}$$

Define $\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \triangleq [p_{\mathrm{t},j_1}^{\mathrm{D}}, \cdots, p_{\mathrm{t},j_{|C_l|}}^{\mathrm{D}}]^T$, where $j_i \in C_l$ with $j_1 < j_2 < \cdots < j_{|C_l|}$.

Our objective is to optimize the power allocation over each RB within C_l at the D2D transmitter for rate maximization. We consider the following two rate objectives to formulate the power allocation problem:

1) Sum-Rate Maximization: We consider the maximization of the sum rate of the lth D2D user and cellular users in the cell over RBs in C_l, while meeting the minimum SINR requirements for cellular users, given by

$$\mathcal{S}0: \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \ \sum_{j \in C_l} \log(1+\mathrm{SINR}_j^{\mathrm{C}}) + \log(1+\mathrm{SINR}_j^{\mathrm{D}})$$
$$\text{subject to} \quad (4), (5), (6).$$

2) D2D-Rate Maximization: We consider maximizing the rate of the lth D2D pair only, under the minimum SINR requirements. Given C_l, we have

$$\mathcal{D}0: \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \ \sum_{j \in C_l} \log(1+\mathrm{SINR}_j^{\mathrm{D}})$$
$$\text{subject to} \quad (4), (5), (6).$$

Using (2) and (3), SINR constraints (4) and (5) can be combined and transformed into the following equivalent constraint in terms of $p_{\mathrm{t},j}^{\mathrm{D}}$

$$p_{\mathrm{t},j}^{\mathrm{D}} \leq \eta_j, \ j \in C_l \tag{7}$$

where

$$\eta_j \triangleq \min\left\{ \frac{p_{\mathrm{r},j}^{\mathrm{C}}/\zeta_{j,\min}^{\mathrm{intra}} - (I_j^C + \sigma^2)}{|h_j^{\mathrm{I}}|^2}, \right. \left. \frac{p_{\mathrm{r},j}^{(k)}/\zeta_{j,\min}^{(k)} - (I_j^{C,(k)} + \sigma^2)}{|h_j^{\mathrm{I},(k)}|^2}, \forall\, k \in S_j \right\}. \tag{8}$$

Thus, problems $\mathcal{S}0$ and $\mathcal{D}0$ can be re-written as follows

$$\mathcal{S}1: \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \ \sum_{j \in C_l} \log(1+\mathrm{SINR}_j^{\mathrm{C}}) + \log(1+\mathrm{SINR}_j^{\mathrm{D}})$$
$$\text{subject to} \quad (6), (7).$$

$$\mathcal{D}1: \quad \max_{\mathbf{p}_{\mathrm{t}}^{\mathrm{D}} \succcurlyeq \mathbf{0}} \ \sum_{j \in C_l} \log(1+\mathrm{SINR}_j^{\mathrm{D}})$$
$$\text{subject to} \quad (6), (7).$$

III. Power Allocation for D2D Resource Sharing

In the following, we first propose an asymptotically optimal solution for the sum-rate maximization problem $\mathcal{S}1$. Next, to decrease the complexity and the required amount of feedback involved in solving $\mathcal{S}1$, we consider D2D rate maximization problem $\mathcal{D}1$ and propose an algorithm to solve it. In the next section, we show that the proposed algorithms can be efficiently applied to a multi-cell multi-D2D scenario, where there are a number of neighboring cells each with a newly allocated D2D pair, and each cell optimizes the D2D resource sharing to maximize its own achievable rate.

A. D2D Admissibility

We first need to determine whether the D2D pair is admissible, *i.e.,* the feasibility of problems $\mathcal{S}1$ and $\mathcal{D}1$. That is, there exists a non-empty set C_l with RBs satisfying constraints (4)-(6). From SINR constraints (4) and (5), we see that C_l is non-empty if and only if there exist $j \in C$, such that

$$\frac{p_{\mathrm{r},j}^{\mathrm{C}}}{\sigma^2 + I_j^C} \geq \zeta_{j,\min}^{\mathrm{intra}} \tag{9}$$

$$\frac{p_{\mathrm{r},j}^{(k)}}{\sigma^2 + I_j^{C,(k)}} \geq \zeta_{j,\min}^{(k)}, \ k \in S_j. \tag{10}$$

By (7), the above necessary and sufficient condition is equivalent to

$$\exists j \in C, \ \text{such that} \ \eta_j \geq 0. \tag{11}$$

If there is no RB available in the cell satisfying condition (11), then the lth D2D pair is not admissible. The set C_l of RBs is given by the central coordinator, where each RB $j \in C_l$ satisfies condition (11). Its exact determination depends on different criteria and methods. In this work, we assume it is given and focus on the D2D power allocation optimization. Note that the central coordinating function resides in the serving eNB of a given user.

B. Sum-Rate Maximization

Note that the sum-rate of cellular users over RBs in C_l, prior to the D2D pair entering the system, is given by $\sum_{j\in C_l}\log(1+\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{\sigma^2+I^C_j})$. It is independent of D2D transmitter power allocation. Thus, the sum-rate maximization problem $\mathcal{S}1$ is equivalent to the problem of maximizing the sum-rate *improvement* due to the addition of the new D2D pair, given by

$$\mathcal{S}2: \quad \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}\succcurlyeq\mathbf{0}} \quad \sum_{j\in C_l}\Big[\log(1+\mathrm{SINR}^{\mathrm{C}}_j)+\log(1+\mathrm{SINR}^{\mathrm{D}}_j) - \log\left(1+\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{I^C_j+\sigma^2_j}\right)\Big]$$

$$\text{subject to} \quad (6),(7).$$

Let $U(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}})$ denote the sum-rate objective function in $\mathcal{S}2$. Substituting the expression of $\mathrm{SINR}^{\mathrm{C}}_j$ and $\mathrm{SINR}^{\mathrm{D}}_j$ in (2) and (3), respectively, into $U(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}})$, we have

$$U(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}})=\sum_{j\in C_l}\Big[\log\left(1+\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{p^{\mathrm{D}}_{\mathrm{t},j}|h^{\mathrm{I}}_j|^2+I^C_j+\sigma^2}\right) - \log\left(1+\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{I^C_j+\sigma^2}\right)+\log\left(1+\frac{|h_j|^2p^{\mathrm{D}}_{\mathrm{t},j}}{I_j+\sigma^2}\right)\Big]. \quad (12)$$

Since $U(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}})$ is not convex with respect to $p^{\mathrm{D}}_{\mathrm{t},j}$, $\mathcal{S}2$ is non-convex. In the following, we first convexify the problem.

