Waveform Diversity and MIMO Radar

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Overview

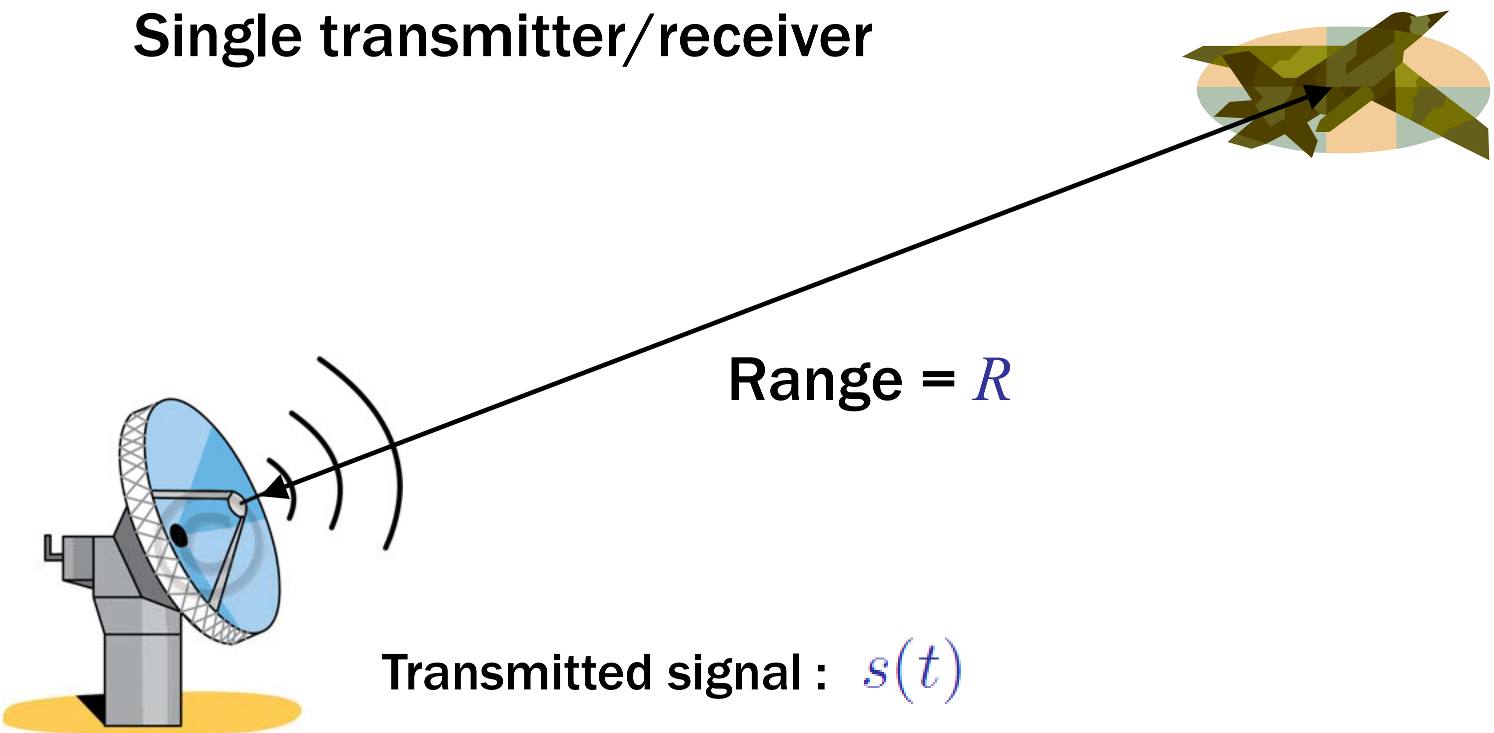
- Radar basics and background
  - waveforms
    - pulse compression
    - ambiguity function
  - phased array radars
  - STAP
  - target models
  - early look at waveform diversity

