

# Maximizing Lifetime in Relay Cooperation through Energy-Aware Power Allocation

Mahdi Hajiaghayi, Min Dong<sup>†</sup>, and Ben Liang

## Abstract

We study the problem of optimal power allocation among relays for lifetime maximization in a dual-hop cooperative network operated by amplify-and-forward relays with battery limitation. We first formulate the optimization problem for global noncausal power allocation and present a solution based on dual decomposition. In the special case of static channels, we provide a closed-form solution for lifetime maximization, which simply requires equally distributing energy over time for each participating relay. Based on this, we then develop a perceived lifetime (PLT) power allocation strategy, which can be viewed as a causal implementation of the noncausal solution by considering only the current channel state information. We also present a minimum weighted total power (MWTP) strategy that does not depend on the prediction of future channel state. PLT and MWTP are compared through analysis and simulation, and it is demonstrated that both result in lifetime performance close to that of the noncausal optimal solution, and that they significantly outperform the conventional strategy of minimizing the total power per transmission, especially when the link conditions are asymmetric or initial energy levels nonuniform among relays. We further extend the proposed power allocation strategies to relay cooperation with multiple sources and discuss how different network configurations affect relay power sharing among the sources.

## Index Terms

Relay network, relay cooperation, lifetime maximization, energy-aware power allocation, perceived lifetime

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## I. INTRODUCTION

Cooperative relaying has been considered a key area of research and development for future-generation wireless networking. It is a promising communication technique to improve coverage, throughput, and reliability, through effective resource sharing in the network. Much research effort has been devoted in the past to understanding the behaviors and benefits of cooperative relaying. Some early works include information theoretic studies following the seminal work in [1], and extensive studies have been conducted in network-layer designs focused on packet forwarding in ad hoc and mesh networks [2]. More recently, the benefit of user-cooperation has been identified from a different perspective, in the new paradigm of cooperation diversity at the physical layer, as a means to improve reception reliability [3][4]. This paradigm has its root in dual-hop relaying, and it has generated much interest in further analyzing the cooperative gain in a variety of dual-hop relay channels and in how to realize such gain with practical schemes [5]- [9]. Two types of relaying strategies are most commonly considered, amplify-and-forward (AF) and decode-and-forward (DF), with the former being the simpler to implement.

In some cooperative relaying applications, it is likely that relays are battery-operated, such as stand-alone relay stations away from the power grid, or peer users in an ad hoc network or multi-hop cellular network. Extending the lifetime of such networks becomes pivotal to maintain uninterrupted data exchange and to reduce the need for replenishing the batteries. There has been much research on energy-efficient packet forwarding in the ad hoc and sensor networking paradigms. However, studies on lifetime maximization for *physical-layer* cooperative relaying have so far been scarce. Existing works on relay power allocation mainly focus on per transmission power usage, either for the optimal power allocation with a given power budget to maximize certain communication metric in the network, or for the minimum level of power consumption per transmission to guarantee some system performance [10]-[20]. These approaches are applicable in the scenarios with tethered power resources. However, when the relays have limited energy, the above results do not necessarily indicate the network lifetime.

In this work, we consider the lifetime of a dual-hop cooperative network operated by battery-limited relays using AF. We explore how appropriate power allocation among relays may lead to prolonging network functionality. The lifetime in this work is defined as the time duration for which the network is able to sustain a minimum data rate. We focus on relay cooperation where

all available relays participate in data forwarding. A slow fading environment is considered; however, we do not assume the communication links have fixed statistics.

#### A. Summary of Results

We first focus on the globally optimal power allocation solution. For a single-source relay network, we formulate the problem of maximizing network lifetime as a global *noncausal* power allocation problem with fixed energy constraints. Even though this results in an integer-programming problem, we show that it can be solved using a dual decomposition method that breaks down the problem into separate subproblems. The noncausal solution can serve as a performance upper-bound for any causal algorithm. To our knowledge, this global solution has not been studied before.

In a special case of static channels, we provide a closed-form optimal power allocation solution for lifetime maximization, which calls for evenly distributing energy over time for each participating relay. This solution is practically appealing for its implementation simplicity and significant reduction on the required feedback overhead.

Based on insights obtained from the closed-form solution in the static-channel case, we then develop a power allocation algorithm for the general time-varying channel case. It can be viewed as a *causal* implementation of the lifetime-maximizing noncausal solution by computing the *perceived lifetime* with the current channel state information (CSI). We term it the perceived-lifetime (PLT) algorithm. PLT essentially maintains the same energy efficiency for each participating relay. We also present a strategy based on the *minimum weighted total power* (MWTP), which is energy-aware but does not depend on the estimation of future CSI. Based on the closed-form solution obtained, we are able to further analyze the behaviors of PLT and MWTP in a two-relay system, deriving new insights on their performance differences. Note that both PLT and MWTP are centralized power allocation strategies. However, we show that, for PLT, the destination only needs to broadcast a single parameter to all relays for power allocation, while unicasting the required power to each relay is needed for MWTP.

Simulation results with more general relaying networks further demonstrate that PLT and MWTP perform well in comparison with the noncausal optimal solution, while PLT is more suitable when asymmetric initial energy levels and/or link conditions are present in the network. Both can significantly outperform the conventional minimum total power (MTP) strategy often

considered for relaying.

Finally, we extend PLT and MWTP to the case with multiple sources, each allocated a fixed share of the transmission resource (e.g., time or frequency). Our simulation results demonstrate how different network configurations affect relay power sharing among multiple sources and the network lifetime.

### *B. Related Works*

The existing studies on cooperative relaying are centered at investigating different relay strategies (e.g., AF and DF) and their performance in relay cooperation and corresponding power allocation. Most current works focus on optimal power allocation without energy limitation. Some of these studies concern optimal power splitting between the source and the relay in single-relay cooperation, for either transmit power minimization given performance requirements or performance maximization for given transmit power, where the performance metrics may be data rate [12][13][20], outage probability [14][16], or bit error rate [15][20]. For multiple relays, optimal power allocation for relay cooperation and selection for data-rate maximization have been considered in [17], distributed relay selection schemes are studied in [10] and [11], and joint relay strategies and resource allocation in OFDM cellular systems for network utility maximization is discussed in [19]. Without energy limitation, these works do not consider the system lifetime.

Studies on lifetime maximization for cooperative relaying have so far been scarce. In [21], the authors studied relay placement and power assignment for DF cooperative relaying in a multi-node network. Their goal was to maximize the minimum node lifetime, under bit-error-rate constraints in an uncoded M-PSK transmission system. Their power allocation was static, based on the channel statistics only. In this work, we consider AF with separate sources and relays, and power allocation is dynamic over time. In [22], power allocation schemes are devised to prolong the lifetime of a single-source AF cooperative network. The authors focused on single-relay selection given some known channel statistics. They studied several selection strategies and the corresponding power allocation algorithm, assuming a finite number of power levels. The network lifetime was defined by the required SNR at the destination to maintain a certain outage probability. In this work, we focus on relay cooperation instead of relay selection, our network lifetime is defined as the duration when a certain data rate is achievable, we consider a continuous range of power levels, and our causal power allocation strategies do not rely on the

knowledge of channel statistics.

The issue of prolonging network lifetime has also been studied in the context of wireless sensor networks (WSN) (see for example [23]-[27] and literature therein), as severe energy limitation makes such issue the most critical in such network to function. A few main differences exist between lifetime maximization we are considering in the relay cooperation and the existing frameworks for WSN lifetime, which make results obtained in WSN not applicable to our problem. The first is regarding the lifetime definition. In WSN, most studies use lifetime defined indirectly as the time until the battery of a node drains out [26], or a portion of sensors are dead [27]. This definition is mostly applied to the single-hop network topology, or non-cooperative transmissions. In cooperative relaying, we consider possibly very distinct link statistics among sources and relays, as well as asymmetric initial energy at relays, as opposed to the identical ones commonly assumed in densely populated sensor networks. Thus, energy depletion of some relays may not result in the end of lifetime of a cooperative network, so long there are relays to forward the data with required performance satisfied. The definition of lifetime for relay cooperation targets directly at the operability of the network to maintain certain performance requirements. In terms of performance focus, most of studies in WSN focus on developing and analyzing efficient routing or medium access protocols to prolong the network lifetime in a multi-hop network, with assumption of perfect physical layer performance (i.e. perfect DF forwarding). Links among nodes is normally abstracted as a weighted graph. In cooperative relaying, the actual physical layer link condition and how cooperation can improve the link condition is the main focus. Power allocation in this case will have direct impact on the network lifetime.

### *C. Paper Organization*

In Section II, we discuss the network model and lifetime formulation. Then we present a noncausal optimization framework and solution for lifetime maximization in Section III. The causal solutions and their analysis, including a closed-form optimal solution for the static channel case, PLT, and MWTP, are presented in Section IV. The multiple-source extensions are given in Section V. Section VI provides the simulation and comparison results, and finally conclusions are given in Section VII.

