

## Equivalence of LC and RC oscillators

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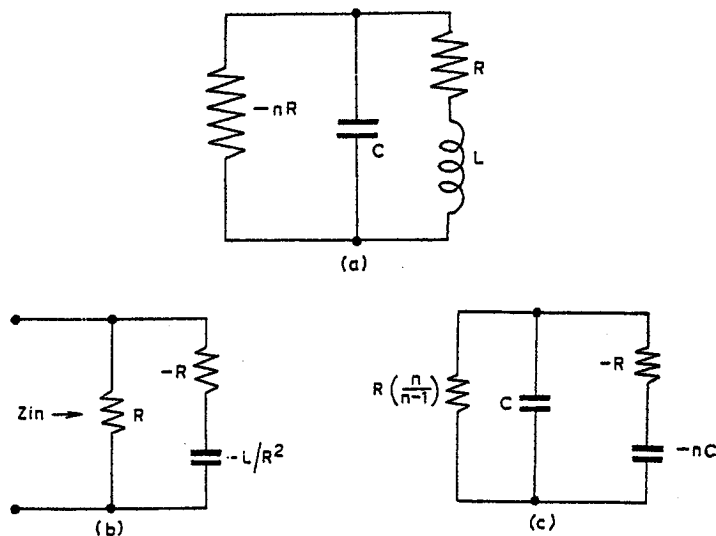
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A general  $RC$  oscillator circuit using positive and negative  $RC$  elements is derived from a well-known  $LC$  oscillator circuit. The active  $RC$  network, thus obtained, is shown to represent the equivalent circuits of many existing  $RC$  oscillators, such as twin  $T$  and Wienbridge oscillators.

### 1. Introduction

The concept of synthesizing oscillators using devices or circuits exhibiting negative resistance seems to have been generally associated with  $LC$  oscillators only and the analysis and synthesis of  $RC$  oscillators as networks having positive and negative  $RC$  elements do not seem to have attracted much attention in scientific literature. This paper considers the synthesis of  $RC$  oscillator as an inductorless synthesis of a  $LC$  oscillator and points out the unifying role of negative impedances in the equivalent circuits of the two types of oscillators. An active  $RC$  network is derived by a transformation of a basic  $LC$  oscillator and it is shown that many existing  $RC$  oscillators can be identified as special cases of this generalized  $RC$  oscillator network.

Fig. 1



(a) A basic  $LC$  oscillator. (b) An active network, equivalent to a lossy inductance ( $Z_{in} = R + sL$ ). (c) A general  $RC$  oscillator.

## 2. Equivalence of $LC$ and $RC$ oscillators

The circuit shown in fig. 1 (a) can be considered as the basic equivalent circuit of many  $LC$  oscillators. The condition for sinusoidal oscillation for this circuit is

$$n = \frac{L}{R^2 C} \quad (n > 1) \quad (1)$$

and the frequency of oscillation is

$$f = \frac{1}{2\pi} \left( \frac{n-1}{nLC} \right)^{1/2} \quad (2)$$

When the impedance of the lossy inductance,  $R + sL$ , in fig. 1 (a) is replaced by the equivalent active  $RC$  network of fig. 1 (b) a basic  $RC$  oscillator circuit results and is shown in fig. 1 (c). The value of the negative capacitance in fig. 1 (c) has been chosen so that the condition for oscillation given by eqn. (1) is satisfied. The frequency of oscillation of the  $RC$  circuit is

$$f = \frac{1}{2\pi} \frac{(n-1)^{1/2}}{nRC} \quad (3)$$

The positive and negative signs in fig. 1 (c) can be interchanged without affecting the general nature of the network.

