

Performance Analysis of Heterogeneous Cellular Networks With HARQ Under Correlated Interference

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Abstract—Hybrid automatic repeat request (HARQ) is widely used in heterogeneous cellular networks (HCNs) to improve communication reliability. The temporal interference correlation caused by the common set of interferers makes the performance of HARQ more complex, especially for Type-II HARQ where the unsuccessful packets are combined with the new one to decode the packet. In general, due to the complexity of network performance analysis, the existing research focused on the performance of HARQ in single-tier wireless networks without considering cell association or base station (BS) load or the case that the combined number of transmissions is no larger than 2. In view of this, we study the performance of HARQ in HCNs jointly considering the temporally correlated interference, flexible cell association, and BS load. To this end, we adopt the popular HCN model, where different types of BSs in HCNs are modeled as K independent Poisson point processes with different densities and transmission powers. Leveraging the tool of stochastic geometry, we derive the success probability and delay-limited throughput for HCNs with Type-I HARQ and Type-II HARQ, respectively, for any number of transmissions. We show that the network performance in multiple time slots is decided by the performance in a single time slot and the temporal interference correlation. Finally, we conduct simulations to validate our analysis and show that the analysis without considering temporal interference correlation overestimates the performance of HCNs with HARQ.

Index Terms—Heterogeneous cellular networks, interference correlation, hybrid automatic repeat request, stochastic geometry, delay-limited throughput.

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I. INTRODUCTION

HETEROGENEOUS cellular networks (HCNs), comprising different kinds of irregular heterogeneous infrastructure elements with conventional BSs, have been put forward to effectively address the immense growth in data demand [1], [2]. As the key to the performance of HCNs, interference has been widely investigated in literature. It is shown that the interference is temporally correlated since it comes from the same set of interferers in different time slots [3]. Such correlation makes the link success events temporally correlated and thus dramatically affecting the network performance. As a result, the effects of interference correlation on network performance should be considered in the performance analysis.

Extensive research has devoted to investigating the coverage probability and average rate of HCNs based on the statistics of interference in a single time slot [4]–[7]. The most popular method is to model different kinds of randomly distributed BSs as multiple independent Poisson Point Processes (PPPs) and analyze the network performance using the tool of stochastic geometry [8]–[14]. Based on this method, the researchers have calculated the success probability and the average rate in HCNs considering strongest-BS association [4], flexible cell association [5], the effects of shadowing [6], and the effects of offloading [7]. Note that all the aforementioned studies focus on instantaneous performance of HCNs without considering the interference correlation, which is insufficient to comprehensively reflect the performance of HCNs under the correlated interference [3], [15].

Therefore, it is well worth studying the correlation of interference in wireless networks. Ganti and Haenggi quantified the interference correlation in ALOHA ad hoc networks, whose nodes are distributed as a PPP and found that there exists temporal and spatial interference correlation even under independent channel fading [16]. Based on their work, people started to investigate the impacts of interference correlation on network performance, which fall into the following three categories: the investigations on temporal interference correlation [17]–[20], spatial interference correlation [21], [22], and local delay [23]–[25]. In this paper, we focus on the temporal interference correlation, which is informally defined as the relationship among the interference in different time slots.

Such correlation brings the phenomenon that if a transmission fails in the current time slot, there is a high probability that the subsequent transmissions also fail in the next several time slots. Thus, it has an important impact on the performance of retransmission schemes.

As an important retransmission scheme, hybrid automatic repeat request (HARQ) is mainly classified into the following two categories: Type-I HARQ and Type-II HARQ [26]. The main difference between these two strategies is that the unsuccessful packets in Type-I HARQ are discarded at the receiver while those in Type-II HARQ are stored to combine with the new one to improve the network performance. The authors in [27] investigated the performance of Type-I HARQ in ad hoc network with correlated interference. However, the results are confined in one-tier Poisson networks without considering cell association or BS load, which can hardly provide valuable insights for HCNs. For Type-II HARQ, maximum-ratio combining (MRC) are usually used to maximize the SIR at receivers. The performance of MRC under spatial interference correlation was investigated under Rayleigh fading in [21] and Nakagami fading in [28]. It was found that the full-correlation assumption underestimates the performance while the no-correlation assumption overestimates it. Considering BS cooperation, the authors in [29] investigated the coverage probability with and without MRC, respectively, and found that the retransmissions always yield time diversity. However, for the performance of MRC, exact results were derived only for the case that the number of combined transmissions $n = 2$ and approximated ones were derived for $n > 2$ [21], [28], [29].

In view of this, we study the performance of HCNs jointly considering cell association, BS load, and temporally correlated interference. Modeling different kinds of BSs as multiple independent PPPs, we obtain the exact expressions of the success probability and the delay-limited throughput for HCNs with Type-I HARQ and Type-II HARQ for general number of retransmissions. The results show that the network performance in multiple time slots are decided by both the instantaneous network performance and the temporal interference correlation. The analysis without considering temporal interference correlation overestimates the performance of HCNs.

II. NETWORK MODEL AND METRICS

A. Multi-Tier Cellular Networks

In this paper, we model a downlink HCN as a K -tier cellular network. Each tier represents a different type of BSs which is distinguished by its BS density λ_k , transmit power P_k , and cell-selection bias B_k . The locations of BSs in the k^{th} tier are assumed as an independent homogeneous PPP ϕ_k . Users are modeled as an independent PPP ϕ_u with density λ_u . Due to the stationarity of homogeneous PPPs, the statistics of the performance is identical at any user over the whole network [4], [5]. Therefore, we focus on the analysis of a typical user located at the origin to study the network performance. Fig. 1 shows a HCN consisting of three tiers of BSs (macro, pico, and femtocell BSs).

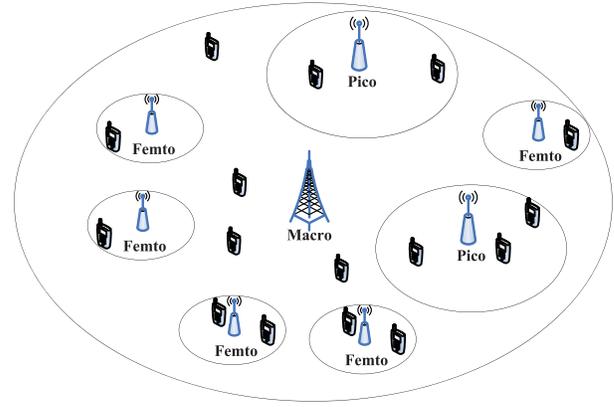


Fig. 1. Illustration of heterogeneous cellular network composed of macro, pico and femtocell BSs and users.

We focus on a fully backlogged system, which is commonly used in the performance analysis of HCNs [4], [5]. For the MAC scheme, we assume all BSs independently transmit packets with probability p . This increases the randomness in MAC, which is beneficial to reducing interference correlation [23]. Note that it is also helpful in decreasing the interference. In HCNs, there exist co-tier and cross-tier interference. Thus, the interference at the typical user associated with BS x_k in the k^{th} tier at time slot t is expressed as

$$I_t(x_k) = \sum_{l=1}^K \sum_{x \in \phi_l, x \neq x_k} P_l 1(x \in \phi_l(t)) h_x(t) g(x), \quad (1)$$

where $\phi_l(t)$ denotes the active BSs of tier- l in time slot t , $1(\cdot)$ is the indicator function, $h_x(t) \sim \exp(1)$ represents the temporally and spatially independent Rayleigh fading, $g(x) = \|x\|^{-\alpha}$ is the standard path loss function with $\alpha > 2$. In this paper, we focus on the interference-limited HCNs where the noise is neglected. Thus, the SIR of the typical user connecting to BS x_k is expressed as follows:

$$\text{SIR}_t(x_k) = \frac{P_k h_{x_k}(t) g(x_k)}{\sum_{l=1}^K \sum_{x \in \phi_l, x \neq x_k} P_l 1(x \in \phi_l(t)) h_x(t) g(x)}. \quad (2)$$

B. Cell Association

In this paper, a user is assumed to connect to the BS providing the *strongest long-term* averaged biased-received-power [5]. Since no mobility or shadowing is considered, the user always connects to the *same* BS, which is given by

$$\text{BS}(z) = \arg \max_{x \in \phi_l, \forall l} P_l \|x - z\|^{-\alpha} B_l, \quad (3)$$

where z is the location of the user and B_l denotes cell-selection bias of the l^{th} tier. Note that B_l represents the association preference of a user to the BSs in tier l .

The association probability A_k is defined as the probability that a user associates with the BSs in the k^{th} tier. It is derived in [5, Lemma 1] as

$$A_k = \frac{\lambda_k (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta}, \quad (4)$$

where $\delta = 2/\alpha$.