1) Convexification of $\mathcal{S}2$: Typically, only those RBs over which the cellular users have a sufficiently high SINR condition are allocated to the D2D user. After D2D reuse, the SINR of the cellular user over such an RB is still relatively high. Therefore, we assume the minimum SINR requirement $\zeta^{\mathrm{intra}}_{j,\min}\gg 1$, for all $j\in C_l$. With this assumption, we can approximate $U(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}})$ in (12) as

$$U(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}})\approx\sum_{j\in C_l}\left[\log\left(\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{p^{\mathrm{D}}_{\mathrm{t},j}|h^{\mathrm{I}}_j|^2+I^C_j+\sigma^2}\right) - \log\left(\frac{p^{\mathrm{C}}_{\mathrm{r},j}}{I^C_j+\sigma^2}\right)+\log\left(1+\frac{|h_j|^2p^{\mathrm{D}}_{\mathrm{t},j}}{I_j+\sigma^2}\right)\right]$$
$$=\sum_{j\in C_l}\log\left(\frac{a_j+b_jp^{\mathrm{D}}_{\mathrm{t},j}}{a_j+c_jp^{\mathrm{D}}_{\mathrm{t},j}}\right). \quad (13)$$

where $a_j\triangleq(\sigma^2+I^C_j)(\sigma^2+I_j)$, $b_j\triangleq(\sigma^2+I^C_j)|h_j|^2$, and $c_j\triangleq(\sigma^2+I_j)|h^{\mathrm{I}}_j|^2$. Thus, the sum-rate improvement maximization problem $\mathcal{S}2$ is approximated as follows:

$$\mathcal{S}3: \quad \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}\succcurlyeq\mathbf{0}} \quad \sum_{j\in C_l}\log\left(\frac{a_j+b_jp^{\mathrm{D}}_{\mathrm{t},j}}{a_j+c_jp^{\mathrm{D}}_{\mathrm{t},j}}\right) \quad (14)$$
$$\text{subject to} \quad (6),(7).$$

Note that, for $b_j\le c_j$, $\log(\frac{a_j+b_jp^{\mathrm{D}}_{\mathrm{t},j}}{a_j+c_jp^{\mathrm{D}}_{\mathrm{t},j}})$ is a decreasing function with respect to $p^{\mathrm{D}}_{\mathrm{t},j}$, while for $b_j>c_j$, it is a strictly increasing function. Hence, if $b_j\le c_j$, for some $j\in C_l$, we have $p^{\mathrm{D}*}_{\mathrm{t},j}=0$ at optimality, *i.e.,* the D2D pair do not use this RB. Define the subset $\tilde{C}_l\triangleq\{j: j\in C_l,\ b_j>c_j\}$. We only need to determine $p^{\mathrm{D}}_{\mathrm{t},j}$, for $j\in\tilde{C}_l$. Therefore, optimization problem $\mathcal{S}3$ is equivalent to

$$\mathcal{S}4: \quad \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}} \quad \sum_{j\in\tilde{C}_l}\log\left(\frac{a_j+b_jp^{\mathrm{D}}_{\mathrm{t},j}}{a_j+c_jp^{\mathrm{D}}_{\mathrm{t},j}}\right)$$
$$\text{subject to} \quad \sum_{j\in\tilde{C}_l}p^{\mathrm{D}}_{\mathrm{t},j}\le P^{\mathrm{D}}_{\max}, \quad (15)$$
$$0\le p^{\mathrm{D}}_{\mathrm{t},j}\le\eta_j, \quad j\in\tilde{C}_l. \quad (16)$$

Now optimization problem $\mathcal{S}4$ is convex, and we can obtain the power allocation solution for it. In the following, we obtain the power solution by dividing the problem into two interference scenarios.

2) High Interference Scenario ($\sum_{j\in\tilde{C}_l}\eta_j\le P^{\mathrm{D}}_{\max}$): For per-RB D2D power constraint (16), if $\sum_{j\in\tilde{C}_l}\eta_j\le P^{\mathrm{D}}_{\max}$, the total power constraint (15) will not be active. Since the objective function is an increasing function of $p^{\mathrm{D}}_{\mathrm{t},j}$, it follows that the optimal $p^{\mathrm{D}*}_{\mathrm{t},j}=\eta_j$, for all $j\in\tilde{C}_l$. From the above discussion, the optimal power solution for $\mathcal{S}3$ is summarized below.

Proposition 1: If $\sum_{j\in\tilde{C}_l}\eta_j\le P^{\mathrm{D}}_{\max}$, the optimal power solution for problem $\mathcal{S}3$ is given by

$$p^{\mathrm{D}*}_{\mathrm{t},j}=\begin{cases}\eta_j & \text{for } j\in\tilde{C}_l\\ 0 & \text{for } j\in C_l\backslash\tilde{C}_l.\end{cases} \quad (17)$$

Note that this case happens when the value of each η_j is relatively small, for $j\in\tilde{C}_j$. From the expression of η_j in (8), this indicates a high interference scenario where, prior to the addition of the D2D pair, per-RB SINR for the corresponding cellular user is already close to the minimum requirement ($\zeta^{\mathrm{intra}}_{j,\min}$ and/or $\zeta^{(k)}_{j,\min}$). This would limit the D2D transmit power $p^{\mathrm{D}}_{\mathrm{t},j}$ at each shared RB.

3) Moderate Interference Scenario ($\sum_{j\in\tilde{C}_l}\eta_j>P^{\mathrm{D}}_{\max}$): In contrast to the previous scenario, when the interference for the corresponding cellular users is moderate, we have $\sum_{j\in\tilde{C}_l}\eta_j>P^{\mathrm{D}}_{\max}$. Both total power constraint (15) and per-RB power constraint (16) are active. In this case, for any feasible power solution $\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}$ of $\mathcal{S}4$, we must have a non-empty subset $\tilde{C}'_l\subseteq\tilde{C}$, such that $p^{\mathrm{D}}_{\mathrm{t},j}<\eta_j$, for $j\in\tilde{C}'_l$. Furthermore, at optimality, the total power constraint (15) must be satisfied with equality. Otherwise, without violating (15) and (16), we can increase $p^{\mathrm{D}}_{\mathrm{t},j}$ for $j\in\tilde{C}'_l$ to further increase the value of the objective function in $\mathcal{S}4$, resulting in a contradiction. Thus, in this case, $\mathcal{S}4$ can be rewritten as

$$\mathcal{S}5: \quad \max_{\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}} \quad \sum_{j\in\tilde{C}_l}\log\left(\frac{a_j+b_jp^{\mathrm{D}}_{\mathrm{t},j}}{a_j+c_jp^{\mathrm{D}}_{\mathrm{t},j}}\right)$$
$$\text{subject to} \quad \sum_{j\in\tilde{C}_l}p^{\mathrm{D}}_{\mathrm{t},j}=P^{\mathrm{D}}_{\max} \text{ and } (16).$$

Since optimization problem $\mathcal{S}5$ is convex, we apply the KKT optimality condition [26] to obtain the optimal power solution. Let $\lambda>0$ denote the Lagrangian multiplier corresponding to the total power equality constraint in $\mathcal{S}5$. The optimal power solution is given in the following proposition.