Overview (2)

- MIMO Radar
  - importance of diversity
  - virtual array representation
  - theoretical analyses
    - target models
    - diversity order
  - STAP with distributed sensors
- MIMO and Waveform Diversity
  - MIMO ambiguity function
  - waveform design
  - fast-time & slow-time MIMO

I : Radar Basics

Single transmitter/receiver

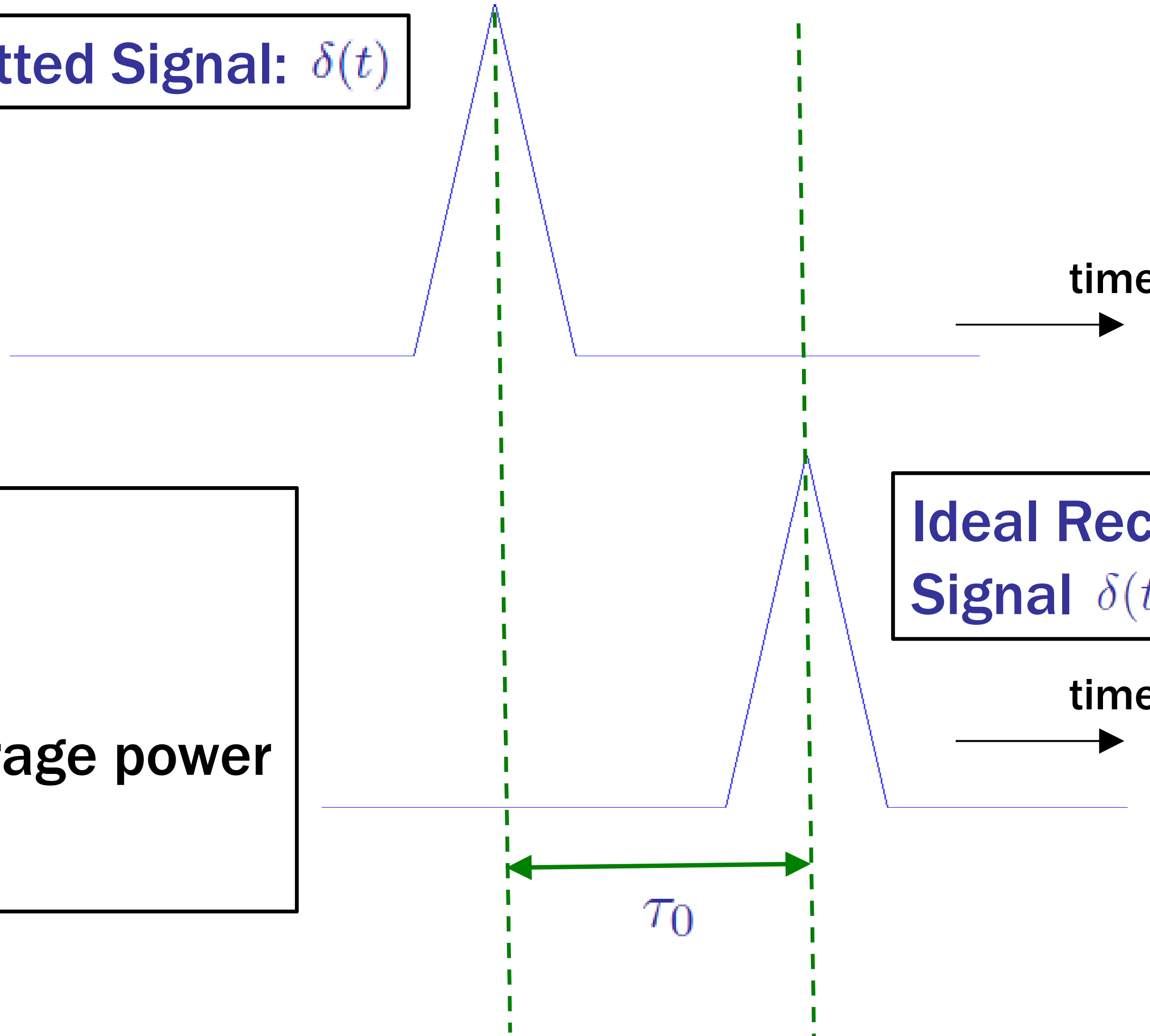


Transmitted signal : $s(t)$

Received signal : $As(t - \tau_0) + n(t) \quad \tau_0 = 2R/c$