Furthermore, the probability density function of the distance between the typical user and its serving BS belonging to the k th tier is expressed as [5, Lemma 3]

$$f_{X_k}(x) = \frac{2\pi\lambda_k \cdot x \cdot \exp\left(-\pi \sum_{l=1}^K \lambda_l \left(\frac{P_l B_l}{P_k B_k}\right)^\delta x^2\right)}{A_k}. \quad (5)$$

We consider both open-access and closed-access modes in this paper [4]. For open access, the typical user is allowed to associate with any tier without any restriction. For closed access, the typical user is only allowed to access to a subset of tiers.

C. Load on Tagged BS

We call the serving BS x_k of the typical user as a tagged BS. The number of users served by the tagged BS x_k is denoted as ψ_k and its approximated distribution is given in [7]:

$$\mathbb{P}(\psi_k = n + 1) = \frac{3.5^{3.5}}{n!} \frac{\Gamma(n+4.5)}{\Gamma(3.5)} \left(\frac{\lambda_u A_k}{\lambda_k}\right)^n \left(3.5 + \frac{\lambda_u A_k}{\lambda_k}\right)^{-(n+4.5)}, \quad (6)$$

where $\Gamma(x) = \int_0^\infty \exp(-t) t^{x-1} dt$ is the gamma function and A_k denotes the association probability which is given by (4). Accordingly, the mean load is given by:

$$\bar{N}_k = \mathbb{E}[\psi_k] = 1 + 1.28 \frac{\lambda_u A_k}{\lambda_k}. \quad (7)$$

For tractability, we assume the users in each BS are allocated equal time-frequency resources. Under this assumption, the downlink rate in bps of the typical user associated with BS x_k is

$$\mathfrak{R}_k = \frac{W}{\psi_k} \log_2(1 + \text{SIR}(x_k)), \quad (8)$$

where W denotes the total effective bandwidth in Hz.

D. HARQ Techniques

In the considered downlink HCNs, the HARQ protocol is adopted to enhance the reliability and efficiency of communication [26]. Specifically, BSs first transmit an encoded packet with rate R decided by some specific application requirement to its associated user. The user sends an acknowledgment (ACK) to its associated BS x_k if it decodes this packet successfully. Otherwise, it sends a nonacknowledgment (NACK) to this BS. After receiving a NACK message, BS x_k sends the same coded packet to its user with probability p in each time slot until it receives an ACK feedback or the maximum number of transmissions is reached.

In this paper, we quantify the performance of type-I HARQ and HARQ-CC, respectively. For the type-I HARQ protocol, users drop the unsuccessful packets and decode the packet received in each retransmission independently. While for the HARQ-CC protocol, users combine the previously received packets with the latest received packets for signal decoding.

E. Performance Metric

1) *Delay-Limited Throughput*: As an important performance metric, delay-limited throughput is defined as, for a randomly chosen user, the expected ratio between the rate of

a coded packet and the number of transmissions of the packet until it is successfully transmitted. Since the delay-limited throughput accounts for the times of retransmission, it gives an accurate rate of transmission and has been widely studied for HARQ in various communication systems [27].

Following the definition of delay-limited throughput and the fact that the associated BS will send the same coded packet to its user until it receives ACK feedback or the maximum number of transmissions is reached, the delay-limited throughput for the typical user conditioned on the event that its tagged BS belongs to the k th tier, i.e., DLT_k , is given as

$$\text{DLT}_k = \sum_{m=1}^M \frac{R}{m} \cdot \text{SP}_k^m, \quad (9)$$

where R is the rate for the encoded packet which is decided by its application requirement, and SP_k^m is the success probability for a user with its tagged BS belonging to the k th tier in the m th transmission, and M denotes the maximum number of transmissions. Note that we call the first transmission and all the retransmissions as transmissions in this paper.

In the context of our system model, HCNs consist of K tiers of BSs and a user connects to at most one BS in HCNs. Based on the total probability theorem, delay-limited throughput for a randomly chosen user in HCNs is given as

$$\text{DLT} = \sum_{k=1}^K \text{DLT}_k \cdot A_k, \quad (10)$$

where DLT_k is given in (9) and A_k is the association probability given in (4).

From (9), we know that SP_k^m is the key to investigating the delay-limited throughput.

2) *Success Probability*: The success probability in the m th transmission SP^m is the probability that the transmission fails in the first $m - 1$ transmissions and succeeds in the m th transmission. Such definition follows from the HARQ protocol, where the m th transmission happens only when the previous $m - 1$ transmissions fail. In addition, based on flexible cell association, a user connects to at most one BS in HCNs. Thus, according to the total probability theorem, the success probability for a randomly chosen user in the m th transmission is given as

$$\text{SP}^m = \sum_{k=1}^K \text{SP}_k^m \cdot A_k, \quad (11)$$

where SP_k^m is the success probability for a user connected to the k th tier BS in the m th transmission.

III. TYPE-I HARQ ANALYSIS

In this section, we study the success probability and the delay-limited throughput in HCNs with Type-I HARQ under both open and closed access.

A. Open Access

1) *Success Probability*: In order to derive the success probability in the M th transmission, we now compute the joint

success probability (JSP) which is defined as the probability that a user successfully receives packets from its associated BS in M successive transmissions. To this end, we first derive the conditional JSP in Lemma 1, given that the typical user associates with the BS x_k in the k th tier. By definition, the conditional success probability, JSP_k^M , is expressed as

$$\text{JSP}_k^M(\beta) \triangleq \mathbb{P}(\text{SIR}_{t_1}(x_k) > \beta, \dots, \text{SIR}_{t_M}(x_k) > \beta). \quad (12)$$

Lemma 1: Given SIR threshold β , the joint success probability for a user connecting to the k th tier BS is

$$\begin{aligned} \text{JSP}_k^M(\beta) &= \frac{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta}{\delta \sum_{l=1}^K \lambda_l (P_l B_l)^\delta Q_M(\delta, p, \frac{B_k \beta}{B_l}) + \sum_{l=1}^K K \lambda_l (P_l B_l)^\delta}, \end{aligned} \quad (13)$$

where $Q_M(\delta, p, \frac{B_k \beta}{B_l}) = \sum_{j=1}^M \binom{M}{j} \frac{(-1)^{j+1}}{j-\delta} p^j \left(\frac{B_k \beta}{B_l}\right)^j$.
 ${}_2F_1(j, j-\delta; j-\delta+1; \frac{-B_k \beta}{B_l})$ and $\delta = 2/\alpha$.

Proof: See Appendix A for details. ■

When $M = 1$, the success probability in a single time slot is expressed as:

$$\text{JSP}_k^1(\beta) = \frac{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta}{\delta \sum_{l=1}^K \lambda_l (P_l B_l)^\delta Q_1(\delta, p, \frac{B_k \beta}{B_l}) + \sum_{l=1}^K \lambda_l (P_l B_l)^\delta}.$$

Remark 1: Joint success probability in multiple time slots is jointly decided by the success probability in a single time slot $\text{JSP}_k^1(\beta)$ and diversity polynomial $Q_M(\delta, p, \frac{B_k \beta}{B_l})$ [17] capturing the temporal interference correlation.

Remark 2: Given SIR threshold β and $B_k = 1, \forall k$, the joint success probability for a user connecting to tier k is simplified as

$$\text{JSP}_k^M(\beta) = \frac{1}{1 + \delta Q_M(\delta, p, \beta)},$$

which is independent with BS density and transmit power. This is because when the unbiased cell association ($B_k = 1, \forall k$) is adopted, users connect to the BS providing the strongest received power. In this case, increasing BS density and transmit power enhance the power and interference in the same factor.

Based on the derived joint success probability, we calculate the success probability given the SIR threshold β in the following Lemma.

Lemma 2: Given SIR threshold β , the success probability for a user connecting to the k th tier BS is expressed as

$$\begin{aligned} \text{SP}_k^M(\beta) &= \sum_{j=1}^M (-1)^{j+1} \binom{M}{j} \text{JSP}_k^j(\beta) \\ &\quad - \sum_{j=1}^{M-1} (-1)^{j+1} \binom{M-1}{j} \text{JSP}_k^j(\beta), \end{aligned} \quad (14)$$

where $\text{JSP}_k^j(\beta)$ is the conditional joint success probability given in Lemma 1.

Proof: See Appendix B for details. ■

The success probability is a linear function of the joint success probability. Thus, the conclusions in Remark 1 and Remark 2 still hold for the success probability in Lemma 2.

Lemma 2 shows the success probability given SIR threshold β . According to Eq. (8), the SIR threshold required to successfully decode an encoded packet with rate R is defined as $\beta_{n+1} \triangleq 2^{\frac{R}{W} \cdot (n+1)} - 1$ given the condition that the number of users connecting to the BS is $\psi_k = n + 1$. De-conditioning on ψ_k , we derive the success probability for a user associated with the k th tier BS denoted by SP_k^M . Further, the success probability for a randomly chosen user in HCNs, denoted by SP^M , is given by the total probability theorem.