Proposition 2: If $\sum_{j\in\tilde{C}_l}\eta_j > P^{\mathrm{D}}_{\max}$, the optimal power solution for problem $\mathcal{S}3$ is given by

$$p^{\mathrm{D}*}_{\mathrm{t},j} = \left[\frac{-\beta_j + \sqrt{\beta_j^2 - 4\kappa_j(1-\frac{\gamma_j}{\lambda^*})}}{2\kappa_j}\right]_0^{\eta_j}, \ \forall j \in C_l \quad (18)$$

where $\kappa_j \triangleq \frac{b_j c_j}{a_j^2}$, $\beta_j \triangleq \frac{b_j + c_j}{a_j}$, $\gamma_j \triangleq \frac{b_j - c_j}{a_j}$, and the optimal λ^* is obtained such that $\sum_{j\in\tilde{C}_l} p^{\mathrm{D}*}_{\mathrm{t},j} = P^{\mathrm{D}}_{\max}$.

Proof: The Lagrangian for $\mathcal{S}5$ is given by

$$\begin{aligned}\mathcal{L}(\mathbf{p}^{\mathrm{D}}_{\mathrm{t}}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \lambda) \triangleq & \sum_{j\in\tilde{C}_l} \log\left(\frac{a_j + b_j p^{\mathrm{D}}_{\mathrm{t},j}}{a_j + c_j p^{\mathrm{D}}_{\mathrm{t},j}}\right) \\ & + \sum_{j\in\tilde{C}_l} \lambda_{1j}(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) - \sum_{j\in\tilde{C}_l} \lambda_{2j} p^{\mathrm{D}}_{\mathrm{t},j} \\ & - \lambda\left(\sum_{j\in\tilde{C}_l} p^{\mathrm{D}}_{\mathrm{t},j} - P^{\mathrm{D}}_{\max}\right) \end{aligned} \quad (19)$$

where $\boldsymbol{\lambda}_1 \triangleq [\lambda_{11}, \lambda_{12}, \cdots, \lambda_{1|\tilde{C}_l|}]^T$, $\boldsymbol{\lambda}_2 \triangleq [\lambda_{21}, \lambda_{22}, \cdots, \lambda_{2|\tilde{C}_l|}]^T$, and λ_{1j} and λ_{2j} are the Lagrange multipliers for the per-RB lower and upper power constraints for $p^{\mathrm{D}}_{t,j}$ in (16), respectively. The KKT optimality condition [26] can be written as follows:

$$\frac{a_j b_j - a_j c_j}{(a_j + b_j p^{\mathrm{D}}_{\mathrm{t},j})(a_j + c_j p^{\mathrm{D}}_{\mathrm{t},j})} - \lambda_{1j} + \lambda_{2j} - \lambda = 0, \ j \in \tilde{C}_l \quad (20)$$

$$\sum_{j\in\tilde{C}_l} p^{\mathrm{D}}_{\mathrm{t},j} = P^{\mathrm{D}}_{\max} \quad (21)$$

$$0 \le p^{\mathrm{D}}_{\mathrm{t},j} \le \eta_j, j \in \tilde{C}_l \quad (22)$$

$$\lambda_{1j}(p^{\mathrm{D}}_{\mathrm{t},j} - \eta_j) = 0, \ j \in \tilde{C}_l \quad (23)$$

$$\lambda_{2j} p^{\mathrm{D}}_{\mathrm{t},j} = 0, \ j \in \tilde{C}_l \quad (24)$$

$$\lambda_{1j} \ge 0, \lambda_{2j} \ge 0, \ j \in \tilde{C}_l. \quad (25)$$

From the above conditions, we have three cases for $\{\lambda^*_{1j}, \lambda^*_{2j}\}$:

i) $\lambda^*_{1j} > 0$ and $\lambda^*_{2j} = 0$: From (23), we have $p^{\mathrm{D}*}_{\mathrm{t},j} = \eta_j$.

ii) $\lambda^*_{1j} = 0$ and $\lambda^*_{2j} > 0$: From (24), we have $p^{\mathrm{D}*}_{\mathrm{t},j} = 0$.

iii) $\lambda^*_{1j} = 0$ and $\lambda^*_{2j} = 0$: From (20), we have the following quadratic equation for $p^{\mathrm{D}}_{t,j}$

$$\lambda b_j c_j (p^{\mathrm{D}}_{\mathrm{t},j})^2 + \lambda a_j (b_j + c_j) p^{\mathrm{D}}_{\mathrm{t},j} + (\lambda a_j^2 - a_j b_j + a_j c_j) = 0$$

or equivalently

$$\kappa_j (p^{\mathrm{D}}_{\mathrm{t},j})^2 + \beta_j p^{\mathrm{D}}_{\mathrm{t},j} + (1 - \frac{\gamma_j}{\lambda}) = 0$$

where $\kappa_j \triangleq \frac{b_j c_j}{a_j^2} > 0$, $\beta_j \triangleq \frac{b_j + c_j}{a_j} > 0$, and $\gamma_j \triangleq \frac{b_j - c_j}{a_j} > 0$ (since $j \in \tilde{C}_l$). Since $\beta_j/\kappa_j > 0$, the only root of the above equation that can be valid for a power solution is

$$p^{\mathrm{D}*}_{\mathrm{t},j} = \frac{-\beta_j + \sqrt{\beta_j^2 - 4\kappa_j(1-\frac{\gamma_j}{\lambda})}}{2\kappa_j} \quad (26)$$

which is non-negative if and only if $0 < \lambda \le \gamma_j$.

Algorithm 1 Sum-Rate Maximization Algorithm

For all $j \in C_l$, compute
- a_j, b_j and c_j;
- κ_j, β_j and γ_j.

Set $\tilde{C}_l = \{j : j \in C_l, b_j > c_j\}$.
For all $j \in C_l \backslash \tilde{C}_l$, set $p^{\mathrm{D}*}_{\mathrm{t},j} = 0$.
if $\sum_{j\in\tilde{C}_l}\eta_j \le P^{\mathrm{D}}_{\max}$ **then**
  For all $j \in \tilde{C}_l$, set $p^{\mathrm{D}*}_{\mathrm{t},j} = \eta_j$.
else
  Obtain $p^{\mathrm{D}*}_{\mathrm{t},j}$ in (18), for $j \in \tilde{C}_l$ and use the bisection method to obtain $\lambda^* > 0$ such that $\sum_{j\in\tilde{C}_l} p^{\mathrm{D}*}_{\mathrm{t},j} = P^{\mathrm{D}}_{\max}$.
end if
Calculate $R_l = \sum_{j\in C_l}\left[\log(\frac{a_j + b_j p^{\mathrm{D}}_{\mathrm{t},j}}{a_j + c_j p^{\mathrm{D}}_{\mathrm{t},j}}) + \log(1 + \frac{p^{\mathrm{C}}_{\mathrm{r},j}}{\sigma^2 + I^C_j})\right]$.