## 3. Twin $T$ and Wienbridge oscillators

We shall now show that the equivalent circuits of many  $RC$  oscillators can be represented by this network. An ideal  $RC$  oscillator using a unity gain amplifier and a twin  $T$  network is shown in fig. 2 (a) (in practice an emitter follower can replace the unity gain amplifier). By considering the two  $T$  networks one by one (Su 1965), we can easily derive the equivalent

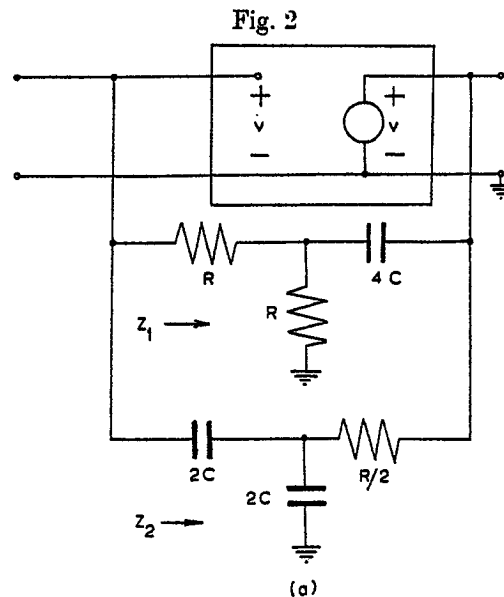
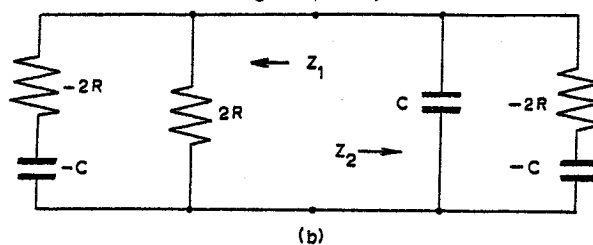
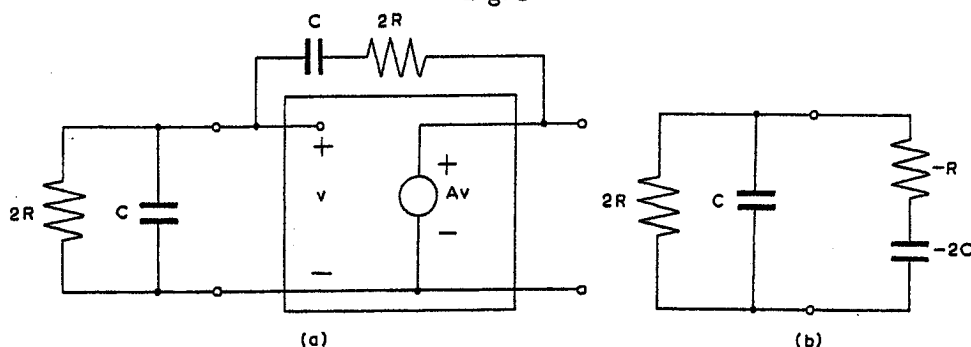


Fig. 2. (cont.)



(a) An ideal twin  $T$   $RC$  oscillator using a unity gain amplifier. (b) Equivalent circuit of twin  $T$  oscillator.

Fig. 3



(a) An ideal Wienbridge oscillator ( $A=3$ ). (b) Equivalent circuit of Wienbridge oscillator.

circuit shown in fig. 2 (b). It is evident that the circuit reduces to that of fig. 1 (c) with  $n=2$ .

An ideal Wienbridge oscillator is shown in fig. 3 (a). Since the voltage gain of the amplifier,  $A=3$  for oscillation, the equivalent circuit easily follows as shown in fig. 3 (b) and can once again be identified with the general oscillating network of fig. 1 (c). Similarly some other  $RC$  oscillators, such as the Wienbridge-type oscillator using current amplifier (Hooper and Jackets 1956) and  $RC$  oscillators using negative impedance converters (Patranabis and Sen 1971, Pasupathy 1966) can also be easily shown to reduce to the equivalent circuit of fig. 1 (c).

#### 4. Conclusion

Apart from the insight that the basic  $RC$  oscillator circuit of fig. 1 (c) offers about the equivalence of  $LC$  and  $RC$  oscillators, a direct synthesis of the circuit with different active networks or devices simulating the negative impedances would give rise to many novel  $RC$  oscillators suitable for low frequency and integrated circuit applications.

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