Theorem 1: The success probabilities for a user connecting the k th tier BS and for a randomly chosen user in HCNs with Type-I HARQ are given in (15) and (16) as shown at top of the next page.

Proof: The success probability for a user connected to a BS belonging to the k th tier is given as

$$\begin{aligned} \text{SP}_k^M &\stackrel{(a)}{=} \mathbb{E}_{\psi_k} [\mathbb{P}(\text{SIR}_{t_1}(x_k) < \beta_{n+1}, \dots, \text{SIR}_{t_{M-1}}(x_k) < \beta_{n+1}, \\ &\quad \text{SIR}_{t_M}(x_k) \geq \beta_{n+1})] \\ &\stackrel{(b)}{=} \sum_{n \geq 0} \text{SP}_k^M(\beta_{n+1}) \cdot \mathbb{P}(\psi_k = n + 1), \end{aligned} \quad (17)$$

where (a) follows from the fact that the typical user successfully decode the encoded packet with rate R only when $\text{SIR}_t(x_k) \geq \beta_{n+1}$ according to (8) and $\beta_{n+1} = 2^{\frac{R}{W} \cdot (n+1)} - 1$, n is the number of other users associated with the tagged BS except the typical user, (b) follows by the definition of SP_k^M .

Based on the law of total probability, we further derive the success probability for a randomly chosen user in HCNs. ■

The success probability can be further simplified at the cost of sacrificing its accuracy if the number of users in BS x_k is assumed equal to its mean given by Eq. (7).

Corollary 1: Success probability with the mean load approximation for a user connecting to the k -tier BS is:

$$\begin{aligned} \overline{\text{SP}}_k^M &= \sum_{j=1}^M (-1)^{j+1} \binom{M}{j} \text{JSP}_k^j \left(2^{\frac{R}{W} \cdot \overline{N}_k} - 1 \right) \\ &\quad - \sum_{j=1}^{M-1} (-1)^{j+1} \binom{M-1}{j} \text{JSP}_k^j \left(2^{\frac{R}{W} \cdot \overline{N}_k} - 1 \right), \end{aligned} \quad (18)$$

where $\overline{N}_k = 1 + 1.28 \frac{\lambda_u A_k}{\lambda_k}$ is the mean number of users in BS.

Proof: Given the mean number of users in BS x_k denoted as \overline{N}_k , the required SIR threshold for the encoded packet with rate R is given by $\overline{\beta}_k = 2^{\frac{R}{W} \cdot \overline{N}_k} - 1$. Substituting $\overline{\beta}_k$ into (13), we get the result in Corollary 1. ■

Based on the total probability theorem, we derive the success probability with the mean load approximation for a randomly chosen user in HCNs.

2) *Delay-Limited Throughput:* Based on the derived success probability, we obtain the delay-limited throughput for HCNs with type-I HARQ as follows.

Theorem 2: Delay-limited throughputs for a user connecting to the k th tier BS and for a randomly chosen user in HCNs with Type-I HARQ are given in (19) and (20), shown at the top of the next page.

$$\begin{aligned} \text{SP}_k^M &= \sum_{n \geq 0} \left(\sum_{j=1}^M (-1)^{j+1} \binom{M}{j} \text{JSP}_k^j(\beta_{n+1}) - \sum_{j=1}^{M-1} (-1)^{j+1} \binom{M-1}{j} \text{JSP}_k^j(\beta_{n+1}) \right) \\ &\quad \cdot \frac{3.5^{3.5} \Gamma(n+4.5)}{n! \Gamma(3.5)} \left(\frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^n \left(3.5 + \frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^{-(n+4.5)}, \end{aligned} \quad (15)$$

$$\begin{aligned} \text{SP}^M &= \sum_{k=1}^K \sum_{n \geq 0} \left(\sum_{j=1}^M (-1)^{j+1} \binom{M}{j} \mathcal{F}(\beta_{n+1}) - \sum_{j=1}^{M-1} (-1)^{j+1} \binom{M-1}{j} \mathcal{F}(\beta_{n+1}) \right) \\ &\quad \cdot \frac{3.5^{3.5} \Gamma(n+4.5)}{n! \Gamma(3.5)} \left(\frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^n \left(3.5 + \frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^{-(n+4.5)}, \end{aligned} \quad (16)$$

where $\beta_{n+1} = 2^{\frac{R}{W} \cdot (n+1)} - 1$, $\text{JSP}_k^j(\beta)$ is the conditional joint success probability given in Lemma 1 where $\beta = \beta_{n+1}$, and $\mathcal{F}(\beta) = \frac{\lambda_k (P_k B_k)^\delta}{\delta \sum_{l=1}^K \lambda_l (P_l B_l)^\delta Q_j(\delta, p, \frac{B_k \beta}{B_l}) + \sum_{l=1}^K \lambda_l (P_l B_l)^\delta}$.

$$\begin{aligned} \text{DLT}_k^M &= \sum_{n \geq 0} \sum_{m=1}^M \frac{R}{m} \left(\sum_{j=1}^m (-1)^{j+1} \binom{m}{j} \text{JSP}_k^j(\beta_{n+1}) - \sum_{j=1}^{m-1} (-1)^{j+1} \binom{m-1}{j} \text{JSP}_k^j(\beta_{n+1}) \right) \\ &\quad \cdot \frac{3.5^{3.5} \Gamma(n+4.5)}{n! \Gamma(3.5)} \left(\frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^n \left(3.5 + \frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^{-(n+4.5)}, \end{aligned} \quad (19)$$

and

$$\begin{aligned} \text{DLT}^M &= \sum_{k=1}^K \sum_{n \geq 0} \sum_{m=1}^M \frac{R}{m} \left(\sum_{j=1}^m (-1)^{j+1} \binom{m}{j} \mathcal{F}(\beta_{n+1}) - \sum_{j=1}^{m-1} (-1)^{j+1} \binom{m-1}{j} \mathcal{F}(\beta_{n+1}) \right) \\ &\quad \cdot \frac{3.5^{3.5} \Gamma(n+4.5)}{n! \Gamma(3.5)} \left(\frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^n \left(3.5 + \frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta} \right)^{-(n+4.5)}, \end{aligned} \quad (20)$$

where $\beta_{n+1} = 2^{\frac{R}{W} \cdot (n+1)} - 1$, $\text{JSP}_k^m(\beta_{n+1})$ is the joint success probability given in Lemma 1 where $\beta = \beta_{n+1}$, and $\mathcal{F}(\beta) = \frac{\lambda_k (P_k B_k)^\delta}{\delta \sum_{l=1}^K \lambda_l (P_l B_l)^\delta Q_j(\delta, p, \frac{B_k \beta}{B_l}) + \sum_{l=1}^K \lambda_l (P_l B_l)^\delta}$.

Proof: We obtain the delay-limited throughput for a user connecting the k th tier BS by directly substituting (15) into (9). The delay-limited throughput of the overall network is obtained according to the law of total probability. ■

Similarly to the success probability, the delay-limited throughput can be simplified with the mean load assumption.

Corollary 2: In view of the mean load approximation, delay-limited throughput for a user connecting to the k th tier BS is given by:

$$\begin{aligned} \overline{\text{DLT}}_k^M &= \sum_{m=1}^M \frac{R}{m} \left[\sum_{j=1}^m (-1)^{j+1} \binom{m}{j} \text{JSP}_k^j(\beta_{\overline{N}_k}) \right. \\ &\quad \left. - \sum_{j=1}^{m-1} (-1)^{j+1} \binom{m-1}{j} \text{JSP}_k^j(\beta_{\overline{N}_k}) \right], \end{aligned} \quad (21)$$

where $\beta_{\overline{N}_k} = 2^{\frac{R}{W} \cdot \overline{N}_k} - 1$.

Remark 3: Increasing BS density leads to less user served by each BS (since $\overline{N}_k = 1 + 1.28 \frac{\lambda_u A_k}{\lambda_k} = 1 + 1.28 \frac{\lambda_u (P_k B_k)^\delta}{\sum_{l=1}^K \lambda_l (P_l B_l)^\delta}$) and lower required SIR threshold ($\overline{\beta}_k = 2^{\frac{R}{W} \cdot \overline{N}_k} - 1$), thus

making more users successfully communicate with its associated BS and improving the delay-limited throughput.

B. Closed Access

In this sub-section, we evaluate the performance of HCNs under closed-access scenario where a typical user is allowed to associate with only a subset of tiers [4]. Note that we call a tier as a restricted tier if the user is not allowed to associated with this tier. The following corollary shows the success probability and the delay-limited throughput in closed access networks.