Hence, from Cases i-iii, the optimal power allocation, for all $j \in C_l$, is given by

$$p^{\mathrm{D}*}_{\mathrm{t},j} = \left[\frac{-\beta_j + \sqrt{\beta_j^2 - 4\kappa_j(1-\frac{\gamma_j}{\lambda^*})}}{2\kappa_j}\right]_0^{\eta_j}.$$

■

The optimal λ^* should ensure $\sum_{j\in\tilde{C}_l} p^{\mathrm{D}*}_{\mathrm{t},j} = P^{\mathrm{D}}_{\max}$. We note that $p^{\mathrm{D}}_{\mathrm{t},j}$ in (26) is a decreasing function of λ. Thus, we can use the bisection method to efficiently determine λ^*. We summarize the power allocation solution for the sum-rate maximization problem in Algorithm 1. In this algorithm, the interference condition is first verified. For a high-interference condition, *i.e.,* $\sum_{j\in\tilde{C}_l}\eta_j \le P^{\mathrm{D}}_{\max}$, a closed-form solution is available as in (17). For a moderate-interference condition, *i.e.,* $\sum_{j\in\tilde{C}_l}\eta_j > P^{\mathrm{D}}_{\max}$, a semi-closed form solution is available as in (17).

C. D2D-Rate Maximization

To obtain the power allocation solution for sum-rate maximization, full information about all the interfering cellular users in the main and neighboring cells are needed, including received power level, received interference level, and channel coefficients. For the interfering cellular users in the main cell, the above mentioned information is needed to verify the feasibility of SINR constraint (4) and also to calculate the objective function of $\mathcal{S}1$. For the interfering cellular users in the neighboring cells, the information is needed to verify the feasibility of the SINR constraint (5).

To avoid the above amount of information exchange, we can interpret power constraint (7) as per-RB power constraints set by the eNB in the main cell. In other words, η_j, for all $j \in C_l$, is set by the eNB such that SINR constraints (4) and (5) are satisfied. In this case, the D2D transmitter directly receives the value of η_j, for $j \in C_l$, from the eNB of its own cell.[2] To determine η_j at the eNB, for uplink resource sharing,

[2]After the initial setting of η_j by the eNB, any change in the value of η_j can be reported using limited feedback. Note that in practice, a quantized value of η_j is sent to the D2D transmitter. It is expected that the accuracy of η_j will affect the cell throughput, with more feedback bits providing more accurate knowledge of η_j and resulting in less throughput loss. Our quantitative simulation study shows that using around 10-12 feedback bits gives negligible performance degradation.

the eNB needs to know the received powers from the cellular users of its own cell, the received inter-cell interference, and the channel between the D2D transmitter and the eNB, all of which are available at the eNB; in addition, the eNB also needs those from the the neighboring cells, which can be shared among the eNBs through backhaul. For downlink resource sharing, those quantities are obtained through feedback to the respective eNBs of the cell of interests and the neighboring cells, and are shared among eNBs through backhaul.

To further decrease the amount of information exchange, we may only consider maximizing the rate of the D2D pair under the same total power and individual power constraints as in optimization problem $\mathcal{D}1$. In fact, knowing η_j's, no additional information is needed from the interfering cellular users in the main cell to solve $\mathcal{D}1$.

Considering the rate of the lth D2D pair as the objective, in the following, we obtain the optimal power allocation for $\mathcal{D}1$. Again, the solution is given for two interference scenarios.

1) High Interference Scenario ($\sum_{j\in C_l}\eta_j \le P^{\rm D}_{\max}$): In this case, the total power constraint (6) will not be active. As the objective function is an increasing function of $p^{\rm D}_{{\rm t},j}$, it follows that the optimal $p^{\rm D*}_{{\rm t},j} = \eta_j$. We summarize the solution in the following proposition.

Proposition 3: If $\sum_{j\in C_l}\eta_j \le P^{\rm D}_{\max}$, the optimal solution for power allocation problem $\mathcal{D}1$ is given by $p^{\rm D*}_{{\rm t},j} = \eta_j$, for $j \in C_l$.

Proof: The same discussion as we had in the sum-rate maximization method is valid to prove the proposition. ■

2) Moderate Interference Scenario ($\sum_{j\in C_l}\eta_j > P_{\max}$): If $\sum_{j\in C_l}\eta_j > P^{\rm D}_{\max}$, then the total power constraint (14) must be satisfied with equality; Otherwise, it will be possible to increase some $p^{\rm D}_{{\rm t},j}$'s without violating the constraints, which leads to an increase in the objective function and results in contradiction. In this case, optimization problem $\mathcal{D}1$ can be rewritten as

$$\begin{aligned}\mathcal{D}2: \quad & \max_{\mathbf{p}^{\rm D}_{\rm t}} \quad \sum_{j\in C_l}\log\left(1+\frac{|h_j|^2 p^{\rm D}_{{\rm t},j}}{I_j+\sigma^2}\right)\\ & \text{subject to} \quad \sum_{j\in\tilde{C}_l} p^{\rm D}_{{\rm t},j} = P^{\rm D}_{\max}, \qquad (27)\\ & \qquad\qquad 0 \le p^{\rm D}_{{\rm t},j} \le \eta_j,\ j\in C_l. \qquad (28)\end{aligned}$$

Since $\mathcal{D}2$ is convex, we again can use the KKT optimality condition [26] to obtain the optimal power solution. Let $\mu > 0$ denote the Lagrangian multiplier corresponding to the total power equality condition in $\mathcal{D}2$. The power solution is provided in the following proposition.

Proposition 4: If $\sum_{j\in C_l}\eta_j > P^{\rm D}_{\max}$. The optimal power allocation at the D2D transmitter is given by

$$p^{\rm D*}_{{\rm t},j} = \left[\frac{1}{\mu^*} - \frac{\sigma^2+I_j}{|h_j|^2}\right]_0^{\eta_j}. \qquad (29)$$

where the optimal μ^* is obtained such that $\sum_{j\in C_l} p^{\rm D*}_{{\rm t},j} = P^{\rm D}_{\max}$.

Proof: Similar to the proof of Proposition 2, by applying the KKT optimality condition to optimization problem $\mathcal{D}2$, the optimal power allocation in (29) can be obtained and we omit the details of the steps. ■

Algorithm 2 D2D-Rate Maximization Algorithm

1: **if** $\sum_{j\in C_l}\eta_j \le P^{\rm D}_{\max}$ **then**
2: For all $j\in C_l$, set $p^{\rm D}_{{\rm t},j} = \eta_j$.
3: **else**
4: Using the bisection method, obtain $p^{\rm D}_{{\rm t},j}$ in (29) and $\mu > 0$ such that $\sum_{j\in C_l} p^{\rm D}_{{\rm t},j} = P^{\rm D}_{\max}$.
5: **end if**
6: Calculate $R_l = \sum_{j\in C_l}\log(1+\frac{|h_j|^2 p^{\rm D}_{{\rm t},j}}{\sigma^2+I_j})$.