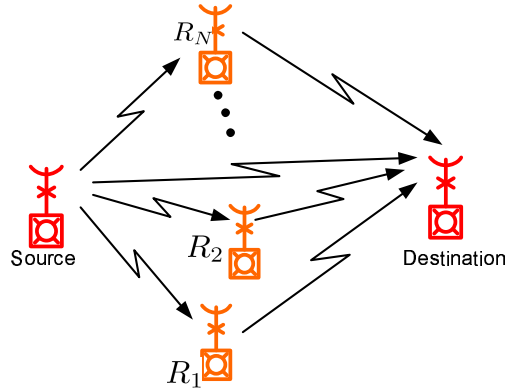


Fig. 1. Dual-hop cooperative network.

## II. PROBLEM FORMULATION

### A. Network Model

We first consider a dual-hop cooperative network where a source node  $s$  transmits data to a destination node  $d$  with the assistance of  $N$  relays, as shown in Fig. 1. An extension to the multiple-source case will be presented in Section V. We constrain ourselves to half-duplex transmission, where a relay node is either in transmission or reception but not simultaneously. Assuming  $x_s(t)$  being the source data to be sent at time  $t$ , a cooperative transmission takes place in two phases. In the first phase, the source node broadcasts its data  $x_s(t)$  to the relays and the destination. In the second phase, the relays forward an amplified version of the received signal from the source to the destination node with its designated power  $P_k(t)$ ,  $k = 1, \dots, N$ . We assume each relay transmits the data using an orthogonal channel (e.g. frequency or time). Such arrangement arises in the case where coherent transmissions among relays are not possible, either in an asynchronous network, or no instantaneous channel state information available at the relays.

The signals received at the  $k$ th relay and the destination in the first phase are given by

$$\begin{aligned} y_{rk}(t) &= \sqrt{P_s} h_{sk}(t) x_s(t) + n_{rk}(t), \\ y_d(t) &= \sqrt{P_s} h_{sd}(t) x_s(t) + n_d(t), \end{aligned} \quad (1)$$

where  $h_{sd}(t)$  and  $h_{sk}(t)$  denote the channel gains between source and destination, and source and relay  $k$ , respectively. They capture the path loss with exponent  $\alpha$ , shadowing, and small-scale

flat fading. The source transmit power is denoted as  $P_s$ . The noise terms  $n_{rk}(t)$ ,  $k = 1, \dots, N$ , and  $n_d(t)$  are the additive white Gaussian noises at time  $t$ . Without loss of generality, we assume the noise variance is the same for all links and denote it as  $\sigma_n^2$ , i.e.  $n_{rk}(t), n_d(t) \sim \mathcal{CN}(0, \sigma_n^2)$ .

The forwarded signal at the destination by the  $k$ th relay in the second phase is given by

$$y_{dk}(t) = \sqrt{\frac{P_k(t)}{P_s|h_{sk}(t)|^2 + \sigma_n^2}} h_{dk}(t) y_{rk}(t) + n_{dk}(t), \quad (2)$$

where  $h_{kd}(t)$  denotes the channel gain between relay  $k$  and the destination, and  $n_{dk}(t)$  is the corresponding additive Gaussian noise with variance  $\sigma_n^2$ . With slight abuse of notation, we still use time  $t$  for the second phase transmission, to indicate that it is part of the transmission (2nd-hop) of the same source data  $x_s(t)$ . Nonetheless, such notation should not cause confusion, as it essentially indicates the time of a complete dual-hop relay transmission.

From (1) and (2), the received SNR from the  $k$ th relay is given by

$$\gamma_k(P_k(t)) = \frac{P_s P_k(t) b_k(t) c_k(t)}{1 + P_s b_k(t) + P_k(t) c_k(t)}, \quad (3)$$

where  $b_k(t) = |h_{sk}(t)|^2/\sigma_n^2$  and  $c_k(t) = |h_{kd}(t)|^2/\sigma_n^2$  are the nominal received signal-to-noise ratio (SNR) with unit transmit power at relay  $k$  and the destination (from relay  $k$ ), respectively. Since relays use orthogonal transmissions, at the destination, the maximum ratio combining (MRC) technique can be used to add coherently the received signal observations, leading to a combined SNR of all received SNRs from the relays. Therefore, the effective end-to-end data rate is given by

$$C(t) = \frac{1}{N+1} \log \left( 1 + P_s a(t) + \sum_{k=1}^N \gamma_k(P_k(t)) \right), \quad (4)$$

where  $a_k(t) = |h_{sd}(t)|^2/\sigma_n^2$  is the nominal received SNR from the direct path, and  $1/(N+1)$  is the bandwidth efficiency factor, reflecting the orthogonal transmissions.

### B. Lifetime for Cooperative Relay

We assume that relays are battery powered. Let  $\underline{\mathcal{E}}(t) = [\mathcal{E}_1(t), \dots, \mathcal{E}_N(t)]$  be the relay residual energy vector with  $\mathcal{E}_k(t)$  being the residual energy of relay  $k$  at time  $t$ . The initial energy is then given by  $\mathcal{E}_k(0)$  for relay  $k$ . A relay gradually depletes its energy as it participates in forwarding the source message. A relay can no longer cooperate if its required energy for the current transmission is more than its residual energy. To define the lifetime of such a relay network, we

use a more direct definition to capture the functionality of the network. That is to maintain the quality of service (QoS) requirement of the end-to-end data transmission subject to a limited energy budget at each relay. Therefore, we define relay network lifetime as the time interval during which the end-to-end data rate is maintained above a minimum required rate  $R$ , i.e

$$T = \max\{t : C(t') \geq R, 0 < t' < t\}. \quad (5)$$

Based on this definition, the source would no longer be able to convey its message with required rate  $R$  through relays after time  $t > T$ . Note that beside the data rate, other performance metrics can be employed, as long as they can be converted to a minimum SNR requirement. Examples include the end-to-end bit error rate requirement or the packet delay constraint. All of such metrics eventually lead to a similar type of power allocation strategy.

The transmission power allocated on a relay at time  $t$  should satisfy the energy constraint:  $P_k(t)\Delta t \leq \mathcal{E}_k(t)$ , where  $\Delta t$  denotes the transmission duration. A network is called *functional* at time  $t$ , if there exists a feasible relay power allocation vector  $\underline{P}(t) = [P_1(t), \dots, P_N(t)]^T$  that satisfies both energy and QoS constraints. A network should be functional during its entire lifetime. We denote the matrix  $\mathbf{P}(T) = [\underline{P}(0), \underline{P}(1), \dots, \underline{P}(T)]$  as the  $N \times T$  power allocation matrix during the network lifetime, with columns and rows corresponding to time and relay nodes, respectively.

Note that, when the network reaches its lifetime, each relay may have some residual energy left which is not sufficient to forward the signal at the required rate. Therefore, the lifetime depends on both the amount of actual energy spent and the transmission power used. Minimizing relay transmission power at each time does not necessarily prolong the network lifetime, as residual energy also needs to be taken into account on how power should be allocated. This fact will be further explained and demonstrated in the simulations. Our objective is then to seek effective power allocation strategies to maximize the lifetime  $T$ .

### III. POWER OPTIMIZATION FOR LIFETIME MAXIMIZATION: NONCAUSAL SOLUTION

We first ignore the causality issue for power allocation, i.e. the allocation vector  $\underline{P}(t)$  can be determined based on future link conditions. Then our lifetime maximization problem essentially is a global optimization problem with entries of  $\mathbf{P}$  as the optimization variables. We assume that the power of relays remain constant within each transmission slot with duration  $\Delta$  and we set



$\Delta = 1s$  without loss of generality. Therefore, our optimization problem can be expressed in a discrete version with  $t$  as slot index and  $T = n\Delta$  as the lifetime. The optimization is then given by

$$\begin{aligned}
& \max_{\mathbf{P}(n)} n && (6) \\
& \text{s.t. (i)} \quad \sum_{t=1}^n P_k(t) \leq \mathcal{E}_k(0), \quad \text{for } k = 1, \dots, N; \\
& \text{(ii)} \quad P_s a(t) + \sum_{k=1}^N \frac{P_s P_k(t) b_k(t) c_k(t)}{1 + P_s b_k(t) + P_k(t) c_k(t)} \geq \gamma_{th}, \\
& \quad \text{for } t = 1, \dots, n; \\
& \text{(iii)} \quad P_k(t) \geq 0, \quad t = 1, \dots, n; \quad k = 1, \dots, N,
\end{aligned}$$

where  $\gamma_{th} \triangleq (2^{(N+1)R} - 1)$  is the SNR threshold for the required rate  $R$ . The first condition ensures that the total expended energy by relay  $k$  during the network's operations will not exceed the initial energy  $\mathcal{E}_k(0)$ , while the second constraint provides the rate requirement as the QoS constraint. Finally, the power variables are non-negative.