I.1: Radar Basics : Ideal

Ideal Transmitted Signal: $\delta(t)$



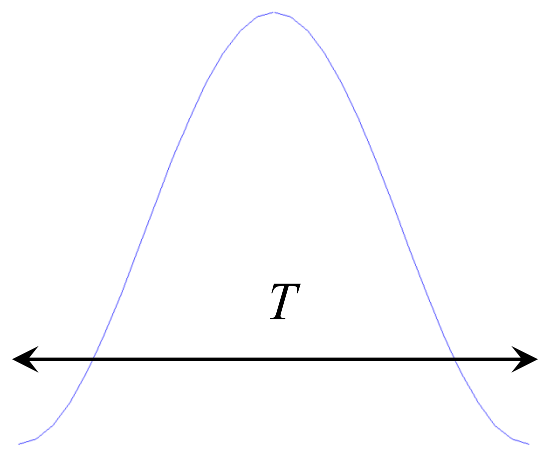
Ideal Received Signal $\delta(t-\tau_0)$

Issues:

- Noise
- Peak-to-average power
- Bandwidth

Radar Basics : Bandlimited Pulses

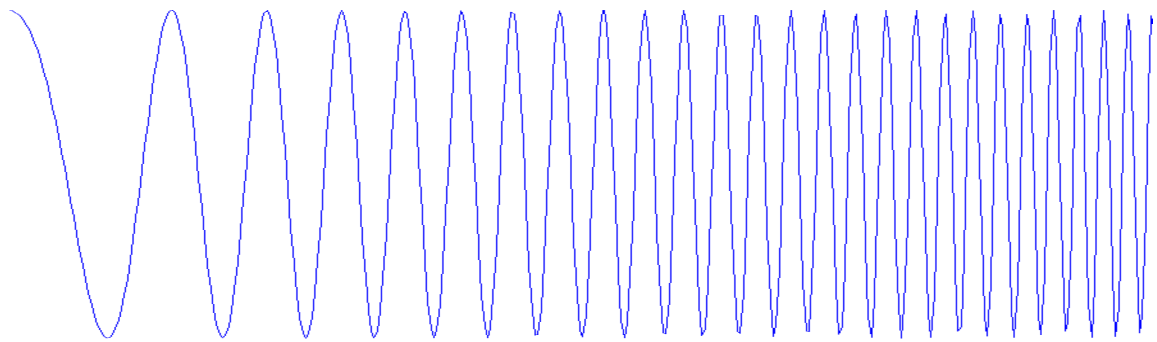
- Transmit a pulse (effectively) limited in time and frequency, e.g.,



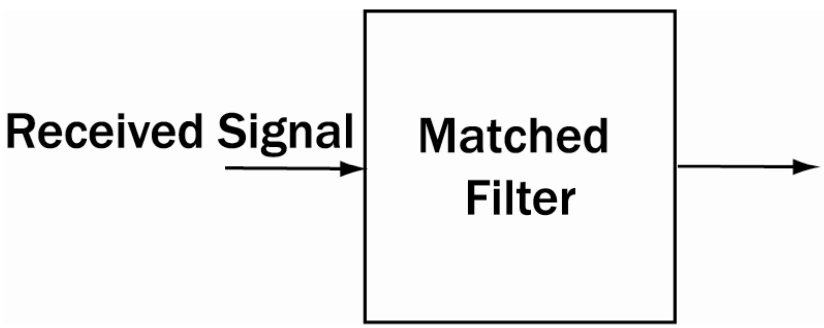
- Range resolution (ΔR) proportional to T