Since $p^{\rm D}_{{\rm t},j}$ is a decreasing function of μ, we can use the bisection method to efficiently determine the optimal μ^*. Algorithm 2 summarizes the steps for the D2D rate maximization method. In Algorithm 2, the interference condition is first determined. For a high-interference condition, *i.e.,* $\sum_{j\in\tilde{C}_l}\eta_j \le P^{\rm D}_{\max}$, a closed-form power allocation solution is obtained. For a moderate-interference condition, *i.e.,* $\sum_{j\in\tilde{C}_l}\eta_j \ge P^{\rm D}_{\max}$, we have a semi-closed form solution as in (29).

Note that Algorithm 2 can be easily implemented distributively at each D2D pair. At the D2D transmitter, the only feedback information needed from the eNB is the value of η_j. As mentioned earlier, the value of η_j is set by the eNB based on measurements from the network, and the D2D transmitter directly receives the value of η_j from the eNB of its own cell. Thus, our power allocation algorithm for D2D-rate maximization can be implemented at each D2D pair in a distributed manner, with a minimum level of required feedback exchange.

D. Multi-D2D and Multi-Cell Scenario

So far, we have derived the D2D transmission power allocation solutions for both sum-rate and D2D-rate maximization, considering a new D2D pair requesting resource in the cell of interest. Our proposed power allocation algorithms can be applied to a more realistic multi-cell network with multiple D2D pairs requesting resource in each cell.

Note that since RB assignments among D2D pairs are orthogonal within a cell, once the RB assignments are given, our power allocation algorithms can be readily applied to the power allocation of multiple D2D pairs in a cell. Furthermore, our power allocation algorithms are per-cell based algorithms, and thus they can be implemented within each cell, as long as the required information for each algorithm (*e.g.,* inter-cell interference, channel state, or η_j) is available at the eNB of each cell (through measurement or feedback).

By implementing the same power allocation algorithm (either Algorithms 1 or 2) in all cells, we guarantee that all constraints (6) and (7), *i.e.,* total power constraint for D2D pairs and the minimum SINR requirement for each cellular user, can be satisfied for all active D2D pairs and cellular users.

IV. Simulation Results

A. Network Setup

We consider a cellular network with one-tier interference, where the cellular network consists of total seven cells, with one main cell and six neighboring cells. To determine inter-cell interference more accurately, we apply a common wrap-around method in simulation, such that each cell has six interfering neighboring cells. We set the number of cellular users in each cell to be $N = 10$. The cellular users and D2D users are randomly located in each cell with uniform distribution. We assume that there are 100 RBs available in each cell with frequency reuse factor of 1, *i.e.,* all neighboring cells have access to the same RBs. Each cellular user is randomly assigned 10 RBs that are unique from other cellular users in the cell.

We consider both downlink and uplink D2D resource sharing scenarios. For the downlink case, we assume equal power allocation among RBs at the eNB. For the uplink case, we assume each cellular user has a power control mechanism which compensates for path-loss effects.

We assume that the D2D pairs are entering the network at a random time. To model this, we use a two-state Markov model to indicate the active or inactive state of a D2D pair. The number of active D2D pairs in a cell at any time slot is denoted by N_d. The average number of active D2D pairs in a cell is given by $\bar{N}_d = E[N_d]$, which will be indicated in the figures. When a D2D pair changes its state from inactive to active, it requests resource for transmission. For each D2D pair, we set the requested number of RBs to be $N_l = 10$. At each time slot, for each cell, we check the state of each D2D pair, and form a queue of D2D pairs whose states become active (from inactive). For these D2D pairs in the queue, we determine their RB assignments (C_l) sequentially, and then their power allocation ($\mathbf{p}_t^{\mathrm{D}}$) using our proposed algorithms sequentially. As mentioned earlier, in a multi-cell environment, the power allocation in each cell affect neighboring cells and vise versa. To accurately determine inter-cell interference at the main cell, in our simulation, we apply the above procedure to determine the power allocation of D2D pairs in each of the neighboring cells sequentially, and then in the main cell. This is to ensure the inter-cell interference from neighboring cells to the main cell is as accurate as possible, and the throughput performance at the main cell is plotted.

RB Assignment to Each D2D Pair: As mentioned in Section II-A, we assign the set C_l of RBs to the lth D2D pair who requests RBs. We determine C_l as follows: 1) For RBs in C, find the subset C_l^{fes} of all feasible RBs based on the D2D admissibility condition in (9) and (10); and 2) If $|C_l^{\mathrm{fes}}| \geq N_l$, choose N_l RBs with first N_l highest values of η_j's, for $j \in C_l^{\mathrm{fes}}$. Otherwise, take the entire feasible set $C_l = C_l^{\mathrm{fes}}$. It can be shown that, by choosing these RBs, the largest possible feasible set for $\mathcal{S}1$ and $\mathcal{D}1$ can be achieved.

Unless we explicitly specify, the default values of key system parameters are listed in Table II. For the link between any two nodes, we use a simple path loss model $K_0 D^{-\alpha}$, where we have the pass loss constant $K_o = 0.01$, and the pathloss exponent $\alpha = 4$. We assume that the link between each cellular user and its BS experiences Rician fading [27] with the K-factor being set to 2, and that all inference links and the D2D links experience Rayleigh fading. Unless otherwise mentioned, we set the minimum SINR requirement for all cellular users over all RBs to $\zeta_{j,\min}^{\mathrm{intra}} = \zeta_{j,\min}^{(k)} = \zeta_{\min} = 3\mathrm{dB}$, for all $j \in C_l$ and $k \in S_j$.

TABLE II: Default values

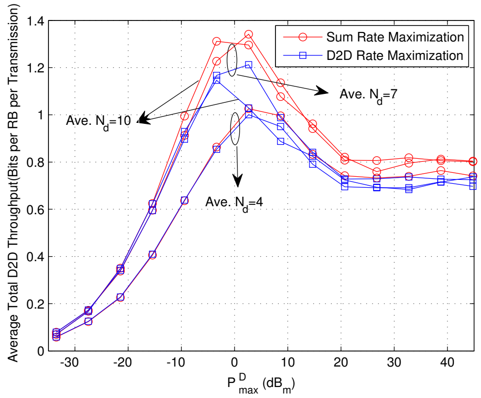
$N = 10$
Number of RBs per cellular user= 10
RB bandwidth=12x15 KHz
Cell radius $(R_c) = 100m$
D2D distance $(d) = 20m$
Ave. number of D2D pairs per cell$(\bar{N}_d) = 7$
$P_{\max}^{\mathrm{D}} = 8.5\mathrm{dB_m}$
$\zeta_{j,\min}^{\mathrm{intra}} = 3\mathrm{dB}, \forall j$
$\zeta_{j,\min}^{(k)} = 3\mathrm{dB}, \forall j$ and $\forall k \in S_j$
BTS power= $28.5\mathrm{dB_m}$(Downlink Case)
Ave. cellular SNR= $30\mathrm{dB}$(Uplink Case)
Path-loss exponent= 4
Standard deviation for shadowing= 4dB

We average the results over a large number of Monte Carlo runs of random distributed cellular users and channel realizations. We compare the two proposed power allocation algorithms (Algorithms 1 and 2) based on sum-rate maximization and D2D rate maximization, respectively. We plot throughput which is computed as average bits per RB per transmission. In Figs. 2-7, figures which are labeled by (a) show the total throughput for D2D users in the main cell, and figures which are labeled by (b) show the cell throughput of the main cell including cellular users and D2D users. In following, we discuss the effect of different system parameters on the system performance.