Corollary 3: When a typical user is allowed to associate with only a subset $\mathcal{S} \subset \{1, 2, \dots, K\}$, the success probability and the delay-limited throughput for closed access are expressed as

$$\text{SP}_{\text{closed}}^M = \sum_{k \in \mathcal{S}} A_k \cdot \text{SP}_k^M, \quad (22)$$

and

$$\text{DLT}_{\text{closed}}^M = \sum_{k \in \mathcal{S}} A_k \cdot \text{DLT}_k^M. \quad (23)$$

Proof: By definition of closed access, a user is authorized to associate with only a subset of tiers. In the context of our model, if its associated BS lies in the restricted tier, it leads to an outage event *irrespective* of the received SIR from the associated BS [4]. As a result, the success probability of a typical user associated with the restricted tiers is 0. Based on the law of total probability, we obtain the success probability and the delay-limited throughput of the overall network under closed access in Corollary 3. ■

Since the closed access constrains the link connectivity, it intuitively leads to a reduced success probability and delay-limited throughput.

IV. HARQ-CC ANALYSIS

In this section, we calculate the success probability and the delay-limited throughput for HCNs with HARQ-CC, where the users combine the current received packet with the former received packets for signal decoding. To achieve the maximum effective SIR, maximal ratio combining is used in this paper. Thus, the corresponding effective SIR in the M th transmission is given as $\Gamma^{(M)} = \sum_{m=1}^M \Gamma_m$ where Γ_m denotes the SIR in the m th transmission. Note that when $M = 1$, the success probability is calculated based on the result in Theorem 1, since there is no packets combination, i.e., $\Gamma^{(1)} = \Gamma_1$. When $M \geq 2$, it is challenging to calculate the performance of HCNs with HARQ-CC, since the SIR in each time slot is correlated.

To address this challenge, we first focus on the conditional success probability $\widetilde{\text{SP}}_{x_k|\phi}^M(\beta)$ given the locations of BSs ϕ . In this case, the random variables SIR of the typical user in different time slots are independent since the the locations of BSs ϕ are known [25]. Then, we calculate the success probability by deconditioning on ϕ in two steps, first with respect to the locations of interferers and then with respect to the link distance between the user and its associated BS.

Theorem 3: Given SIR threshold β , the success probability for a user connecting to the k th tier BS in HCNs with HARQ-CC in the M th ($M \geq 2$) transmission is given as

$$\begin{aligned} \widetilde{\text{SP}}_k^M &= \int_0^\infty \left[\int_0^\beta \int_0^{\beta-\gamma_1} \cdots \int_0^{\beta-\sum_{m=1}^{M-2} \gamma_m} \right. \\ &\quad \left. \times Z_1 Z_2 \prod_{m=1}^{M-2} B_m C_m \prod_{m=1}^{M-1} d\gamma_m \right] f_{X_k}(x) dx, \quad (24) \end{aligned}$$

where $Z_1 = \sum_{l=1}^K 2\pi \lambda_l \int_{z_l}^\infty \frac{p\theta_k d_k^\alpha r^{1-\alpha}}{(1+\theta_k r^{-\alpha} d_k^\alpha \gamma_{M-1})^2} \cdot \Lambda\left(r, \beta - \sum_{m=1}^{M-1} \gamma_m\right) dr$,

$$Z_2 = \prod_{l=1}^K \exp\left(-2\pi \lambda_l \int_{z_l}^\infty (1 - \Lambda(r, \gamma_{M-1})) \cdot \Lambda\left(r, \beta - \sum_{m=1}^{M-1} \gamma_m\right) r dr\right)$$

$$B_m = \sum_{l=1}^K 2\pi \lambda_l \int_{z_l}^\infty \frac{p\theta_k d_k^\alpha r^{1-\alpha}}{(1 + \theta_k r^{-\alpha} d_k^\alpha \gamma_m)^2} dr,$$

$$C_m = \prod_{l=1}^K \exp\left(-2\pi \lambda_l p \int_{z_l}^\infty \frac{\theta_k r^{1-\alpha} d_k^\alpha \gamma_m}{1 + \theta_k r^{-\alpha} d_k^\alpha \gamma_m} dr\right), \quad \Lambda(x, z) = \frac{p}{1 + \theta_k z d_k^\alpha \|x\|^{-\alpha}} + 1 - p, \quad \text{and } \theta_k = P_l/P_k. \quad \text{Proof: See Appendix C for details.} \quad \blacksquare$$

Unfortunately, a closed form expression for the success probability is impossible in HCNs with HARQ-CC, as the effective SIR in HARQ-CC is the sum of several correlated variables and it is hopeless to derive the corresponding distribution. Luckily, the result in Theorem 3 can be numerically calculated with standard numeric software. While the computational complexity of calculating the success probability increases with the maximum number of transmissions, we propose an analytical approach to study the performance of HCNs with HARQ-CC for any number of transmissions M .

Based on the derived success probability given SIR threshold β , the success probability and the delay-limited throughput for a user associated with the k th tier is calculated according to Eq. (17) and Eq. (9). Further, following by Eq. (11) and Eq. (10), we get the success probability and delay-limited throughput for a randomly selected user in HCNs.

Note that the above calculation is for HCNs with HARQ-CC under open access. Since the access model only affects the link connectivity, the success probability and the delay-limited throughput for closed access are directly calculated according to Eq. (22) and Eq. (23), respectively.

V. NUMERICAL RESULTS

In this section, our focus is on validating our analysis of the success probability and the delay-limited throughput, and showing the effect of system parameters on the network performance. We define the outage probability as the probability that a user cannot successfully receive the packet from its associated BSs after the M th transmission, i.e., $P_{out} = 1 - \sum_{j=1}^M \text{SP}^j$.

Matlab and Monte Carlo method are used for the simulations. In all the simulations, we consider a two-tier HCN where the locations of BSs in each tier are distributed as independent homogeneous PPP. The typical user is located at the origin. The Rayleigh fading channels are independent over different links and different time slots. In each Monte Carlo trial, the locations of BSs and users remain fixed in M time slots. But for different Monte Carlo trials, the locations of BSs and users are generated independently.

A. Analysis Validation and Effects of Interference Correlation

The emphasis in this subsection is on validating the analysis of the success probability and the delay-limited throughput for fixed SIR threshold, i.e., without considering BS load. Given SIR threshold, the success probability of each tier for Type-I HARQ and HARQ-CC are first calculated by Lemma 2 and Theorem 3, respectively. Then, the corresponding delay-limited throughput is obtained by Eq. (9) based on the derived success probability. At last, the success probability and the delay-limited throughput for the overall network are computed according to the total probability theorem.

Fig. 2 and Fig. 3 show the outage probability and the delay-limited throughput of HCNs with Type-I HARQ and HARQ-CC, respectively. Note that the BS load is not considered

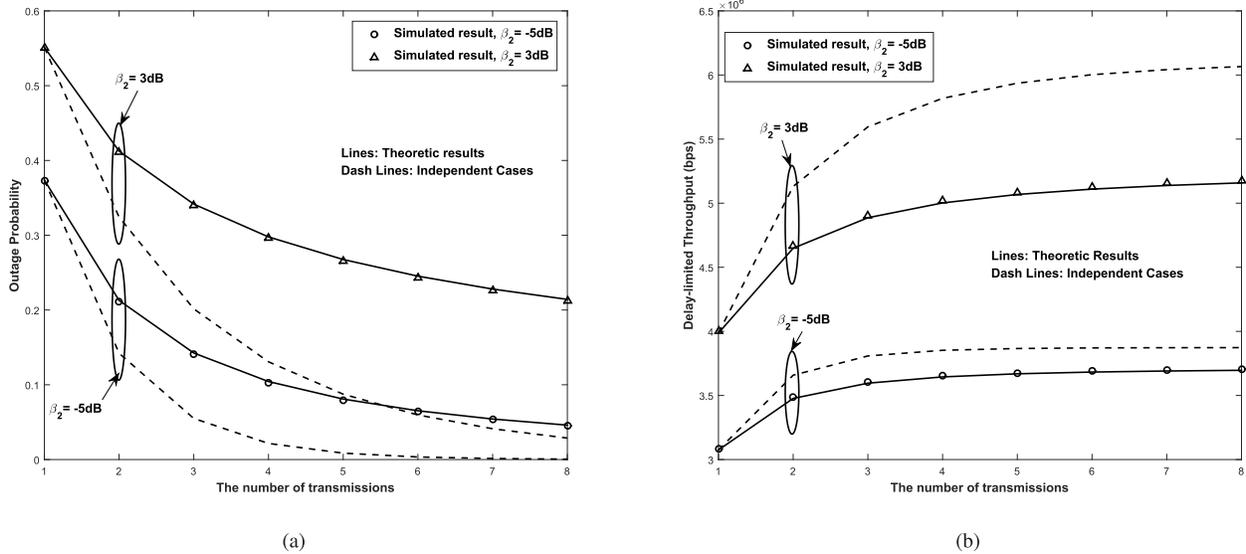


Fig. 2. Outage probability and delay-limited throughput of HCNs with Type-I HARQ under varying number of transmissions ($K = 2$, $W = 10\text{MHz}$, $\alpha = 3$, $\lambda_2 = 10\lambda_1$, $P_1 = 46\text{dBm}$, $P_2 = 30\text{dBm}$, $B_1 = B_2 = 1$, $p = 0.8$, $\beta_1 = -3\text{dB}$).