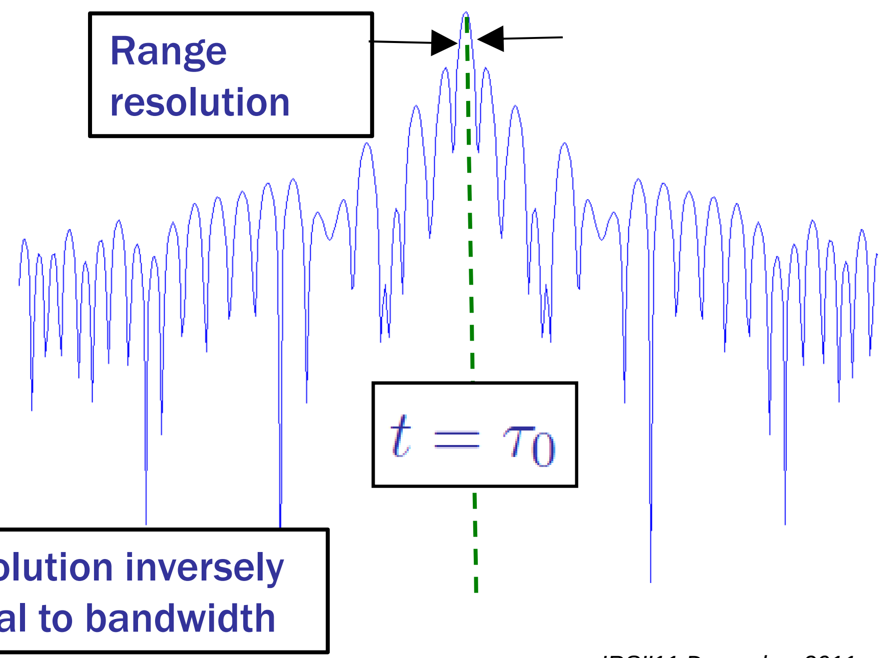
Radar Basics : Bandlimited Pulses (2)

Transmitted Signal:



Receiver





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Radar Basics : Bandlimited Pulses (3)

- Matched filter: $q(t) = s^{\star}(-t)$;: gathers all energy in $q(t)$
- Detect target by finding the maximum of the output of the matched filter
  - target declared present if signal above some threshold
  - target range from round-trip time
- This is equivalent to pulse compression
  - transmitted signal spread over long time
  - receiver creates very narrow signal in time
    - range resolution inversely proportional to bandwidth ($\Delta R \approx c/2B$)
    - improvement in resolution ≈ time-bandwidth product

Radar Basics : Doppler Shift

- What if the target is moving? Doppler shift:

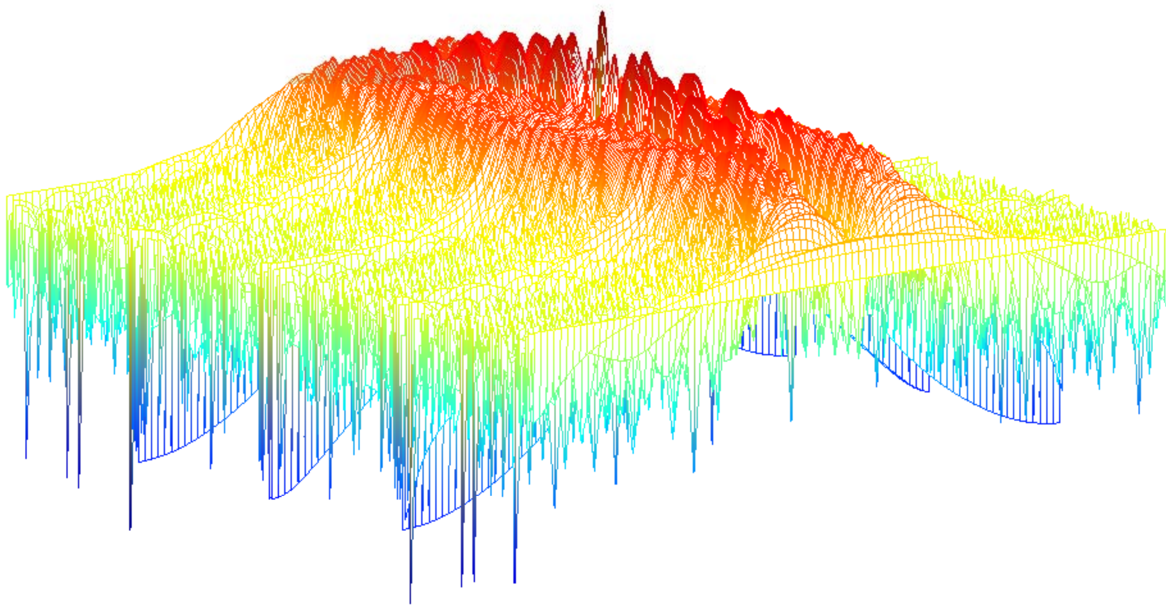
$$f_{d0} = \frac{2v}{\lambda} \qquad r(t) = As(t - \tau_0)e^{j2\pi f_{d0} t} + n(t)$$

  - bank of matched filters $q(t) = s(-t)e^{-j2\pi f_d t}$
    - each matched to a single Doppler frequency f_d
- Detect target by finding the maximum of the output of the matched filters
  - target present if signal above some threshold
  - target range from round-trip time
  - target Doppler from which MF provides the max

Radar Basics : Ambiguity Function

- Range-Doppler resolution determined by the ambiguity function

$$\chi(\tau, f_d) = \int_{-\infty}^{\infty} s(t)s^{\star}(t-\tau)e^{-j2\pi f_d t}dt$$



Radar Basics : Ambiguity Function (2)

- Indicates the spread in delay (τ) and Doppler (f_d) due to the matched filter
  - determines the resolution in range and Doppler

- Key properties:
  - Energy:

$$\chi(0,0) = \int_{-\infty}^{\infty} |s(t)|^2 dt = \mathcal{E}$$

  - Fixed area:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\chi(\tau, f_d)|^2 \, d\tau df_d = |\chi(0,0)|^2 = \mathcal{E}^2$$

Background : Popular Waveforms

- Linear FM: FM modulate a linear signal
  - instantaneous frequency is proportional to time
  - time shift implies a frequency shift…
    - …leading to a coupling in range and Doppler
  - constant envelope signal
  - Doppler tolerant in that characteristics, e.g., sidelobes, not affected by Doppler shift of target
- Phase-coded waveforms
  - Subdivide a long pulse into N “chips”
  - in each chip use a different phase for the transmit waveform

Background : Popular Waveforms (2)

- Resolution function of chip length, not pulse length
- Can choose the phase sequence to e.g., minimize sidelobe levels
- Biphase codes
  - phases of 0 and 180 degrees only
  - Barker codes
    - achieve best peak-to-sidelobe ratio
  - Maximal length sequences
    - low peak sidelobes, high average sidelobes compared to LFM
    - poor spectral characteristics without bandlimiting

Background : Popular Waveforms (3)

- Polyphase codes
  - polyphase Barker codes through exhaustive search
  - Frank, P1 and P2 codes
    - lower sidelobe levels for same length
    - P1 and P2 codes are robust to bandlimiting
    - All have poor Doppler tolerance
      - range sidelobes raise dramatically with Doppler
    - P3 and P4 codes mimic LFM and can be robust to bandlimiting (P4)