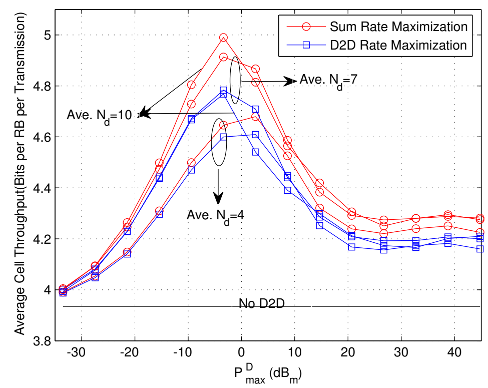
B. Maximum Power and Average Number of D2D Users

Figs. 2 and 3 show how different values of $P_{\max}^{\mathrm{D}}$ affect the throughput performance for uplink and downlink resource sharing, respectively, for different average number $\bar{N}_d$ of active D2D pairs. As it can be seen, as $P_{\max}^{\mathrm{D}}$ increases, the throughput increases to a peak value, then it starts to decrease. When $P_{\max}^{\mathrm{D}}$ is relatively low, the D2D pairs do not cause significant interference to the cellular users in the main cell or D2D/cellular users in the neighboring cells. As $P_{\max}^{D}$ increases, the D2D throughput increases while other users' SINRs still meet the minimum SINR requirements. This results in an increased overall throughput. As $P_{\max}^{\mathrm{D}}$ becomes high (for all cells), further increasing $P_{\max}^{\mathrm{D}}$ causes more intra-interference to cellular users and inter-cell interference to the D2D and cellular users, and their throughputs decrease, which contribute to the overall throughput decrease. As $P_{\max}^{\mathrm{D}}$ becomes even higher, it causes more interference, and the minimum SINR requirements of cellular users in turn limits the power used at D2D users, and the overall throughput starts to flatten out. For uplink resources sharing in Fig. 2, the peak throughput happens around $0\mathrm{dB_m}$, and for downlink resources sharing in Fig. 3, the peak throughput happens around $10\mathrm{dB_m}$.

Furthermore, in Figs. 2-3, we see the effect of different average number $\bar{N}_d$ of active D2D users in each cell on the performance. As we see, the throughput increases with $\bar{N}_d$ when $\bar{N}_d$ is small. However, the improvement diminishes when $\bar{N}_d$ becomes large and close to the number of cellular users in each cell. As we notice, under the D2D rate maximization method, both D2D throughput and cell throughput reduce as we increase $\bar{N}_d$ from 7 to 10. This is because that with

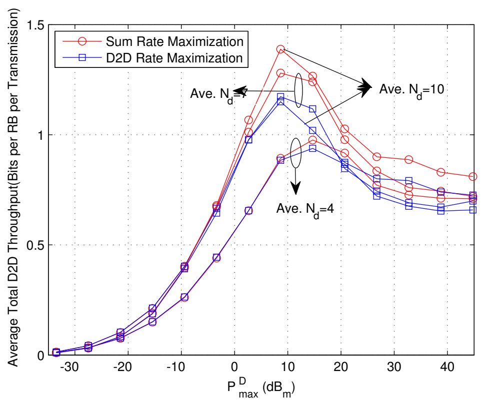


(a) Total throughput for D2D users in the main cell.

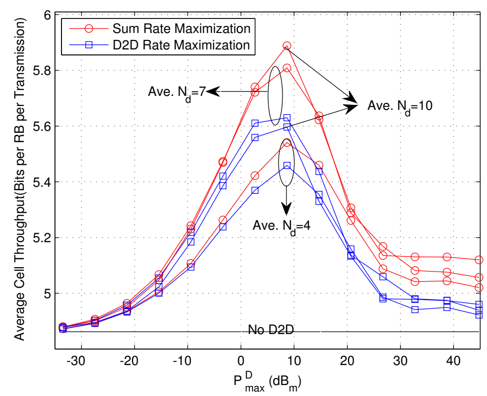


(b) Cell throughput in the main cell.

Fig. 2: The achieved throughput vs. $P_{\max}^{\mathrm{D}}$ for uplink resource sharing.



(a) Total throughput for D2D users in the main cell.



(b) Cell throughput in the main cell.

Fig. 3: The achieved throughput vs. $P_{\max}^{\mathrm{D}}$ for downlink resource sharing.

more D2D users in each cell, a higher interference level from the neighboring cells is experienced. In comparison, under the sum-rate maximization method, $\bar{N}_d$ can be higher without suffering any loss in the D2D throughput and the cell throughput.

As a comparison, we also plot the throughput for the case of no D2D resource sharing in Figs. 2(b) and 3(b). For uplink resource sharing in Fig. 2(b), at the peak value, we observe around 27% and 20% improvements in the cell throughput by the sum-rate maximization (Algorithm 1) and the D2D rate maximization (Algorithm 2), respectively. For downlink resource sharing in Fig. 3, we observe, at the peak value, around 22% and 16% improvement in cell throughput using the sum-rate maximization and the D2D rate maximization methods, respectively.

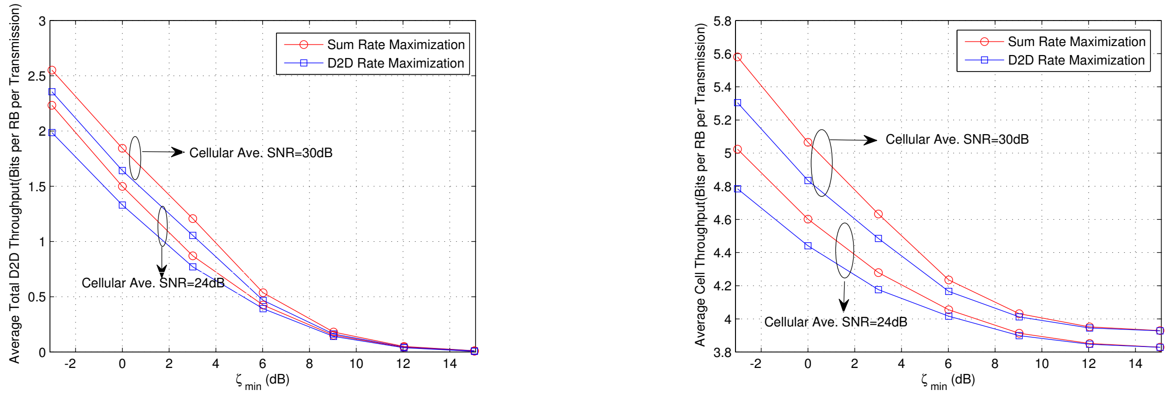
C. Minimum SINR Requirement and Average SNR for Cellular Users

Figs. 4 and 5 show the effect of the minimum SINR requirement $\zeta_{\min}$ for cellular users on the D2D and cell throughputs for uplink and downlink resource sharing, respectively. As it can be seen, when $\zeta_{\min}$ increases, it reduces η_j in (7) and thus reduces the transmit power for D2D users. As a result, the total throughput for D2D users and cell throughput are reduced.