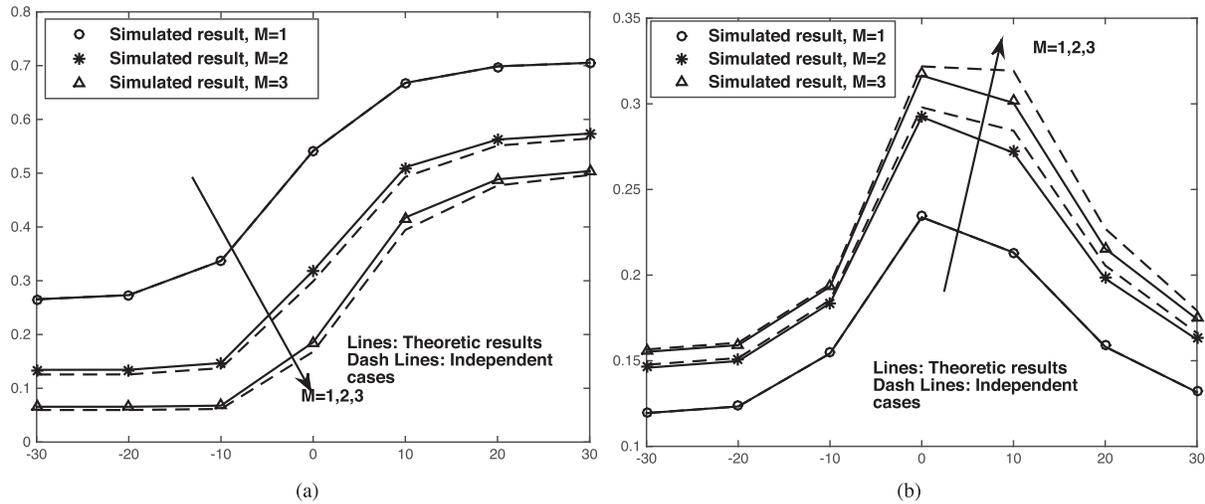


Fig. 3. Outage probability and delay-limited throughput of HCNs with HARQ-CC for varying SIR threshold ($K = 2$, $W = 10\text{MHz}$, $\alpha = 3$, $\lambda_2 = 2\lambda_1$, $P_1 = 4P_2$, $B_1 = B_2 = 1$, $p = 1$, $\beta_1 = -3\text{dB}$).

in these two figures, i.e. the typical user uses the whole resources. First, the good match between our analytical results and simulated results verifies the accuracy of our analysis. Then, we observe that the temporal interference correlation degrades the performance of HCNs with HARQ due to the diversity loss. It means that the analysis without considering the interference correlation leads to an inaccurate performance evaluation. Moreover, from Fig. 2, we find that the performance improvement is large in the first two retransmissions but very small in the following retransmissions. It indicates that increasing the number of retransmission cannot significantly improve the delay-limited throughput. In addition, from Fig. 3(a), we see that the outage probability increases with the increase of SIR threshold. If SIR threshold increases, less users successfully receive the packets and thus the outage probability is increased. However, the successful users receive

packets with higher rate with the increase of SIR threshold. As a result, the delay-limited throughput increases at first but decreases after a certain value as shown in Fig. 3(b).

B. Effects of BS Density

In this subsection, we show the effect of BS density on the performance of HCNs. The analytical success probability and the delay-limited throughput are calculated by Theorem 1 and Theorem 2, respectively. These two metrics with mean load are derived by Corollary 1 and Corollary 2, respectively. Note that the outage probability is given by $P_{out} = 1 - \sum_{j=1}^m SP^j$.

Fig. 4 provides the outage probability and the delay-limited throughput varying with BS density of picocells. From this figure, we observed that HARQ-CC outperforms Type-I HARQ since the former received packets are dropped in Type-I HARQ while they are combined with the current one

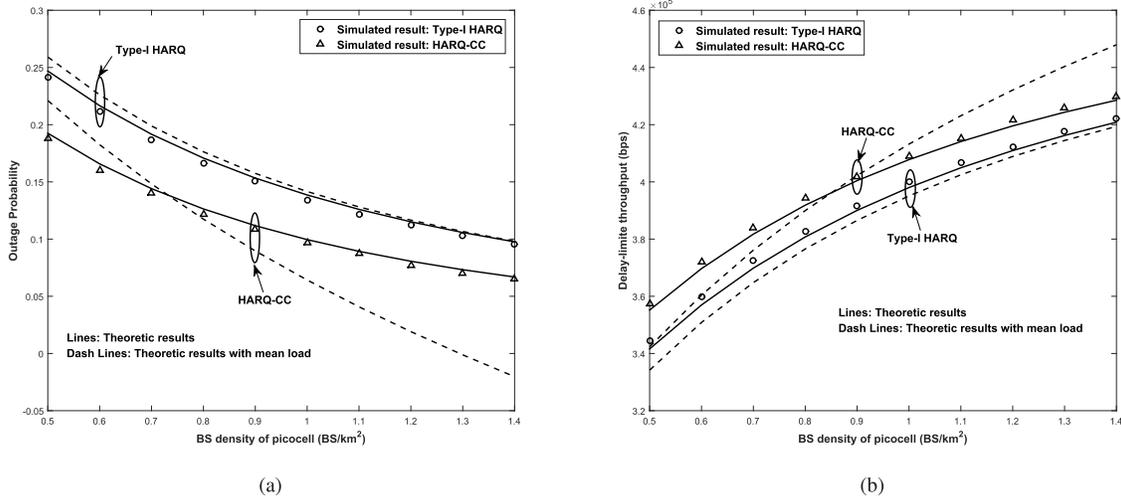


Fig. 4. Outage probability and delay-limited throughput for varying BS density in a two-tier HCNs ($N_{max} = 2$, $W = 10\text{MHz}$, $R_{req} = 500\text{kbps}$, $\alpha = 3$, $\lambda_1 = 0.1$, $\lambda_u = 2$, $P_1 = 46\text{dBm}$, $P_2 = 30\text{dBm}$, $B_1 = 1$, $B_2 = 2$, $p = 1$).

for decoding in HARQ-CC. We also find that the network performance increases with the increase in BS density in terms of the outage probability and the delay-limited throughput. As stated in Remark 3, the more BS there are, the less users are served by each BS, and the lower required SIR threshold is needed for successfully decoding. Therefore, there are more users successfully communicated with its associated BS and the corresponding success probability and delay-limited throughput are improved.

VI. CONCLUSIONS

In this paper, we have jointly considered the temporal interference correlation, flexible cell association, and BS load to investigate the SIR statistics over multiple time slots for HCNs with HARQ. Under the assumptions of multi-tier independent homogeneous PPPs and independent Rayleigh fading, we have derived the success probability and the delay-limited throughput for both Type-I HARQ and HARQ-CC under both open access and closed access scenarios. Further, we have revealed how the system parameters affect the success probability and the delay-limited throughput of each tier and the overall network. Specifically, some important conclusions are as follows: i) The increase in BS density leads to the decrease in the load of each BS and the required SIR threshold for the packet with fixed rate. Thus, it allows more users to transmit successfully with BSs and improves the success probability and the delay-limited throughput. ii) Temporal interference correlation degrades the performance of HCNs with both Type-I HARQ and HARQ-CC due to the reduction of diversity gain.

This work focuses on the network performance in multi-tier independent HCNs, where BSs are independent with each other. An interesting extension is to investigate the network performance in multi-tier HCNs with BS spatial dependence. Furthermore, studying the effects of spatial interference correlation on network performance is also an interesting topic.