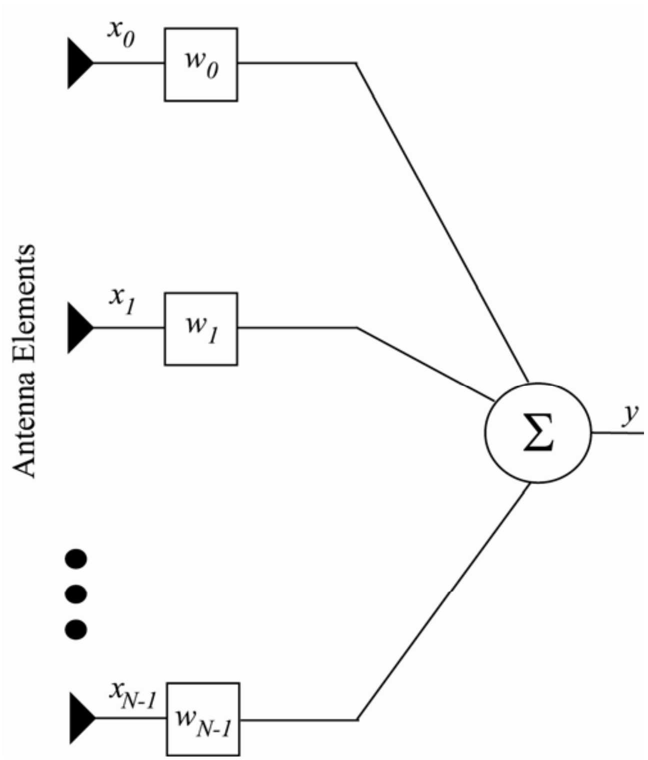
I.2 : Phased Arrays

- So far, we assumed a single transmit antenna with an isotropic pattern
  - energy sprayed in all directions equally
  - radar range improved using directive antennas

$$P_r = P_t \frac{G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}$$

- Phased arrays provide digital control of antenna patterns
  - control location of the mainbeam using phase shifts

Phased Arrays (2)



- Consider a linear, equi-spaced phased array
- d: inter-element spacing
- Controlling phase shift w_k controls the direction of mainbeam
- Can provide gains in SNR of up to N

Phased Arrays (3)

- Received signal

$$\mathbf{x} = \alpha_t \mathbf{s} + \mathbf{n}$$

- $\mathbf{n}$: noise, α_t : target amplitude

$$\mathbf{s} = \left[1, e^{jkd\sin(\phi_t)}, e^{j2kd\sin(\phi_t)}, \ldots, e^{j(N-1)kd\sin(\phi_t)}\right]^T$$

- ϕ_t: look angle, $k = 2\pi/\lambda$ is the wavenumber
- Optimal weights: $w_n = e^{jnkd\sin(\phi_t)} = s_n$
- Optimal because $\mathbf{R} = E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma^2\mathbf{I}$ and the matched filter is optimal in white noise