For Fig. 4, we show the curves under different average received powers for the cellular users in uplink transmission. For Fig. 5, we show the curves under different eNB power in downlink transmission. Increasing the power level for the cellular users causes two different effects which act in different directions. First, the interference level at D2D users increases. Second, η_j in (7) increase, which allows a higher transmit power to be used for D2D users while satisfying the minimum SINR requirement for the cellular users. As we see from Fig. 4 for uplink resource sharing, with a higher power level for cellular users, the throughput increases. For downlink resource sharing in Fig. 5, although the cell throughput increases with increasing eNB power, the D2D throughput is smaller with higher eNB power for smaller value of $\zeta_{\min}$, and it becomes higher when value of $\zeta_{\min}$ increases above 0dB approximately.

D. Cell Size

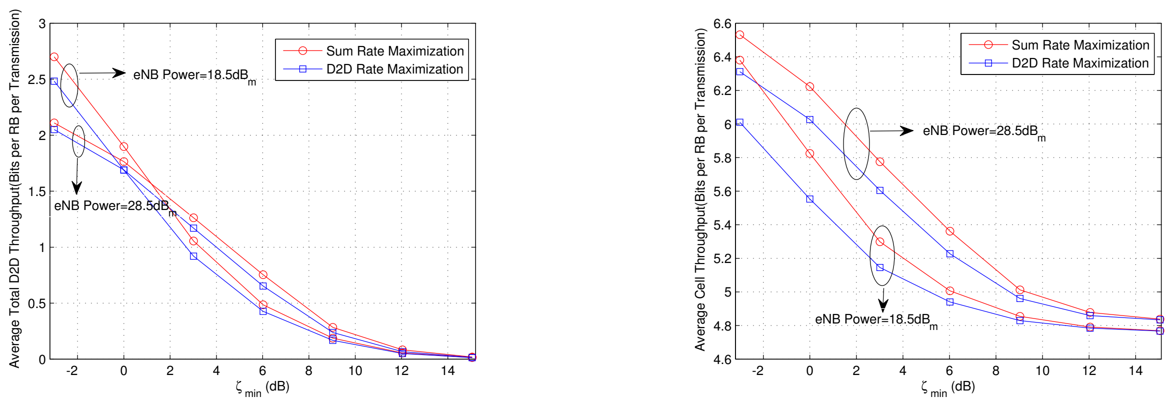
Figs. 6 and 7 show the effect of changing the cell radius, denoted by R_c, on the D2D and cell throughputs for uplink and downlink resource sharing, respectively. As we see, for both uplink and downlink resource sharing, both D2D and



(a) Total throughput for D2D users in the main cell.

(b) Cell throughput in the main cell.

Fig. 4: The achieved throughput vs. minimum SINR requirement for uplink resource sharing.



(a) Total throughput for D2D users in the main cell.

(b) Cell throughput in the main cell.

Fig. 5: The achieved throughput vs. minimum SINR requirement for downlink resource sharing.

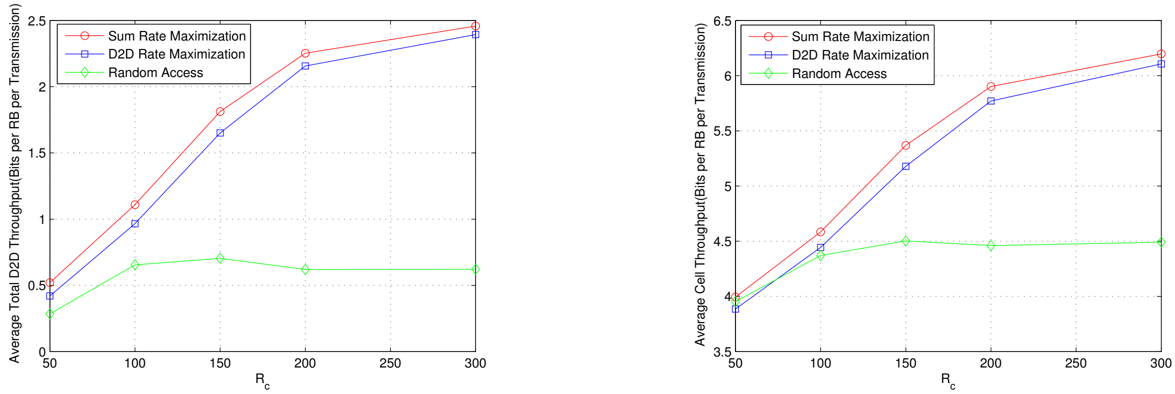
cell throughputs increase as the cell radius increases. As the cell radius increases, the cellular and D2D users are scattered over a larger area, and thus they experience less inter-cell and intra-cell interference, which results improved throughput.

For comparison, we consider a random access method: the D2D users are matched to all the RBs of a randomly chosen cellular user, and each D2D user maximizes its rate using the same power allocation method as the D2D rate maximization method. For each plot, we compare throughput under Algorithms 1 and 2 with that under the random access method. As we see from Figs. 6 and 7, both proposed power allocation algorithms with D2D rate maximization and sum-rate maximization significantly outperform the random access method.

E. Performance Comparison of Different Convex Approximation Methods

To solve the sum-rate maximization problem $\mathcal{S}0$, we have proposed a simple method to convexity the non-convex objective function. There are some well-known successive convexification methods proposed in [24] and [25]. Note that both [24] and [25] are iterative numerical methods and none of them guarantees global optimality. Different from [24] and [25], our proposed solution is asymptotically global optimal as SINR goes to infinity. Furthermore, we provide a semi-closed-form solution, which is simple to implement and requires minimal computation complexity. For both iterative methods in [24] and [25], their performance are very sensitive to the initialization and step-size, which impose challenges for practical implementation in cellular systems with different configurations. However, this is not a concern for our solution.