At first glance, the problem (6) seems to be an integer-programming problem and possibly hard to solve. However, notice that, for a given  $n$ , this problem is transformed into a feasibility problem with convex constraints. The maximum value of  $n$  for which a feasible solution  $\mathbf{P}(n)$  exists can be obtained numerically using bisection search. Specifically, we decompose the problem into two-nested search loops. The outer loop varies  $n$  and the inner loop search for a feasible solution  $\mathbf{P}(n)$  at the given value of  $n$ . As for the feasibility test, we use a dual decomposition method to break down the optimization problem into a set of subproblems that are individually solvable.

Specifically, for a given  $n$ , we define the lagrangian function associated with (6) by

$$\begin{aligned}
\Gamma(\mathbf{P}(n), \boldsymbol{\mu}, n) &= n + \sum_{t=1}^n \mu_t \left( P_s a(t) + \sum_{k=1}^N \gamma_k(P_k(t)) - \gamma_{th} \right) \\
&= n + \sum_{k=1}^N \left( \sum_{t=1}^n \mu_t \gamma_k(P_k(t)) \right) \\
&\quad + \sum_{t=1}^n \mu_t (P_s a(t) - \gamma_{th}), && (7)
\end{aligned}$$

where  $\boldsymbol{\mu} = [\mu_1, \dots, \mu_n]$  are the dual variables, and  $\gamma_k(P_k(t))$  is given in (3). Then, the dual

function is obtained as

$$g(\boldsymbol{\mu}, n) = \begin{cases} \max_{\mathbf{P}(n)} & \Gamma(\mathbf{P}(n), \boldsymbol{\mu}, n) \\ s.t. & \text{constraints (i), (iii) in (6)} \end{cases}. \quad (8)$$

It can be decomposed to  $N$  underlying subproblems corresponding to the  $N$  relays

$$g_k(\boldsymbol{\mu}, n) = \begin{cases} \max_{P_k(1), \dots, P_k(n)} & \sum_{t=1}^n \mu_t \gamma_k(P_k(t)) \\ s.t. & \sum_{t=1}^n P_k(t) \leq \mathcal{E}_k(0), P_k(t) \geq 0 \end{cases} \quad (9)$$

for  $k = 1, \dots, N$ . Note that the interaction among the relays over the duration of lifetime is controlled through the use of dual variables  $\boldsymbol{\mu}$  common to all relays. It controls the relative power allocation over time to ensure the rate requirement is met at each time slot, so that a higher value of  $\mu_t$  induces more power allocated to time  $t$ . Note that having the local knowledge of CSI for all time allows each relay to *individually* solve (9) for a given  $\boldsymbol{\mu}$  and obtain the optimum  $P_k^*(t)$ .

For a given  $n$ , the problem in (6) is convex. Due to the zero-duality-gap property of a convex problem, if (6) is feasible for that given  $n$ , then this feasible solution can be found via its dual problem

$$\begin{aligned} \min_{\boldsymbol{\mu}} g(\boldsymbol{\mu}, n) &= \min_{\boldsymbol{\mu}} \left\{ n + \sum_{k=1}^N g_k(\boldsymbol{\mu}) + \sum_{t=1}^n \mu_t (P_s a(t) - \gamma_{th}) \right\} \\ s.t. & \quad \boldsymbol{\mu} \succeq 0, \end{aligned} \quad (10)$$

where  $\succeq$  denotes the element-wise inequality. To solve (10), the subgradient method can be adopted [28]. In this method, we keep updating  $\boldsymbol{\mu}$  through the following iteration procedure until convergence is achieved.

- 1) Initialize  $\boldsymbol{\mu}^{(0)}$
- 2) Given  $\boldsymbol{\mu}^{(l)}$ , solve (9) for  $k = 1, \dots, N$  to obtain the optimal values of  $\mathbf{P}^{l*}(n)$ . Using the Lagrangian method with KKT conditions,  $P_k^{l*}(t)$  is obtained as

$$P_k^{l*}(t) = \left[ \frac{\sqrt{\mu_t^l \lambda^{-1} P_s b_k(t) c_k(t) (1 + P_s b_k(t))} - 1 - P_s b_k(t)}{c_k(t)} \right]^+ \quad (11)$$

where  $[x]^+ \triangleq \max(0, x)$ , and  $\lambda$  is chosen such that constraint (i) in (6) is satisfied. The derivation of (11) is presented in Appendix B.

3) Set

$$\mu_t^{l+1} = \mu_t^l + \left( \gamma_{th} - P_s a(t) - \sum_{k=1}^N \gamma_k(P_k^{l*}(t)) \right) \nu^l, \quad (12)$$

for  $t = 1, \dots, n$ , where  $\nu^l$  is the step size at the  $l$ th iteration.

4) Let  $l = l + 1$ ; Return to step 2 until convergence.

The above subgradient algorithm is guaranteed to converge to the optimal  $\boldsymbol{\mu}^*$ , if the step size is chosen by the diminish step-size rule. The rule requires the step size to converge to zero and to be nonsummable  $\sum_l \nu^l = \infty$  [28]. We set  $\nu^l = 1/l$  which satisfies the above rule. Once  $\boldsymbol{\mu}^*$  is obtained, the optimal primal variables  $\mathbf{P}^*(n)$  can be calculated from (11). If the calculated  $\mathbf{P}^*(n)$  satisfies all the constraints (i)-(iii) in (6), the problem is feasible for the given  $n$ . Otherwise, if either the iteration procedure diverges, or the solution violates the constraints, the problem is declared to be infeasible for the given  $n$ .

If the problem (6) is feasible for  $n$ , we can be assured that this problem is also feasible for  $n'$ , where  $n' \leq n$ . Therefore, the bisection method can be used as a search method for the outer loop to find the maximum lifetime  $n$  whose corresponding  $\mathbf{P}^*(n)$  satisfies all the constraints of (6).

As mentioned earlier, in order to solve the general optimization problem in (6), knowledge of the CSI of all links for the entire lifetime is required. Therefore, the above solution is noncausal and can only be adopted for the static or very slow fading environments. For general time-varying channels, where future CSI cannot be obtained or estimated, a causal power allocation strategy is required.

#### IV. LIFETIME MAXIMIZATION: CAUSAL ALGORITHMS

To address the issue of causality, we first focus on the static-channel case and present a closed-form lifetime maximizing power allocation solution. Based on this solution, we then provide the causal PLT approach for time-varying channels, which only relies on the present CSI and residual energy. We also present the MWTP approach and compare the two approaches analytically.

##### A. Static Channel Case

We consider the optimization problem in (6) for a special case where channel gains are static over time. We drop the time index  $t$  from  $a_k(t)$ ,  $b_k(t)$  and  $c_k(t)$  for the optimization problem in

(6). However, the optimum power  $P_k(t)$  is still subject to change due to the variation of residual energy  $\mathcal{E}_k(t)$  over time. In the following, we show that, for the static channel case, the previous global solution has a closed-form expression, which corresponds to a simple power allocation strategy amenable to easy implementation.

*Proposition 1:* For static channels, the following power allocation strategy is optimal for the problem in (6)

$$P_k^*(t) = \frac{\mathcal{E}_k(0)}{n^*}, \quad t = 1, \dots, n^*, \quad (13)$$

where the maximum lifetime  $n^*$  is given by

$$n^* = \max \left\{ n : P_s a + \sum_{k=1}^N \frac{\mathcal{E}_k(0) P_s b_k c_k}{n(1 + P_s b_k) + \mathcal{E}_k(0) c_k} \geq \gamma_{th} \right\}. \quad (14)$$

*Proof:* See Appendix A. ■

The power allocation scheme in Proposition 1 essentially suggests that equally distributing the energy of each relay over time (i.e. constant power) maximizes the network lifetime. Note that although this optimal solution turns out to be simple, it is a nontrivial solution. Considering the case where the relays with different initial energy can be positioned anywhere and thus exhibit very different link conditions (i.e.  $b_k$  and  $c_k$ ), it is not immediately obvious that the constant-power solution for all relays would give the maximum lifetime. At the same time, a power allocation approach that minimizes the total power used by the relays at each time is suboptimal for lifetime maximization, which is verified also in the later simulation results.

The solution in Proposition 1 also has the following characteristics:

- P1) It maintains equal energy efficiency among participating relays after each transmission, where the energy efficiency is defined as

$$\eta_k(t) \triangleq \frac{\mathcal{E}_k(t)}{P_k(t)} = n^*. \quad (15)$$

- P2) It minimizes the sum of wasted residual energy at the end of the network lifetime.

From the practical engineering point of view, this allocation scheme is very easy to implement. It requires the exact *same fraction* of the remaining energy to be allocated for each relay. All the information required for each relay is  $n^*$ , which can be broadcasted to each relay.