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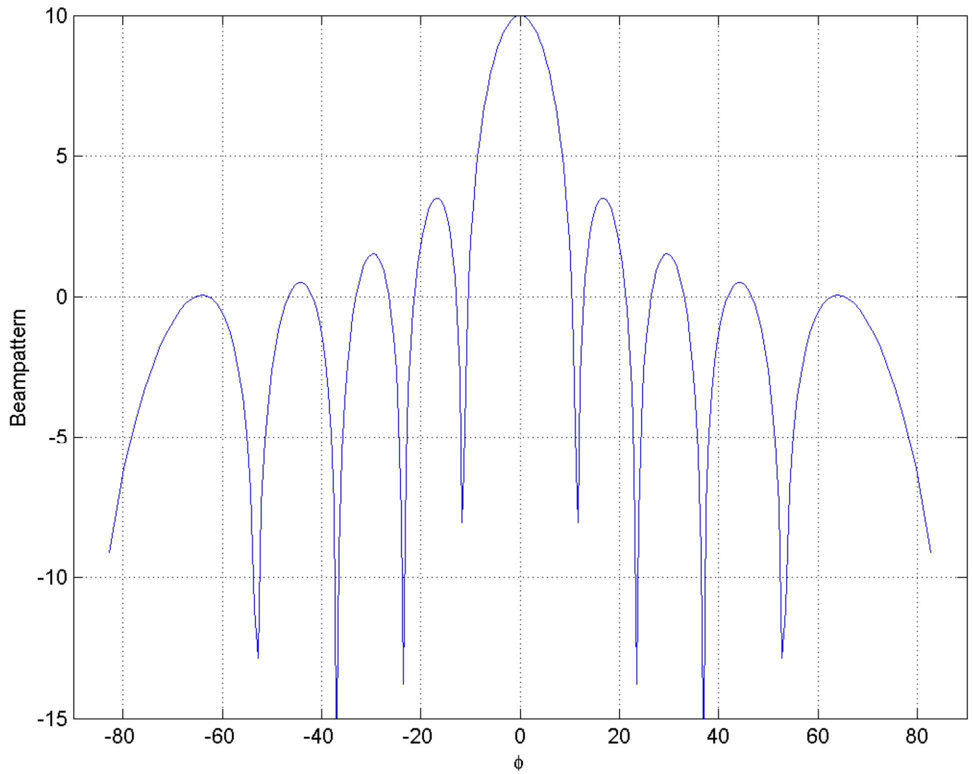
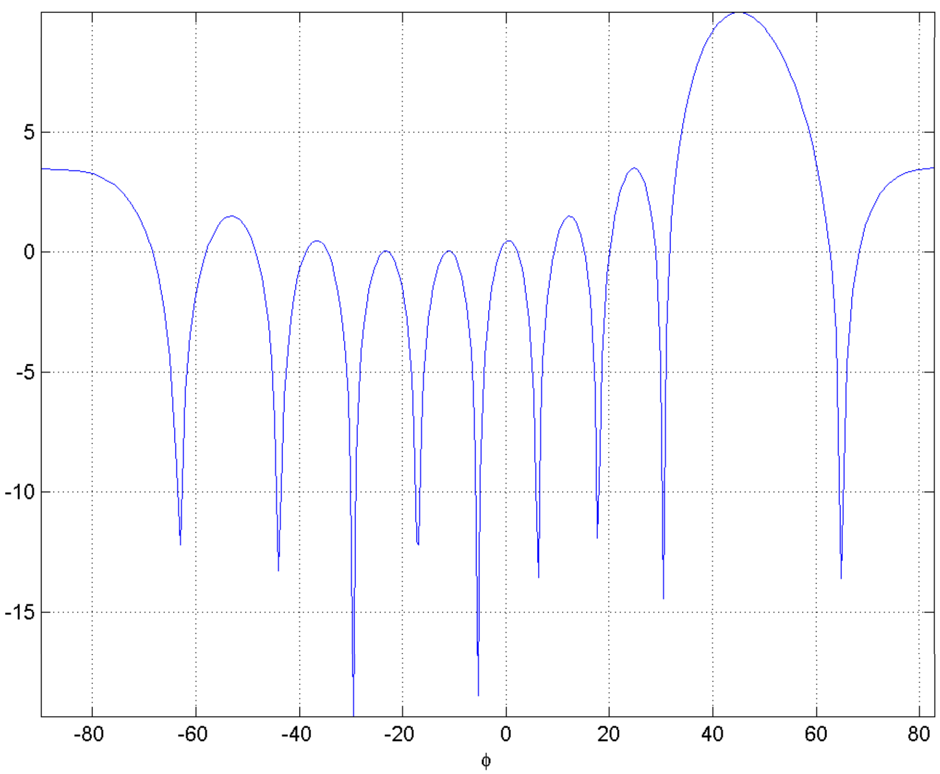
Phased Arrays (4)