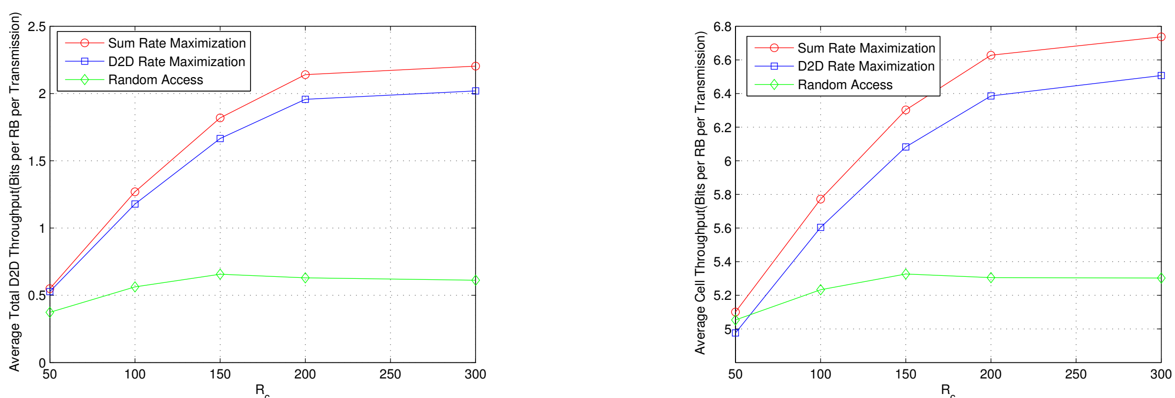
In Fig. 8 (a) and (b), we compare our proposed convexification method with the successive convexification algorithms in [24] and [25] for the sum-rate maximization problem in an uplink single-cell scenario. To implement the successive convexification algorithms in [24] and [25], we have initialized the algorithms with the output of our D2D-rate maximization solution. The number of iterations is set to 35. As we see, for both the D2D users throughput in Fig. 8(a) and the overall cell throughput in Fig. 8(b), all three algorithms have similar performance, with our proposed method providing slightly higher throughput. Furthermore, in Table III, we compare the computational complexity of the three methods. It is based on the MATLAB run time for simulating 40000 LTE frames (equivalent to 200 seconds) under each method. It can be seen from Table III that our proposed algorithm has significantly



(a) Total throughput for D2D users in the main cell

(b) Cell throughput in the main cell.

Fig. 6: The achieved throughput vs. cell radius R_c for uplink resource sharing.



(a) Total throughput for D2D users in the main cell

(b) Cell throughput in the main cell.

Fig. 7: The achieved throughput vs. cell radius R_c for downlink resource sharing.

TABLE III: Simulation Run-Time Comparison

Algorithm	Run Time (s)
Sum-rate maximization method (Algorithm 1)	244 s
Method in [24]	18273 s
Method in [25]	892 s

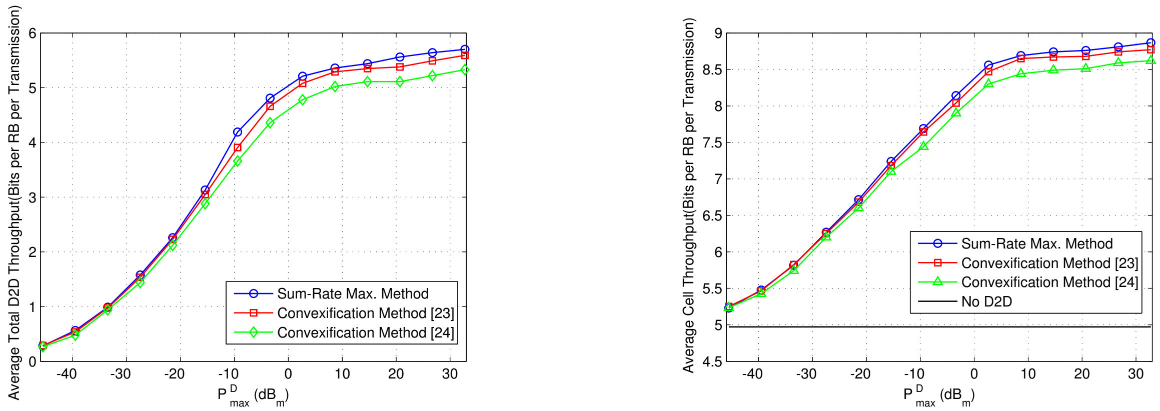
less computational complexity than the methods in [24] and [25].

F. Simulation Summary

An interesting observation from Figs. 2-7 is that the sum-rate maximization algorithm outperforms the D2D rate maximization algorithm from the perspective of total D2D throughput. Note that if we only simulated a single cell, it is clear that the total D2D throughput under the D2D-rate maximization method would be higher than that under the the sum-rate maximization method. For a multi-cell scenario, our simulation indicate, for the sum-rate maximization, some of RBs assigned to a D2D pair are not used (*e.g.,* zero power is allocated to these RBs). Thus, in general, the D2D transmit power usage under the sum-rate maximization method is somewhat lower than that under the D2D rate maximization method. A lower D2D transmit power usage in the neighboring cells causes lower interference to the D2D users in the main cell under the D2D rate maximization method. This results in increased total D2D rate of the sum-rate maximization method. For scenarios where the intra-cell interference is dominant, *e.g.,* large cells, they becomes similar to the single cell case, and the D2D-rate maximization method can perform as good as sum-rate maximization method.

V. Conclusion

In this paper, we have considered optimal power allocation and resource sharing by the D2D users in a cellular network for underlying D2D communication. For a newly entered D2D pair, given its RB assignment, we formulate the optimal D2D power allocation problem as cell sum-rate maximization and D2D rate maximization problems, under cellular user minimum SINR requirements and the total power constraint on each D2D transmitter. For the sum-rate maximization problem, through convexification, we obtain an asymptotically optimal power solution. For the D2D rate maximization problem, we obtain the optimal power allocation in semi-closed-form. In a practical setting, the D2D rate maximization algorithm can be implemented with less needed information exchange than the sum-rate maximization algorithm. We demonstrate



(a) Total throughput for D2D users in the main cell.

(b) Cell throughput in the main cell.

Fig. 8: The achieved throughput vs. $P^{\mathrm{D}}_{\max}$ for uplink resource sharing for different algorithms.

the effectiveness of the two proposed methods in a multi-cell scenario for both uplink and downlink resource sharing in simulation, under various system configurations. We have shown that our D2D-rate maximization algorithm offers up to 20% and 16% cell throughput gains for uplink and downlink resource sharing, respectively, and our sum-rate maximization algorithm offers up to 27% and 22% cell throughput gains for uplink and downlink resource sharing, respectively.

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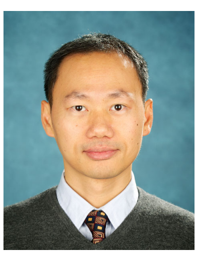

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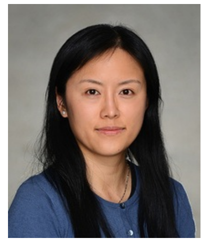

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