### B. Perceived Lifetime (PLT) Power Allocation Strategy for Time-Varying Channels

Now we consider a more practical case when the channels among source, relays, and destination are slowly varying over time, possibly due to movement induced path loss, shadowing, or small-scale flat fading. For each relay, the power to be used at each transmission can only be based on causal information of the channel and residual energy. Based on the optimal power allocation strategy in the static-channel case, we propose the following PLT power allocation approach.

At each time  $t$ , for given channel gains and remaining energy  $(\{a_k(t), b_k(t), c_k(t)\}, \mathcal{E}_k(t))$ , we compute the maximum *perceived lifetime* and the corresponding power allocation:

$$n^*(t) = \max \left\{ n : P_s a(t) + \sum_{k=1}^N \frac{\mathcal{E}_k(t) P_s b_k(t) c_k(t)}{n(1 + P_s b_k(t)) + \mathcal{E}_k(t) c_k(t)} \geq \gamma_{th} \right\}, \quad (16)$$

$$P_k(t) = \frac{\mathcal{E}_k(t)}{n^*(t)}, \quad \text{for } k = 1, \dots, N. \quad (17)$$

Essentially, the PLT algorithm tries to maximize the network lifetime at each transmission stage assuming the current channel gains will not change in the future. At the same time, this strategy also attempts to maintain the two properties P1 and P2, given in the static channel case. In the simulation examples, we will compare the PLT algorithm with the optimal noncausal power allocation solution under time-varying channel scenarios.

### C. Minimum Weighted Total Power (MWTP) Strategy

The PLT algorithm can be viewed as a causal implementation of the noncausal power allocation solution by assuming the future channels stay the same as the current ones. Without assuming the future channel conditions, we now introduce another strategy that directly targets at reducing the current transmission powers among the relays. As mentioned earlier, minimizing the total per transmission power at relays without residual energy consideration is not necessarily prolonging the lifetime. Instead, we propose to minimize a weighted total power per transmission, where

the power allocation at time  $t$  is the solution of the following optimization problem

$$\begin{aligned} \min_{\underline{P}(t)} \quad & \sum_{k=1}^N \frac{P_k(t)}{\mathcal{E}_k(t)} \\ \text{s.t.} \quad & 1) \quad C(t) \geq R \\ & 2) \quad \underline{P}(t) \preceq \underline{\mathcal{E}}(t). \end{aligned} \quad (18)$$

The weight for each relay is the inverse of residual energy at time  $t$ , i.e. more weight is given for the relay with smaller residual energy, inducing the relay to use less power for relay cooperation. From the static-channel case, we observe that to maximize the network lifetime, all relays would deplete energy at the same time. The strategy in (18) is trying to achieve this goal through residual energy weighting.

Since the optimization in (18) is convex, we can solve it by using KKT conditions. Considering the Lagrangian

$$\begin{aligned} \Gamma(P_1(t), \dots, P_N(t), \lambda) = \\ \sum_{k=1}^N \frac{P_k(t)}{\mathcal{E}_k(t)} + \lambda \left( \gamma_{th} - \sum_{k=1}^N \gamma_k(P_k(t)) - P_s a_k(t) \right) \end{aligned} \quad (19)$$

with multiplier  $\lambda$ . By applying KKT conditions similar as in Appendix B, the optimal power  $P_k(t)$  allocated to relay  $k$  is determined as

$$P_k(t) = \min \left\{ \mathcal{E}_k(t), \left( \frac{\sqrt{\lambda \mathcal{E}_k(t) \beta_k(t)} - P_s b_k(t) - 1}{c_k(t)} \right)^+ \right\}, \quad (20)$$

where  $\beta_k(t) = P_s b_k(t) c_k(t) (1 + P_s b_k(t))$ . Parameter  $\lambda$  is chosen such that the minimum rate requirement  $C(t) = R$  is met, where  $C(t)$  is given in (4). Note that, without considering the residual energy, (18) would reduce to the conventional strategy to minimize the total power (MTP). The solution of MTP can be obtained by simply removing  $\mathcal{E}_k(t)$  in (20).

Note that the amounts of feedback required in PLT and MWTP are also different. They both can be implemented in a centralized manner, where the destination collects channel and residual energy information and then computes the appropriate power levels. While in the MWTP and MTP approaches, the destination needs to send the computed power levels to each individual relay at each time slot, the feedback in the PLT approach only requires the destination to broadcast a single parameter  $n^*(t)$  for all relays, where  $n^*(t)$  is the momentarily perceived lifetime value.

#### D. Power Allocation Analysis in the Two-relay Case

To obtain some insightful understanding on how the proposed power allocation schemes behave differently in various cases, we analyze the assigned power for networks two relays ( $N = 2$ ). In the following analysis, we make an assumption that the source-relay channels are sufficiently strong so that  $P_s b_k(t) + 1 \simeq P_s b_k(t)$ ,  $k = 1, 2$ . Furthermore, the SNR threshold  $\gamma_{th}$  (corresponds to the required rate  $R$ ) must satisfy  $\gamma_{th} < P_s a(t) + P_s(b_1(t) + b_2(t))$ . This is because the maximum attainable SNRs through the AF relays 1 and 2 are upper bounded by  $P_s b_1(t)$  and  $P_s b_2(t)$ , respectively; if the assumption is not true, then the required rate could not be maintained, regardless of how much power each relay contributes.

In the following, we compute the power allocation at time  $t$ . For notational simplicity, we drop the dependency of  $t$  in the following calculation. For the PLT approach, from equation (16), the maximum perceived lifetime for the two-relay network is reduced to

$$n^* = \lfloor \frac{-A + \sqrt{B^2 - 4AC}}{2A} \rfloor \quad (21)$$

where

$$\begin{aligned} A &= P_s^2 b_1 b_2 (\gamma_{th} - P_s a), \\ B &= P_s [\mathcal{E}_1 c_1 b_2 (\gamma_{th} - P_s a - P_s b_1) + \mathcal{E}_2 c_2 b_1 (\gamma_{th} - P_s a - P_s b_2)], \\ C &= \mathcal{E}_1 \mathcal{E}_2 c_1 c_2 (\gamma_{th} - P_s a - P_s b_1 - P_s b_2). \end{aligned}$$

The corresponding power allocation solution is simply calculated as  $P_1 = \mathcal{E}_1/n^*$  and  $P_2 = \mathcal{E}_2/n^*$ . We consider the *power ratio factor*,  $\kappa = P_1/P_2$ , that characterizes any approach. For PLT, it is given by

$$\kappa^{\text{PLT}} = \frac{\mathcal{E}_1}{\mathcal{E}_2}. \quad (22)$$

If the MWTP approach is used, from (20), we have

$$P_1 = \left( \frac{P_s b_1}{c_1} \frac{\left( P_s b_2 \sqrt{\frac{\mathcal{E}_1 c_1}{\mathcal{E}_2 c_2}} - P_s b_2 - P_s a + \gamma_{th} \right)}{P_s a + P_s (b_1 + b_2) - \gamma_{th}} \right)^+ \quad (23)$$

and

$$P_2 = \left( \frac{P_s b_2}{c_2} \frac{\left( P_s b_1 \sqrt{\frac{\mathcal{E}_2 c_2}{\mathcal{E}_1 c_1}} - P_s b_1 - P_s a + \gamma_{th} \right)}{P_s a + P_s (b_1 + b_2) - \gamma_{th}} \right)^+, \quad (24)$$

where we assume  $P_i < \mathcal{E}_i$ , for  $i = 1, 2$ . From (23) and (24), we realize that as long as  $\gamma_{th} > P_s a + \max(P_s b_1, P_s b_2)$ ,  $P_1$  and  $P_2$  remain positive regardless what the value of residual energies  $\mathcal{E}_1$  and  $\mathcal{E}_2$  are (note that  $\gamma_{th} < P_s a + P_s(b_1 + b_2)$ ). In this case, the power ratio factor  $\kappa$  is equal to

$$\kappa^{\text{MWTP}} = \frac{b_1 c_2 \left( P_s b_2 \sqrt{\frac{\mathcal{E}_1 c_1}{\mathcal{E}_2 c_2}} - P_s b_2 - P_s a + \gamma_{th} \right)}{b_2 c_1 \left( P_s b_1 \sqrt{\frac{\mathcal{E}_2 c_2}{\mathcal{E}_1 c_1}} - P_s b_1 - P_s a + \gamma_{th} \right)}. \quad (25)$$

In the MTP approach, power allocation is not a function of residual energy, and the ratio  $\kappa$  would reduce to

$$\kappa^{\text{MTP}} = \frac{b_1 c_2 \left( P_s b_2 \sqrt{\frac{c_1}{c_2}} - P_s b_2 - P_s a + \gamma_{th} \right)}{b_2 c_1 \left( P_s b_1 \sqrt{\frac{c_2}{c_1}} - P_s b_1 - P_s a + \gamma_{th} \right)}. \quad (26)$$

While  $\kappa^{\text{PLT}}$  in (22) increases linearly with  $\mathcal{E}_1/\mathcal{E}_2$  at high  $\mathcal{E}_1/\mathcal{E}_2$  region,  $\kappa^{\text{MWTP}}$  in (25) grows in the order of  $\mathcal{O}(\sqrt{\mathcal{E}_1/\mathcal{E}_2})$ . The different order of increment for the PLT and MWTP approaches reveals that when  $\mathcal{E}_1 \gg \mathcal{E}_2$ , PLT allocates more power to relay 1 than MWTP does. Hence, MWTP is more conservative than PLT in terms of assigning power for the relay with higher residual energy. This behavior is observed when  $\kappa^{\text{PLT}} > \kappa^{\text{MWTP}}$ , or equivalently

$$\frac{\mathcal{E}_1}{\mathcal{E}_2} > \frac{b_1 c_2}{b_2 c_1} \cdot \frac{\gamma_{th} - P_s a - P_s b_2}{\gamma_{th} - P_s a - P_s b_1}. \quad (27)$$

The derivation for (27) is provided in Appendix C.