APPENDIX: PROOFS

A. Proof of Lemma 1

Since no mobility and shadowing are considered, the typical user connects to the same BS in M successive time slots. Thus, the joint success probability conditioned on tier k being the serving tier is expressed as:

$$\text{JSP}_k^M \triangleq \mathbb{E}_{x_k} [\mathbb{P}(\text{SIR}_{t_1}(x_k) > \beta, \text{SIR}_{t_2}(x_k) > \beta, \dots, \text{SIR}_{t_M}(x_k) > \beta)],$$

where $\mathbb{P}(\text{SIR}_{t_1}(x_k) > \beta, \text{SIR}_{t_2}(x_k) > \beta, \dots, \text{SIR}_{t_M}(x_k) > \beta)$ (denoted as $\text{JSP}_{x_k}^M$) is the joint success probability of the typical user connecting to a given BS located at x_k (i.e. given the distance between the typical user and its serving BS as $\|x_k\|$). To derive JSP_k^M , we first calculate $\text{JSP}_{x_k}^M$.

$$\begin{aligned} \text{JSP}_{x_k}^M &= \mathbb{P}(\text{SIR}_{t_1}(x_k) > \beta, \text{SIR}_{t_2}(x_k) > \beta, \dots, \text{SIR}_{t_M}(x_k) > \beta) \\ &= \mathbb{P}\left(\frac{P_k h_{x_k}(t_1) g(x_k)}{I_{t_1}(x_k)} > \beta, \dots, \frac{P_k h_{x_k}(t_M) g(x_k)}{I_{t_M}(x_k)} > \beta\right) \\ &= \mathbb{P}\left(h_{x_k}(t_1) > \frac{\beta I_{t_1}(x_k)}{P_k g(x_k)}, \dots, h_{x_k}(t_M) > \frac{\beta I_{t_M}(x_k)}{P_k g(x_k)}\right) \\ &\stackrel{(a)}{=} \mathbb{E}_{I_t} \left[\exp\left(-\frac{\beta (I_{t_1}(x_k) + I_{t_2}(x_k) + \dots + I_{t_M}(x_k))}{P_k g(x_k)}\right) \right] \\ &\stackrel{(b)}{=} \mathbb{E}_{I_t} \left[\exp\left(-\frac{\beta}{P_k g(x_k)} \sum_{l=1}^K \sum_{\substack{x \in \phi_l \\ x \neq x_k}} P_l g(x) \sum_{i=1}^M 1(x \in \phi_l(t_i)) h_x(t_i)\right) \right] \\ &= \prod_{l=1}^K \mathbb{E}_{\phi_l} \left[\prod_{\substack{x \in \phi_l \\ x \neq x_k}} \mathbb{E}_{1(x \in \phi_l), h} \left[\prod_{i=1}^M \exp\left(-\frac{\beta P_l g(x)}{P_k g(x_k)} \right. \right. \right. \\ &\quad \left. \left. \left. \times (1(x \in \phi_l(t_i)) h_x(t_i)) \right) \right] \right] \end{aligned}$$

$$\begin{aligned}
&\stackrel{(c)}{=} \prod_{l=1}^K \mathbb{E}_{\phi_l} \left[\prod_{\substack{x \in \phi_l \\ x \neq x_k}} \mathbb{E}_h \left[\prod_{i=1}^M \left(\exp \left(-\frac{\beta P_l g(x)}{P_k g(x_k)} \cdot 1 \cdot h_x(t_i) \right) P(x \in \phi_l(t_i)) \right. \right. \right. \\
&\quad \left. \left. \left. + \exp \left(-\frac{\beta P_l g(x)}{P_k g(x_k)} \cdot 0 \cdot h_x(t_i) \right) P(x \notin \phi_l(t_i)) \right) \right] \right] \\
&= \prod_{l=1}^K \mathbb{E}_{\phi_l} \left[\prod_{\substack{x \in \phi_l \\ x \neq x_k}} \mathbb{E}_h \left[\prod_{i=1}^M \left[p \cdot \exp \left(-\frac{\beta P_l g(x)}{P_k g(x_k)} \cdot h_x(t_i) \right) + 1 - p \right] \right] \right] \\
&\stackrel{(d)}{=} \prod_{l=1}^K \mathbb{E}_{\phi_l} \left[\prod_{\substack{x \in \phi_l \\ x \neq x_k}} \left(\frac{p}{1 + \frac{\beta P_l g(x)}{P_k g(x_k)}} + 1 - p \right)^M \right] \\
&\stackrel{(e)}{=} \prod_{l=1}^K \exp \left(-\lambda_l \int_{\mathbb{R}^2 \setminus \mathbf{b}(o, z_l)} \left[1 - \left(\frac{p}{1 + \frac{\beta P_l g(x)}{P_k g(x_k)}} + 1 - p \right)^M \right] dx \right), \tag{25}
\end{aligned}$$

where (a) follows from the independence of Rayleigh fading channels, (b) comes from the expression of interference $I_l(x_k) = \sum_{l=1}^K \sum_{x \in \phi_l, x \neq x_k} P_l 1(x \in \phi_l(t)) h_x(t) g(x)$, (c) comes from taking the expectation with respect to the indicator function $1(x \in \phi_l(t_i))$, (d) follows by taking the average with respect to fading channels $h_x(t)$, (e) comes from the probability generating functional of PPP. Since the user connects to the strongest BS in terms of the long-term averaged biased-received-power, the closest interferer in l th tier is at least at a distance $z_l = \left(\frac{P_l B_l}{P_k B_k} \right)^{1/\alpha} \cdot \|x_k\|$.

Denoting $\theta = \frac{\beta P_l}{P_k g(x_k)}$, $\text{JSP}_{x_k}^M$ can be rewritten as

$$\text{JSP}_{x_k}^M = \prod_{l=1}^K \exp \left(-2\pi \lambda_l \int_{z_l}^{\infty} \left[1 - \left(\frac{p}{1 + \theta r^{-\alpha}} + 1 - p \right)^M \right] r dr \right). \tag{26}$$

We denote $F_M = 2\pi \int_{z_l}^{\infty} \left[1 - \left(\frac{p}{1 + \theta r^{-\alpha}} + 1 - p \right)^M \right] r dr$. Next, we calculate F_M as follows:

$$\begin{aligned}
F_M &= 2\pi \int_{z_l}^{\infty} \left[1 - \left(\frac{pr^\alpha}{r^\alpha + \theta} + 1 - p \right)^M \right] r dr \\
&\stackrel{(a)}{=} \delta\pi \int_{z_l^\alpha}^{\infty} \left[1 - \left(1 - \frac{p\theta}{u + \theta} \right)^M \right] u^{\delta-1} du \\
&\stackrel{(b)}{=} \delta\pi \sum_{j=1}^M \binom{M}{j} (-1)^{j+1} p^j \int_{z_l^\alpha}^{\infty} \frac{u^{\delta-1}}{(1 + \frac{u}{\theta})^j} du \\
&\stackrel{(c)}{=} \delta\pi \sum_{j=1}^M \binom{M}{j} (-1)^{j+1} p^j \frac{z_l^{\alpha(\delta-j)}}{\theta^{-j}(j-\delta)} \\
&\quad \cdot {}_2F_1 \left(j, j-\delta; j-\delta+1; \frac{-\theta}{z_l^\alpha} \right) \\
&= \delta\pi \left(\frac{P_l B_l}{P_k B_k} \right)^\delta \|x_k\|^2 \sum_{j=1}^M \binom{M}{j} \frac{(-1)^{j+1}}{j-\delta} p^j \left(\frac{B_k \beta}{B_l} \right)^j \\
&\quad \cdot {}_2F_1 \left(j, j-\delta; j-\delta+1; -\frac{B_k \beta}{B_l} \right) \tag{27}
\end{aligned}$$

where (a) comes from $u = r^\alpha$, (b) follows from the binomial expansion of the calculation of $\left(1 - \frac{p\theta}{u + \theta} \right)^M$, (c) comes from the calculation of the integral $\int_{z_l^\alpha}^{\infty} \frac{u^{\delta-1}}{(1 + \frac{u}{\theta})^j} du$ and $\theta = \frac{\beta P_l}{P_k g(x_k)}$. According to [30, eq. (3.194.2)], we obtain that

$$\begin{aligned}
&\int_{z_l^\alpha}^{\infty} \frac{u^{\delta-1}}{(1 + \frac{u}{\theta})^j} du \\
&= \frac{z_l^{\alpha(\delta-j)}}{\theta^{-j}(j-\delta)} \cdot {}_2F_1 \left(j, j-\delta; j-\delta+1; \frac{-\theta}{z_l^\alpha} \right). \tag{28}
\end{aligned}$$

Substituting (27) into (26) and denoting

$$\begin{aligned}
Q_M \left(\delta, p, \frac{B_k \beta}{B_l} \right) &= \sum_{j=1}^M \binom{M}{j} \frac{(-1)^{j+1}}{j-\delta} p^j \left(\frac{B_k \beta}{B_l} \right)^j \\
&\quad \cdot {}_2F_1 \left(j, j-\delta; j-\delta+1; \frac{-B_k \beta}{B_l} \right), \tag{29}
\end{aligned}$$

we obtain $\text{JSP}_{x_k}^M$ as

$$\text{JSP}_{x_k}^M = \exp \left(-\pi \sum_{l=1}^K \lambda_l \delta \left(\frac{P_l B_l}{P_k B_k} \right)^\delta Q_M \left(\delta, p, \frac{B_k \beta}{B_l} \right) \|x_k\|^2 \right). \tag{30}$$

Averaging over the distance $\|x_k\|$ given by (5), the joint success probability of each tier is obtained as

$$\text{JSP}_k^M = \mathbb{E}_{x_k} \left[\text{JSP}_{x_k}^M \right] = \int_0^\infty \text{JSP}_{x_k}^M \cdot f_{X_k}(x) dx. \tag{31}$$

Substituting (30) and (5) in (31), we obtain the joint success probability of each tier.