Fig. 2 depicts the behavior of PLT, MWTP, and MTP, by showing the variation of  $P_1/P_2$  with respect to  $\mathcal{E}_1/\mathcal{E}_2$  in a static-channel example. The network setup is as follows: asymmetric cooperative network with two relays located at  $R_1(20m, 20m)$  and  $R_2(-20m, 15m)$ , and source and destination respectively positioned at  $S(30m, 0m)$  and  $D(0m, 0m)$ . The channel path-loss exponent is  $\alpha = 2$ , noise variance is set to  $\sigma_n^2 = 10^{-4}$ , and the rate threshold value is  $R = 1.9\text{bits/s/Hz}$ . The figure shows that the PLT and MWTP power-ratio curves coincide at  $\mathcal{E}_1/\mathcal{E}_2 \simeq 0.38$ , which is confirmed in (27). Then, the curve under MWTP grows sublinearly with  $\mathcal{E}_1/\mathcal{E}_2$ , as compared to the linear growth under PLT.

Since MWTP is more conservative, if the channel for  $R_1$  happens to deteriorate in the future, it could outperform PLT, as it happens to track the future channel state in the right direction. Conversely, if the channel for  $R_1$  improves in the future, MWTP will track the channel state in the wrong direction and perform worse. Overall, since none of the schemes deliberately predicts



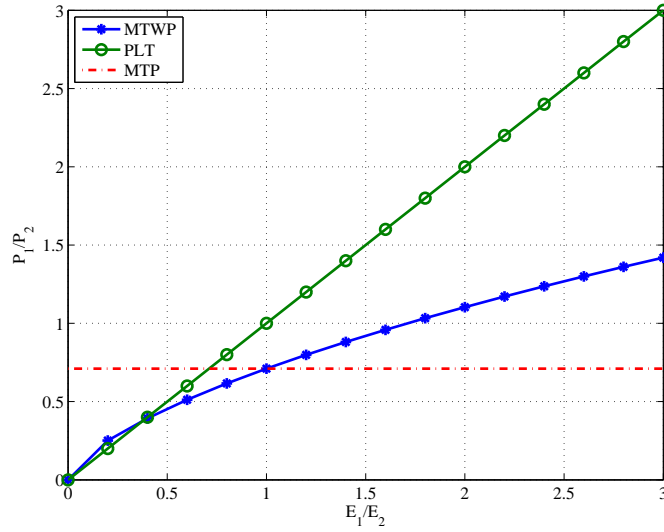


Fig. 2. Power ratio of three power allocation schemes in a two-relay network.

the future CSI, there is no obvious advantage between PLT and MWTP in terms of performance. In various simulations, we observe that PLT often outperforms MWTP.

## V. LIFETIME MAXIMIZATION FOR MULTIPLE SOURCES

Our analysis for the single-source scenario can be extended to a multiple-source network. We assume a relay network with  $M$  sources. These sources may transmit data to either their corresponding destinations or to a common single destination. Assume  $N$  relays in the network, and all sources share them for data transmission. Each source needs to maintain its minimum rate requirement for as long as possible, and the network lifetime is defined as the duration where the required rates for all sources are satisfied. We assume orthogonal channels are used among sources for their data transmission, and each source is assigned a fixed amount of channel resource. For example, the network bandwidth (or time) maybe divided into fixed subbands (or time frames) and assigned to the sources, so that the communication between each source its destination via relays is performed over that subband (or time frame) only. In this multiple-source setup, the question is how each relay at time  $t$  should distribute its power among the sources with the objective of prolonging lifetime<sup>1</sup>.

<sup>1</sup>Optimal allocation of resources jointly among sources and relays is outside the scope of this paper and is left open for future research.

We can generalize (6) to the multiple-source case with a noncausal global power allocation solution. The following two changes must be incorporated into the formulation: 1) the dimension of the optimization variable  $\mathbf{P}(n)$  is increased by one to contain the power allocation vectors for all sources; and 2)  $(M - 1)$  minimum rate constraints corresponding to  $(M - 1)$  new sources are added to constraint (ii) of (6). The modified new optimization problem can again be transformed into a feasibility problem with convex constraint sets for a given  $n$ :

$$\begin{aligned}
 & \max_{\mathbf{P}(n)} n & (28) \\
 \text{s.t.} & (i) \quad \sum_{t=1}^n \sum_{j=1}^M P_{kj}(t) \leq \mathcal{E}_k(0), \quad \text{for } k = 1, \dots, N; \\
 & (ii) \quad P_s a_j(t) + \sum_{k=1}^N \gamma_{kj}(P_{kj}(t)) \geq \gamma_{th}^{(j)}, \\
 & \quad \text{for } t = 1, \dots, n; \quad j = 1, \dots, M; \\
 & (iii) \quad P_{kj}(t) \geq 0, \quad t = 1, \dots, n; \quad k = 1, \dots, N; \quad j = 1, \dots, M,
 \end{aligned}$$

where  $\gamma_{th}^{(j)}$  is destination SNR corresponding to the rate requirement of source  $j$ , and  $P_{kj}(t)$  denotes the power contributed by relay  $k$  to assist source  $j$  to convey its message at time  $t$ . Similarly to (6), we can solve (28) by using a two-nested loop search, and we omit the details.

In the following, we focus on the *causal* PLT approach. Since the PLT strategy is devised based on the optimal power allocation in the static-channel case, we first present the solution for the static-channel case and then explain how it is applied to the time-varying case.

By adapting the result in Proposition 1, we reach the following equation for power allocation among relays in the multiple-source scenario,

$$\sum_{j=1}^M P_{kj} = \frac{\mathcal{E}_i(0)}{n^*} \quad k = 1, \dots, N, \quad (29)$$

where  $n^*$  is the maximum lifetime, and each  $P_{kj}$  can be shown to be constant over the entire lifetime. Effectively, from each relay's point of view, all sources can be considered as one combined source. Thus, similar to the single-source case, the energy of each relay should be equally distributed over time to maximize the network lifetime.

In addition, we need to find the optimal power sharing of each relay among multiple sources

at time  $t$ . By combining (29) and the rate constraints in (14), we have the following optimization:

$$\begin{aligned}
 n^* &= \max_{P_{kj}} n & (30) \\
 \text{s.t. (i)} \quad & n \sum_{j=1}^M P_{kj} \leq \mathcal{E}_k(0), \quad k = 1, \dots, N \\
 \text{(ii)} \quad & P_s a_j + \sum_{k=1}^N \gamma_{kj}(P_{kj}) \geq \gamma_{th}^{(j)} \quad j = 1, \dots, M \\
 \text{(iii)} \quad & P_{kj} \geq 0 \quad k = 1, \dots, N; j = 1, \dots, M
 \end{aligned}$$

To solve (30), we can relax the integer restriction on  $n$  and attempt to find the optimal real value of lifetime  $\tau^*$  and then set  $n^* = \lfloor \tau^* \rfloor$ . Since (30) is convex, an adaptation of dual decomposition method along with the bisection search, similar to the two-nested search loops in Section III, can also be employed here. In the inner-loop search, if one replaces the parameter  $t$  with  $j$ , which corresponding to the rate constraint of each source, the  $N$  decomposed subproblems in (9) would become

$$g_k(\boldsymbol{\mu}, n) = \begin{cases} \max_{P_{k1}, \dots, P_{kM}} \sum_{j=1}^M \mu_j \gamma_k(P_{kj}) \\ \text{s.t.} \quad n \sum_{j=1}^M P_{kj} \leq \mathcal{E}_k(0), P_{kj} \geq 0 \end{cases} \quad (31)$$

for  $k = 1, \dots, N$ . Note that in this case, the convergence in the inner loop is achieved much faster since the number of variables to be determined is only  $N \times M$  compared with that in (7), which is  $N \times n$  (typically  $n \gg M$ ).

Based on the above solution of static channels, the PLT strategy for time-varying channels is as follows. Given the CSI and residual energy at time  $t$ , compute the perceived lifetime and corresponding power allocation assuming the current channels remain unchanged in the future:

$$\begin{aligned}
 [n(t)^*, \mathbf{P}(t)^*] &= \\
 & \arg \max_{\{P_{kj}(t)\}, n(t)} \left\{ n(t) : \text{(i)-(iii) of (30) satisfied at time } t \right\} & (32)
 \end{aligned}$$

Finally, one may also generalize the MWTP approach (and similarly MTP) to the multiple-

source scenario. The power allocation at time  $t$  can be computed from the following optimization:

$$\mathbf{P}(t)^* = \arg \min_{P_{kj}(t)} \left\{ \sum_{k=1}^N \frac{\sum_{j=1}^M P_{kj}(t)}{\mathcal{E}_k(t)} : \right. \quad (33)$$

$$\left. \text{(i)-(iii) of (30) satisfied at time } t \right\}.$$

Since (33) is convex, it can be solved with standard convex optimization procedure to find the optimal  $P_{ij}^*(t)$ .

## VI. SIMULATION RESULTS

To evaluate the performance of PLT, MWTP, and MTP in static and slow-varying channel conditions, we present simulation results in these section for different relay cooperation setups.

### A. Single Source

1) *Network Setup*: Fig.3 shows two types of network setup we consider for the single-source simulations: a) fixed node locations and static or slow-fading channel environment, and b) a moving source creating time-varying channels due to path-loss variation. In all presented results, the path-loss exponent is assumed  $\alpha = 2$ , and the noise level  $\sigma_n^2 = 10^{-4}\text{W} = -40\text{dBW}$ . We further assume that the source power and the transmission time slot are normalized to unity, i.e  $P_s = 1\text{W}$  and  $\Delta = 1\text{s}$ .

2) *Performance under Static Channels*: We first consider the static-channel case and use Fig.3(a) for the network setup. A source-destination pair is fixed at  $S(-16m, 0m)$  and  $D(14m, 0m)$ . Four relay nodes are placed at coordinates  $R_1(10m, -7m)$ ,  $R_2(-13m, 7m)$ ,  $R_3(0m, 10m)$  and  $R_4(0m, -10m)$ . Their initial energies are set to  $\underline{\mathcal{E}}(0) = [10\text{KJ}, 2\text{KJ}, 20\text{KJ}, 2\text{KJ}]$ . Because of the static-channel assumption, no movement is considered in this experiment. Fig.4 depicts the achieved lifetime vs. the minimum required rate under PLT, MWTP, and MTP. We observe that as the required rate increases, the network lifetime decreases due to the increase of power consumption. The PLT scheme, which gives the optimal allocation for the static-channel case, clearly outperforms both MWTP and MTP. Moreover, the MWTP scheme achieves considerably longer lifetime than that of MTP. For example, at the rate requirement of  $R = 1.3\text{bits/s/Hz}$ , the lifetime of MWTP is almost doubled compared with that of MTP.

To better understand the above improvement, in Fig.5, we plot the residual energy of each relay when the network's lifetime is reached under the three PA schemes. We observe that, in

the MTP scheme, because  $R_2$  has the most favorable channel condition, the energy of  $R_2$  has been used predominantly to satisfy the required rate. Upon energy depletion at  $R_2$ , the other three relays fail to maintain the required rate, and the network reaches its lifetime. The MWTP scheme, on the other hand, manages to allocate some power at  $R_4$  to help relaying while still relay on  $R_2$  as the main node for relaying. When the network lifetime is reached, a substantial amount of energy remains in  $R_1$  and  $R_3$ , which is not sufficient for the network to provide the required rate. Noted that the ratio of wasted energy under the MTP is even higher than that under the MWTP. In contrast, we observe that all relays in PLT deplete their energy at the same time, demonstrating that the energy of each relay has been efficiently used. Such concurrent energy depletion does not typically occur in the other two schemes.

3) *Performance under Time-Varying Channels:* We consider the scenario with slow time-varying channels. Since the channel variation can be factored into large-scale variation and small-scale multipath fading, we test the performance under small-scale variation and large-scale variation separately. First, we consider slow fading over all links due to small-scale multipath fading. The node placement in Fig.3(a) is considered. To model the slow-fading channels, we employ Jakes' fading model with normalized fading rate  $f_d T_s = 0.005$ , where  $f_d$  is the Doppler frequency and  $T_s$  is a symbol duration [29]. Fig.6 demonstrates the average lifetime vs. the required rate under each power allocation scheme. As expected, we see that the lifetime under noncausal optimal allocation serves as an upper bound for PLT, MWTP, and MTP, while the performance of PLT and MWTP are closer to this upper bound. This demonstrates the importance of considering the residual energy in power allocation.

Next, we consider the case of a moving source causing large-scale path-loss variation of the channels. The network setup is shown in Fig.3(b) where the source  $S$  transmits data to destination  $D$  through two relay nodes  $R_1$  and  $R_2$ . Assume that the source moves with a constant speed  $v = 0.3\text{m/s}$  in random directions confined in a circle with center  $S(-30\text{m}, 0\text{m})$  and radius  $r = 4\text{m}$ , and two relays are placed at  $R_1(-20\text{m}, 5\text{m})$  and  $R_2(-20\text{m}, -15\text{m})$ . Initial energy levels  $\underline{\mathcal{E}}(0) = [1\text{KJ}, 1\text{KJ}]$  are assigned to  $R_1$  and  $R_2$ . Using Monte Carlo simulation, the average lifetime of the power allocation schemes vs. the required rate is plotted in Fig.7. Comparing the performance under each scheme, we see that PLT nearly achieves the maximum lifetime under the noncausal optimal solution. Compared with MTP, both PLT and MWTP provides significant lifetime improvement. For example, at  $R = 1.8\text{bps/Hz}$ , the lifetime increment is approximately

30%.

4) *Effect of Asymmetric Initial Energy:* In the next experiment, we study the effect of the initial energy levels on the lifetime of different power allocation schemes. We again use the network setup in Fig.3(b). In this experiment, the locations of all nodes including relays, source, and destination are fixed. To remove the effect from the asymmetrical relay links, we place  $R_1$  to  $(-20m, 15m)$ , and fix the source at  $S(-30m, 0m)$ . We vary the ratio of initial energy,  $\beta = \frac{\mathcal{E}_1(0)}{\mathcal{E}_2(0)}$ , from 1 to 10, while keeping the sum initial energy unchanged, i.e.,  $\mathcal{E}_1(0) + \mathcal{E}_2(0) = 2KJ$ . In Fig.8, we plot the network lifetime for two rate requirements:  $R = 1.7$  or  $1.9\text{bits/s/Hz}$ . For  $\beta = 1$ , the network setup becomes completely symmetrical both in terms of the relays residual energy and the links' channel conditions. Due to this symmetry, it is not hard to prove that the PLT, MWTP, and MTP would result in the same power allocation solution and identical lifetime performance. This fact is verified in Fig. 8 at  $\beta = 1$ . As  $\beta$  increases, i.e. the asymmetry increases, the lifetime gap among the three schemes becomes larger, with PLT giving the best performance. Results in Fig.7 and Fig.8 demonstrate that using the energy-aware power allocation schemes are particularly more effective in the asymmetric network setup which is common in practice.

### B. Multiple Sources

We now consider a multiple-source relay network with time-varying channels. The network setup is shown in Fig.9, where the two relays are fixed while the two sources are assumed to move at speed  $v = 0.3m/s$  independently in random directions inside two circles with radius  $r = 3m$  and centered at  $S_1(-20m, 10m)$  and  $S_2(-20m, -10m)$ . We assume that the two sources have the same rate requirement. We further assume that the first relay,  $R_1$ , has double the initial energy compared to the second relay,  $R_2$ . Fig. 10 shows the lifetime vs. the rate requirement under different power allocation schemes. This figure again demonstrates that by incorporating the residual energy through either PLT or MWTP, the network lifetime can be significantly increased.

For the multiple-source case, it is of interest to investigate how each relay splits its power among existing sources. Fig. 11 and Fig. 12 illustrate the relay power distribution of  $R_1$  and  $R_2$  for the network setup given in Fig. (9) with fixed  $S_1$  and  $S_2$  at  $(-20m, 10m)$  and  $(-20m, -10m)$ , respectively. The rate threshold value for both users is set at  $R = 2.3\text{bits/s/Hz}$ . The ratio  $\beta = \mathcal{E}_1/\mathcal{E}_2$  is varied from 0.5 to 5. Note that  $\mathcal{E}_1$  and  $\mathcal{E}_2$  are the instantaneous residual energy levels

(not necessarily initial energy). Using the same notation as before,  $P_{ki}$  denotes the amount of power spent by relay  $k$  to help source  $i$ . From Fig. 11 and Fig. 12, we see that for  $\beta = 1$  (a symmetric setup), using any of the three schemes would result in the same power splitting for both relays. As  $\beta > 1$ , for both PLT and MWTP,  $R_1$  with higher remaining energy  $\mathcal{E}_1$  will spend more power than  $R_2$ , with  $R_1$  in PLT using more power than that in MWTP. This is similar to the single-source case in Section IV-D. In terms of power splitting for different sources, we see that, for  $R_1$ , as  $\beta$  increases, the percentage of power allocated to  $S_2$  increases. Since  $S_1$ 's required rate is met by using a relatively fixed power  $P_{11}$  from  $R_1$ , the additional power of  $R_1$  is devoted to support  $S_2$  to satisfy its required rate. On the other hand,  $R_2$  with much less remaining energy would be less willing to spend the power for the farther source  $S_1$ .