B. Proof of Lemma 2

Proof: Based on the derived joint success probability, we first calculate the joint outage probability, which is defined as the probability that the transmissions fail in M successive attempts. For simple notation, we denote $S_k(t_j)$ as the event that the transmission succeeds at the j th attempt and $\bar{S}_k(t_j)$ as the event that the transmission fails at the j th attempt. According to the inclusion-exclusion formula, the probability that the data packets are transmitted successfully at least one time in m attempts is

$$P_k^{1|M} \triangleq \mathbb{P}(\cup_{j=1}^M S_k(t_j)) = \sum_{j=1}^M (-1)^{j+1} \binom{M}{j} \text{JSP}_k^{(j)}. \tag{32}$$

Since the event that the transmissions fail in m successive attempts is complementary to the event that they succeed at least once in m attempts, the joint outage probability is

$$P_k^{0|M} \triangleq \mathbb{P}(\cap_{j=1}^M \bar{S}_k(t_j)) = 1 - P_k^{1|M}. \tag{33}$$

Now, we compute the outage probability given $M-1$ failures, which is defined as the probability that the transmission

fails in the M th attempt conditional on the event that it fails in the first $M - 1$ attempts. By definition, the outage probability given $M - 1$ failures is expressed as

$$\begin{aligned} \mathbb{P}(\bar{S}_k(t_M) | \cap_{j=1}^{M-1} \bar{S}_k(t_j)) &= \frac{\mathbb{P}(\cap_{j=1}^M \bar{S}_k(t_j))}{\mathbb{P}(\cap_{j=1}^{M-1} \bar{S}_k(t_j))} \\ &= \frac{1 - P_k^{1|M}}{1 - P_k^{1|M-1}}. \end{aligned} \quad (34)$$

Next, we calculate the success probability given $M - 1$ failures, which is defined as the probability that the transmission succeeds in the M th attempt conditional on the event that it fails in the first $M - 1$ attempts. Based on the definition, the success probability given $M - 1$ failures is

$$\begin{aligned} \mathbb{P}(S_k(t_M) | \cap_{j=1}^{M-1} \bar{S}_k(t_j)) &= 1 - \mathbb{P}(\bar{S}_k(t_M) | \cap_{j=1}^{M-1} \bar{S}_k(t_j)) \\ &= 1 - \frac{1 - P_k^{1|M}}{1 - P_k^{1|M-1}}. \end{aligned} \quad (35)$$

According to the definition of success probability, we get

$$\begin{aligned} SP_k^{(M)} &\triangleq \mathbb{P}(\bar{S}_k(t_1) \cap \bar{S}_k(t_2) \cap \dots \cap \bar{S}_k(t_{M-1}) \cap S_k(t_M)) \\ &= \mathbb{P}(S_k(t_M) | \cap_{j=1}^{M-1} \bar{S}_k(t_j)) \cdot \mathbb{P}(\cap_{j=1}^{M-1} \bar{S}_k(t_j)). \end{aligned} \quad (36)$$

Substituting (33) and (35) into (36), we obtain the success probability of HCNs. ■

C. Proof of Theorem 3

Denote the locations of BSs from all tiers as $\phi = \cup_{i=1}^K \phi_i$. Conditioning on ϕ , the success probability of the typical user connecting to a given BS x_k in the M th time slot is given by

$$\begin{aligned} \widetilde{SP}_{x_k|\phi}^M(\beta) &\triangleq \mathbb{P}\left(\sum_{m=1}^M \Gamma_m(x_k) \geq \beta, \sum_{m=1}^{M-1} \Gamma_m(x_k) < \beta | \Phi\right) \\ &\stackrel{(a)}{=} \int_0^\infty \mathbb{P}\left(\Gamma_M(x_k) > \beta - \Gamma^{(M-1)}, \Gamma^{(M-1)} < \beta | \phi, \Gamma^{(M-1)}\right) \\ &\quad \cdot f_{\Gamma^{(M-1)}|\phi}(\gamma) d\gamma \\ &= \int_0^\beta \mathbb{P}\left(\Gamma_M(x_k) > \beta - \sum_{m=1}^{M-1} \Gamma_i(x_k) | \phi\right) \cdot f_{\Gamma^{(M-1)}|\phi}(\gamma) d\gamma, \end{aligned} \quad (37)$$

where (a) comes from taking expectation with respect to the variable $\Gamma^{(M-1)}$.

Note that the SIR received at each slot, i.e., $\Gamma_m, \forall m$, is independent conditioned on ϕ [25]. Thus, the PDF of $\Gamma^{(M-1)} = \sum_{m=1}^{M-1} \Gamma_i$ is directly derived as follows via mathematical

induction starting from the well-known result for the sum of two independent random variables.

$$\begin{aligned} f_{\Gamma^{(M-1)}|\phi}(\gamma) &= \int \int \dots \int \prod_{m=1}^{M-2} f_{\Gamma_m|\phi}(\gamma_m) d\gamma_m \cdot f_{\Gamma_{M-1}|\phi}(\beta - \sum_{m=1}^{M-2} \gamma_m) \\ &= \int \int \dots \int \prod_{m=1}^{M-2} f_{\Gamma_m|\phi}(\gamma_m) d\gamma_m \cdot f_{\Gamma_{M-1}|\phi}(\gamma_{M-1}). \end{aligned} \quad (38)$$

Substituting (38) into (37), $\widetilde{SP}_{x_k|\phi}^M(\beta)$ is rewritten as

$$\begin{aligned} \widetilde{SP}_{x_k|\phi}^M(\beta) &= \int \int \dots \int \mathbb{P}\left(\Gamma_M(x_k) > \beta - \sum_{m=1}^{M-1} \Gamma_m(x_k) | \phi, \Gamma^{(M-1)}\right) \\ &\quad \prod_{m=1}^{M-1} f_{\Gamma_m|\phi}(\gamma_m) d\gamma_m. \end{aligned} \quad (39)$$

Now, we calculate the CCDF of $\Gamma_m(x_k)$ conditioning on ϕ , which is expressed as

$$\begin{aligned} \mathbb{P}(\Gamma_m(x_k) > \gamma_m | \phi) &= \mathbb{P}\left(\frac{P_k h_{x_k}(t_m) g(x_k)}{I_i(x_k)} > \gamma_m | \phi\right) \\ &\stackrel{(a)}{=} \mathbb{E}\left[\exp\left(-\frac{\gamma_m I_m(x_k)}{P_k g(x_k)}\right) | \phi\right] \\ &= \mathbb{E}_h \left[\prod_{l=1}^K \prod_{x \in \phi_l, x \neq x_k} \exp\left(-\frac{\gamma_m P_l 1(x \in \phi_l(t_m)) h_x(t_m) g(x)}{P_k g(x_k)}\right) | \phi \right] \\ &\stackrel{(b)}{=} \prod_{l=1}^K \prod_{x \in \phi_l, x \neq x_k} \mathbb{E}\left[\exp\left(-\frac{\gamma_m P_l 1(x \in \phi_l(t_m)) h_x(t_m) g(x)}{P_k g(x_k)}\right) | \phi\right] \\ &\stackrel{(c)}{=} \prod_{l=1}^K \prod_{x \in \phi_l, x \neq x_k} \frac{p}{1 + \frac{P_l \gamma_m ||x||^{-\alpha} ||x_k||^\alpha}{P_k}} + 1 - p \\ &\stackrel{(d)}{=} \prod_{l=1}^K \prod_{x \in \phi_l, x \neq x_k} \frac{p}{1 + \theta_k \gamma_m d_k^\alpha ||x||^{-\alpha}} + 1 - p, \end{aligned} \quad (40)$$

where (a) comes from the Rayleigh fading channels, (b) follows by taking the independence of ϕ_k , (c) comes from averaging with respect to Rayleigh fading channel and the indicator function $1(x \in \phi_l(t_i))$, (d) follows by denoting $\theta_k = P_l/P_k$, $d_k = ||x_k||$.