## VII. CONCLUSION

Energy-aware power allocation for network lifetime maximization was considered in this paper for a dual-hop cooperative network operated by battery-limited AF relays. For a given minimum rate requirement, the problem was first formulated as convex optimization with a global noncausal solution. In the case of static channels, a closed-form and easy-to-implement solution was shown. Inspired by this solution, we presented a causal implementation of this optimal solution for the time-varying scenario, *i.e.*, the PLT scheme. Furthermore, we proposed an alternative MWTP scheme, which is independent of future CSI prediction, and analytically compared its performance to that of the PLT scheme. Unlike the traditional scheme of minimizing total power per transmission, both MWTP and PLT exploit the CSI as well as the state of residual energy to assign power to each relay. We further extended these schemes to relay cooperation with multiple sources. In simulation, we considered the cases of both static channels and slow-varying channels that are either due to small-fading or moving nodes causing path-loss variation. Both schemes demonstrated a significant lifetime improvement over the MTP scheme. The energy-aware schemes are particularly effective under asymmetry in terms of either relay link condition or the initial energy, which is common in practical relay networks.

Note that, in the PLT scheme, a simple prediction is applied based on current CSI assuming the future CSI is the same as the current one. For channels with time correlations, we could exploit the current and past channel to better predict the future channel gain, and possibly improve the performance. The issue of how much performance improvement vs. the complexity of computing

power allocation as compared to the PLT needs to be studied in details. How to modify the power allocation updating method to improve the performance is currently under our investigation as the future work.

## APPENDIX A

### PROOF OF PROPOSITION 1

For the solution presented in Section III to solve (6), we first look at the inner loop with a given  $n$ . Consider the subgradient iteration method for solving (10). We initialize the Lagrange multiplier  $\boldsymbol{\mu}$  with an  $n \times 1$  vector of 1's, i.e.  $\boldsymbol{\mu}^{(0)} = \mathbf{1}$ . The maximum of  $g_k(\boldsymbol{\mu}^{(0)}, n)$  in (9) should be determined through finding the optimal  $\{P_k^{0*}(t)\}_{t=1}^n$ . Since the channels  $b_k(t)$  and  $c_k(t)$  are static, the optimal  $P_k^{0*}(t)$  shown in (11) is therefore constant over  $t$  as well, and it reduces to the following expression

$$P_k^{0*}(t) = \frac{\mathcal{E}_k(0)}{n}. \quad (34)$$

For the update of  $\boldsymbol{\mu}^l$  in (12), since  $a(t)$  and  $P_k^{0*}(t)$  are constant for all  $t = 1, \dots, n$ ,  $\mu_t^1$  is the same for all  $t = 1, \dots, n$  as well, and  $\boldsymbol{\mu}^1 = \mu_1^1 \mathbf{1}$ . In the next iteration with  $\boldsymbol{\mu}^1$ , we see that  $P_k^{1*}(t)$  in (11) is again constant over  $t$ . Following the updating procedure, we see that  $\boldsymbol{\mu}^l$  will always contain equal entries, and  $P_k^{l*}(t)$  has the expression in (34) for every iteration until convergence. Therefore, for a given  $n$ , we have the optimal  $P_k^*(t) = \frac{\mathcal{E}_k(0)}{n}$ , for  $t = 1, \dots, n$ .

In the outer loop, we then search for the maximum  $n^*$  that satisfies the rate constraint given in (ii) of (6). Since we have shown that, for any feasible  $n$ , the optimal power allocation  $P_k^*(t) = \frac{\mathcal{E}_k(0)}{n}$ , the maximum  $n^*$  then can be found as shown in (14).

## APPENDIX B

### PROOF OF (11)

The Lagrangian function of (9) with the multiplier  $\lambda$  is written as

$$\Gamma_k(P_k(1), \dots, P_k(n), \lambda) = \sum_{t=1}^n \mu_t \left( \frac{P_s P_k(t) b_k(t) c_k(t)}{1 + P_s b_k(t) + P_k(t) c_k(t)} + P_s a(t) \right) - \lambda \left( \sum_{t=1}^n P_k(t) - \mathcal{E}_k(0) \right), \quad (35)$$



for  $k = 1, \dots, N$ , where  $\gamma_k$  in (9) was substituted by (3). Then by applying the KKT conditions as follow

$$\frac{\partial \Gamma}{\partial P_k^*(t)} \begin{cases} = 0, & \text{if } 0 < P_k^*(t) < \mathcal{E}_k(t) \\ \geq 0, & \text{if } P_k^*(t) = 0 \\ \leq 0, & \text{if } P_k^*(t) = \mathcal{E}_k(t), \end{cases} \quad (36)$$

the optimal  $P_k^{l*}(t)$  can be obtained as

$$P_k^{l*}(t) = \left[ \frac{\sqrt{\mu_t^l \lambda^{-1} P_s b_k(t) c_k(t) (1 + P_s b_k(t))} - 1 - P_s b_k(t)}{c_k(t)} \right]^+, \quad (37)$$

where  $\lambda$  is chosen such that the constraint (i) in (6) is met.

## APPENDIX C

### PROOF OF (27)

By comparing the two equations (22) and (25), we can find the intersection point of MWTP and PLT. Defining a new variable  $x = \sqrt{\frac{\mathcal{E}_1}{\mathcal{E}_2}}$  and performing ratio cross-multiplication on the equations (22) and (25), we have

$$x^2(\gamma_{th} - P_s a - P_s b_1) b_2 c_1 - b_1 c_2 (\gamma_{th} - P_s a - P_s b_2) = 0. \quad (38)$$

Solving this leads to the right-hand-side of (27). After this point, as  $\mathcal{E}_1/\mathcal{E}_2$  increases,  $\kappa^{\text{PLT}}$  grows faster than  $\kappa^{\text{MWTP}}$ .

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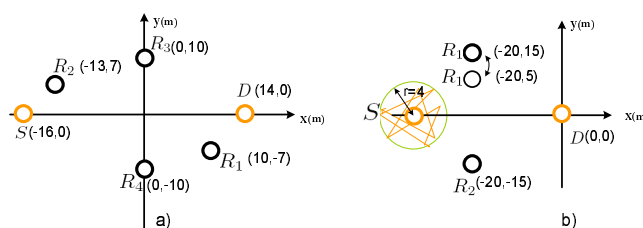


Fig. 3. Simulation configurations: a) used for static channel and slow small-scale fading experiments in Fig.4 - Fig.6; b) used for source-moving experiment with large-scale path-loss variation in Fig.7 and Fig.8.

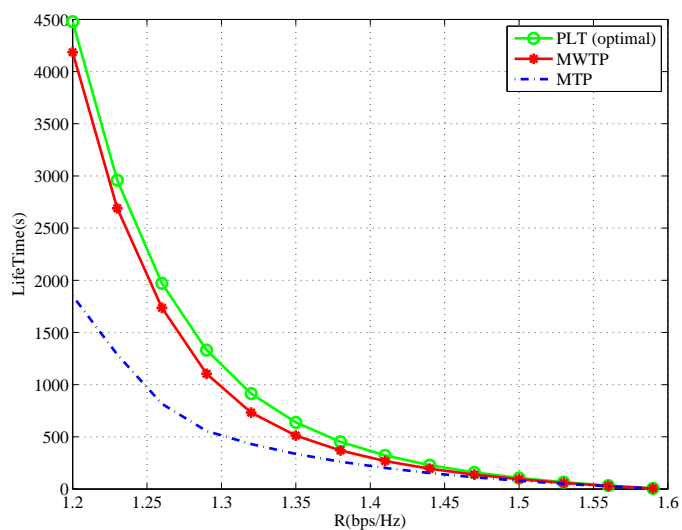


Fig. 4. Lifetime vs. rate requirement for static-channel case with the network setup in Fig.3(a).