Differentiating $\mathbb{P}(\Gamma_m(x_k) \leq \gamma_m | \phi) = 1 - \mathbb{P}(\Gamma_m(x_k) > \gamma_m | \phi)$ with respect to γ_m , we derive the PDF of Γ_m as

$$\begin{aligned} f_{\Gamma_m}(\gamma_m) &= \sum_{l=1}^K \sum_{\substack{x_p \in \phi_l \\ x_p \neq x_k}} \frac{p \theta_k ||x_p||^{-\alpha} d_k^\alpha}{(1 + \theta_k ||x_p||^{-\alpha} d_k^\alpha \gamma_m)^2} \\ &\quad \cdot \prod_{l=1}^K \prod_{\substack{x_q \in \phi_l \\ x_q \neq x_k, x_p}} \left(\frac{p}{1 + \theta_k \gamma_m d_k^\alpha ||x_q||^{-\alpha}} + 1 - p \right). \end{aligned} \quad (41)$$

To simplify the expression, we denote $\Lambda(x, \gamma) = \frac{p}{1 + \theta_k \gamma d_k^\alpha \|x\|^{-\alpha}} + 1 - p$. Then, we obtain $\widetilde{\text{SP}}_{x_k}^M$ via substituting (40) and (41) into (39). Further, deconditioning on ϕ , we get:

$$\begin{aligned} \widetilde{\text{SP}}_{x_k}^M &= \int \int \cdots \int \mathbb{E}_\phi \left[\prod_{l=1}^K \prod_{\substack{x \in \phi_l \\ x \neq x_k}} \Lambda \left(x, \beta - \sum_{m=1}^{M-1} \gamma_m \right) \right. \\ &\quad \cdot \sum_{l=1}^K \sum_{\substack{x_p \in \phi_l \\ x_p \neq x_k}} \frac{p \theta_k \|x_p\|^{-\alpha} d_k^\alpha}{(1 + \theta_k \|x_p\|^{-\alpha} d_k^\alpha \gamma_{M-1})^2} \\ &\quad \times \prod_{l=1}^K \prod_{\substack{x_q \in \phi_l \\ x_q \neq x_k, x_p}} \Lambda(x_q, \gamma_{M-1}) \\ &\quad \cdot \prod_{m=1}^{M-2} f_{\Gamma_m | \phi}(\gamma_m) d\gamma_m d\gamma_{M-1} \Big] \\ &= \int \int \cdots \int \mathbb{E}_\phi \left[\sum_{l=1}^K \sum_{\substack{x_p \in \phi_l \\ x_p \neq x_k}} \frac{p \theta_k \|x_p\|^{-\alpha} d_k^\alpha}{(1 + \theta_k \|x_p\|^{-\alpha} d_k^\alpha \gamma_{M-1})^2} \right. \\ &\quad \cdot \Lambda \left(x_p, \beta - \sum_{m=1}^{M-1} \gamma_m \right) \Big] \\ &\quad \times \mathbb{E}_\phi \left[\prod_{l=1}^K \prod_{\substack{x_q \in \phi_l \\ x_q \neq x_k, x_p}} \Lambda(x_q, \gamma_{M-1}) \cdot \Lambda \left(x_q, \beta - \sum_{m=1}^{M-1} \gamma_m \right) \right] \\ &\quad \cdot \mathbb{E}_\phi \left[\prod_{m=1}^{M-2} (f_{\Gamma_m}(\gamma_m) d\gamma_m) \right] d\gamma_{M-1}. \end{aligned} \quad (42)$$

To derive $\widetilde{\text{SP}}_{x_k}^M$, we need to further calculate the following three functions.

$$\begin{aligned} &\mathbb{E}_\phi \left[\sum_{l=1}^K \sum_{\substack{x_p \in \phi_l \\ x_p \neq x_k}} \frac{p \theta \|x_p\|^{-\alpha} d^\alpha}{(1 + \theta \|x_p\|^{-\alpha} d^\alpha \gamma_{M-1})^2} \cdot \Lambda \left(x_p, \beta - \sum_{m=1}^{M-1} \gamma_m \right) \right] \\ &\stackrel{(a)}{=} \sum_{l=1}^K \int_{\mathbb{R}^2 \setminus \text{b}(o, z_l)} \frac{p \theta \|x_p\|^{-\alpha} d^\alpha}{(1 + \theta \|x_p\|^{-\alpha} d^\alpha \gamma_{M-1})^2} \\ &\quad \cdot \Lambda \left(x_p, \beta - \sum_{m=1}^{M-1} \gamma_m \right) \lambda_l dx_p \\ &\stackrel{(b)}{=} \sum_{l=1}^K 2\pi \lambda_l \int_{z_l}^\infty \frac{p \theta r^{1-\alpha} d^\alpha}{(1 + \theta r^{-\alpha} d^\alpha \gamma_{M-1})^2} \\ &\quad \cdot \Lambda \left(r, \beta - \sum_{m=1}^{M-1} \gamma_m \right) dr \end{aligned} \quad (43)$$

where (a) comes from the Campbell-Mecke formula and $\text{b}(o, z_l)$ denotes a ball of radius $z_l = \left(\frac{P_l B_l}{P_k B_k} \right)^{1/\alpha} \cdot \|x_k\|$ centered at the origin denoted by o , (b) follows from changing two-dimensional coordinates to polar coordinates.

Then, leveraging the probability generating functional for stationary PPPs and changing two-dimensional coordinates to polar coordinates, we get the following equation:

$$\begin{aligned} &\mathbb{E}_\phi \left[\prod_{l=1}^K \prod_{\substack{x_q \in \phi_l \\ x_q \neq x_k, x_p}} \Lambda(x_q, \gamma_{M-1}) \cdot \Lambda \left(x_q, \beta - \sum_{m=1}^{M-1} \gamma_m \right) \right] \\ &= \prod_{l=1}^K \exp \left(-2\pi \lambda_l \int_{z_l}^\infty (1 - \Lambda(r, \gamma_{M-1})) \right. \\ &\quad \cdot \Lambda \left(r, \beta - \sum_{m=1}^{M-1} \gamma_m \right) r dr \Big) \end{aligned} \quad (44)$$

Furthermore, $\mathbb{E}_\phi \left[\prod_{m=1}^{M-2} f_{\Gamma_m}(\gamma_m) d\gamma_m \right]$ is given as

$$\begin{aligned} &\mathbb{E}_\phi \left[\prod_{m=1}^{M-2} f_{\Gamma_m}(\gamma_m) d\gamma_m \right] \\ &= \prod_{m=1}^{M-2} \mathbb{E}_\phi \left[\sum_{l=1}^K \sum_{\substack{x_p \in \phi_l \\ x_p \neq x_k}} \frac{p \theta_k \|x_p\|^{-\alpha} d_k^\alpha}{(1 + \theta_k \|x_p\|^{-\alpha} d_k^\alpha \gamma_m)^2} \right. \\ &\quad \times \prod_{l=1}^K \prod_{\substack{x_q \in \phi_l \\ x_q \neq x_k, x_p}} \left(\frac{p}{1 + \theta_k \gamma_m d_k^\alpha \|x_q\|^{-\alpha}} + 1 - p \right) \Big] \\ &\stackrel{(a)}{=} \prod_{m=1}^{M-2} \left\{ \sum_{l=1}^K 2\pi \lambda_l \int_{z_l}^\infty \frac{p \theta_k d_k^\alpha r^{1-\alpha}}{(1 + \theta_k r^{-\alpha} d_k^\alpha \gamma_m)^2} dr \right\} \\ &\quad \times \mathbb{E}_\phi \left[\prod_{l=1}^K \prod_{\substack{x_q \in \phi_l \\ x_q \neq x_k, x_p}} \left(\frac{p}{1 + \theta_k \gamma_m d_k^\alpha \|x_q\|^{-\alpha}} + 1 - p \right) \right] \\ &\stackrel{(b)}{=} \prod_{m=1}^{M-2} \left[\sum_{l=1}^K 2\pi \lambda_l \int_{z_l}^\infty \frac{p \theta_k d_k^\alpha r^{1-\alpha}}{(1 + \theta_k r^{-\alpha} d_k^\alpha \gamma_m)^2} dr \right. \\ &\quad \times \prod_{l=1}^K \exp \left(-2\pi \lambda_l p \int_{z_l}^\infty \frac{\theta_k r^{1-\alpha} d_k^\alpha \gamma_m}{1 + \theta_k r^{-\alpha} d_k^\alpha \gamma_m} dr \right) \Big] \end{aligned} \quad (45)$$

where (a) comes from Campbell-Mecke formula and changing two-dimensional coordinates to polar coordinates, (b) is from the probability generating functional for stationary PPPs and changing two-dimensional coordinates to polar coordinates.

Substituting (43), (44), and (45) into (42), we obtain the expression of $\widetilde{\text{SP}}_{x_k}^M$. Note that the integration limits for random variable γ_m are from $\beta_k - \sum_{j=1}^{m-1} \gamma_j$ because the m th transmission will happen only when all the prior transmissions fail. Then, deconditioning on the link distance between the user and its associated BS, i.e., $\|x_k\|$, we derive the success probability given in Theorem 3.

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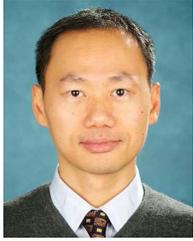
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