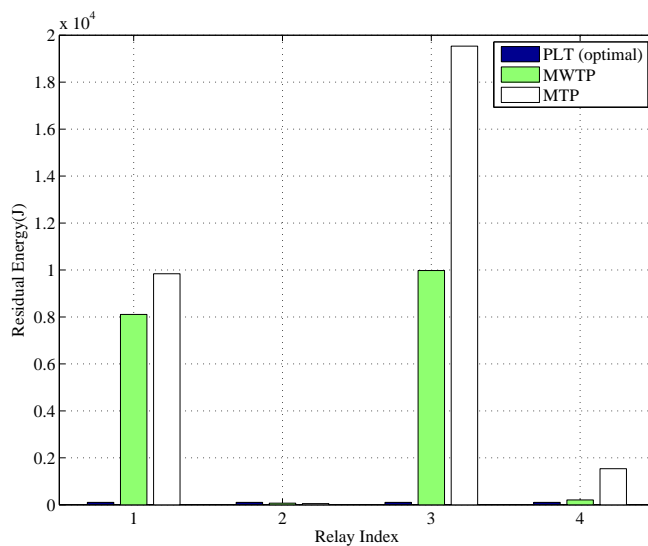


Fig. 5. Residual energy of each relay at the end of lifetime with the initial energy  $\mathcal{E}(0) = [10\text{KJ}, 2\text{KJ}, 20\text{KJ}, 2\text{KJ}]$ , and the required rate  $R = 1.3\text{bps/Hz}$ .

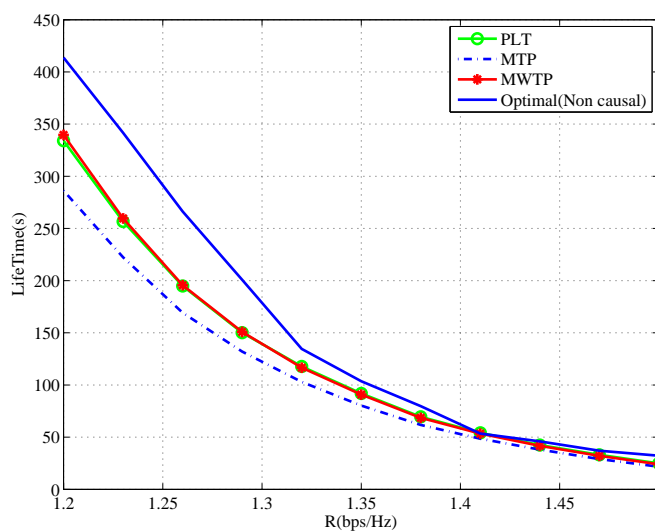


Fig. 6. Lifetime vs. rate requirement for slow-fading channel case with the network setup in Fig.3(a) ( $f_d T_s = 0.005$ ).

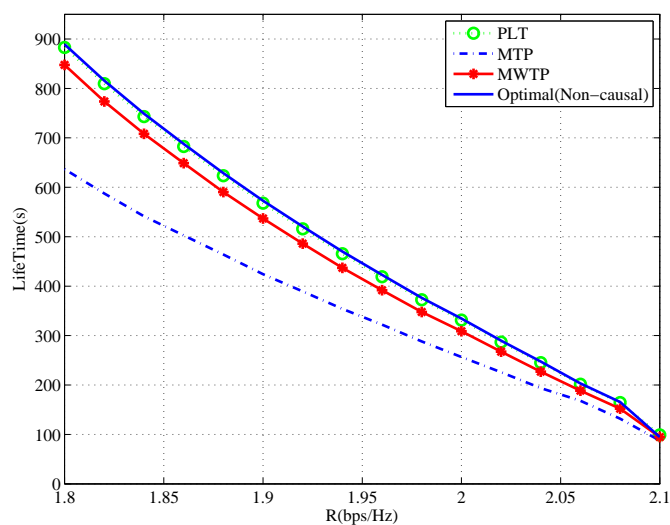


Fig. 7. Lifetime vs. rate requirement for the moving-source setup in Fig.3(b).

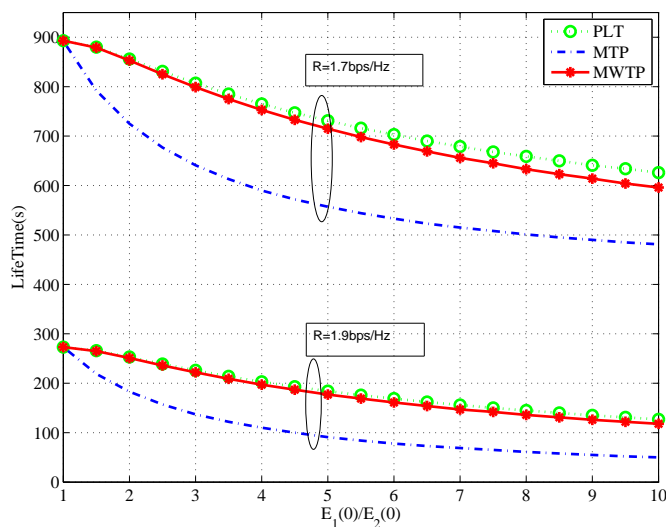


Fig. 8. Effect of initial energy ratio on lifetime for static channel with two relays.

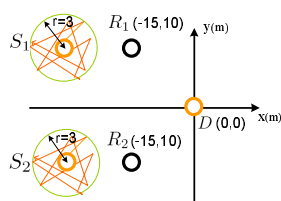


Fig. 9. Simulation configuration: multiple-source experiment setup in Fig. 10.

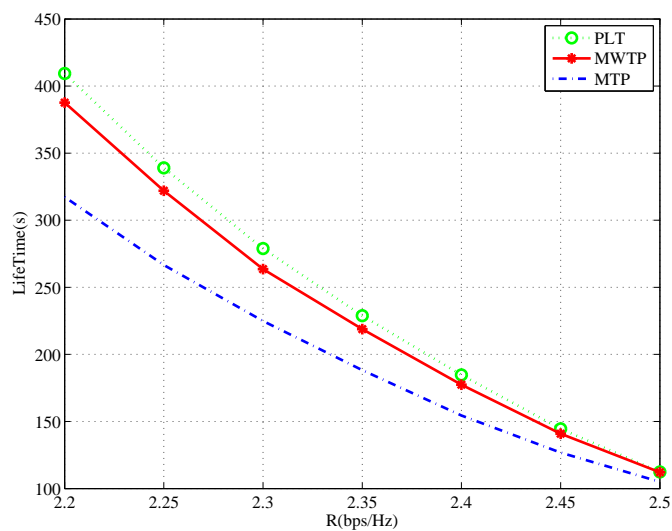


Fig. 10. Lifetime vs. rate requirement for  $M = 2$  and  $N = 2$ .

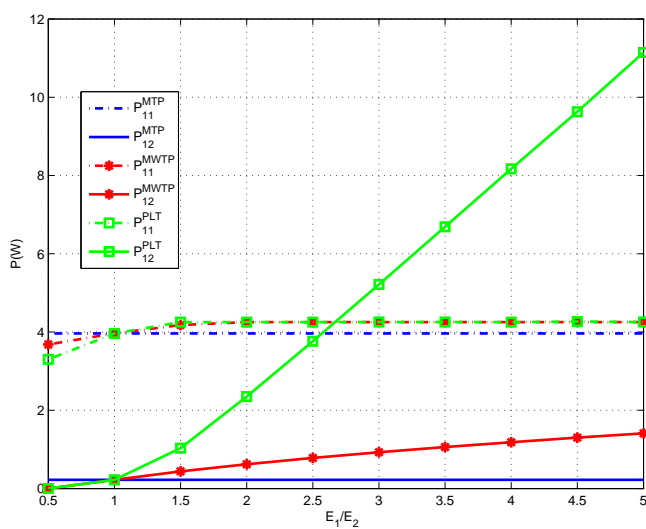


Fig. 11. Relay 1's power distribution among two sources vs. residual energy ratio for static-channel case with  $N = 2$ .

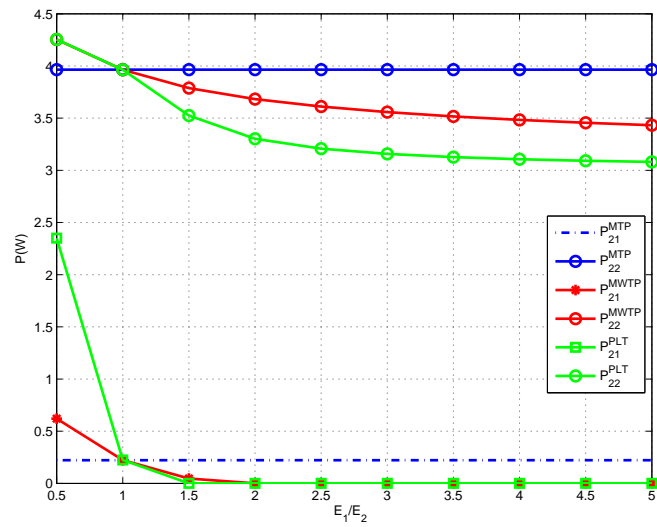


Fig. 12. Relay 2's power distribution among two sources vs. residual energy ratio for static-channel case with  $N = 2$ .