The weights cause a beampattern - with a peak at the look direction





Look angle = 0^{o}

Look angle = 45^{o}

Inter-element spacing = $\lambda/2$

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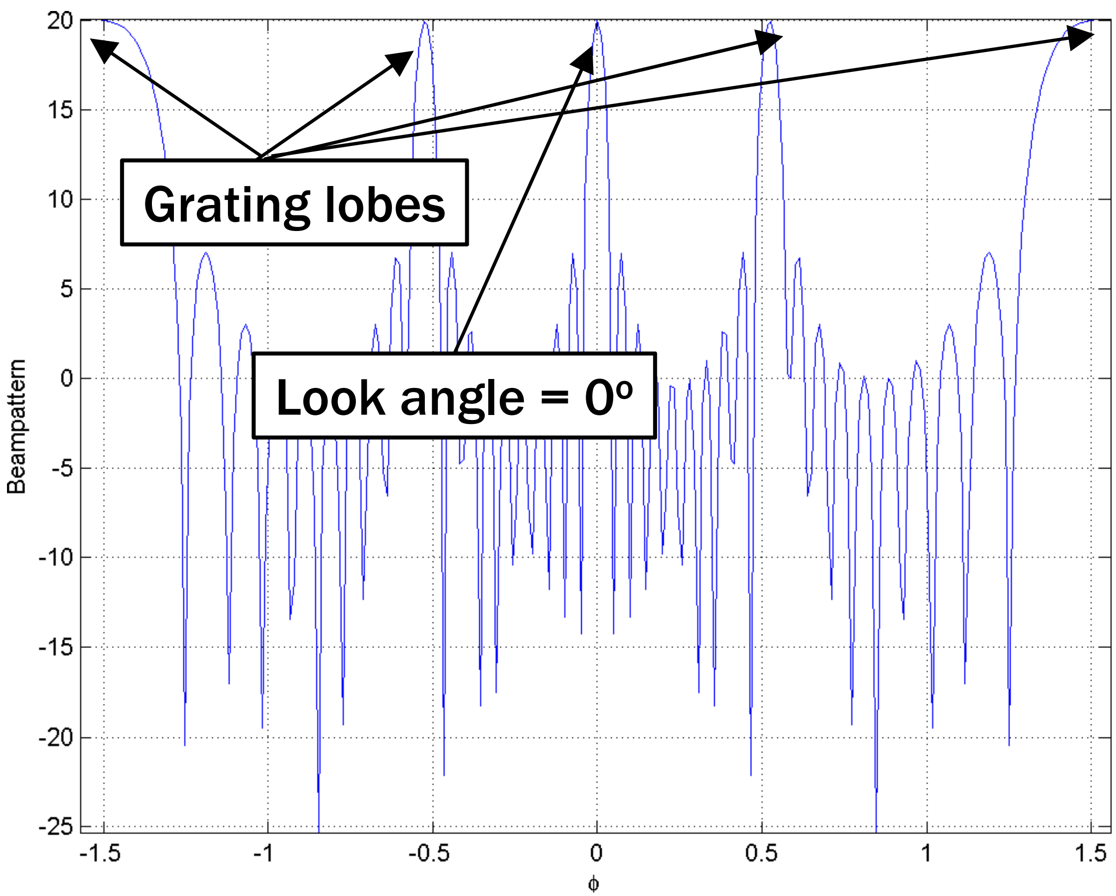
Phased Arrays (5)

- However, depending on the spacing, the entire space may not
  - be visible (closely spaced elements – not really relevant to MIMO radar)
  - be uniquely identifiable: grating lobes with widely spaced elements
  - grating lobes are caused by coherent addition at multiple angles
  - use unequally spaced elements



Inter-element spacing = 2λ

Phased Arrays (6)

- So far, we have focused on detection of a single target in noise
  - what if there is interference?
    - e.g., clutter, external sources of interference
  - can use an array to suppress interference while maintaining gain on the target
  - key: knowing the target signature that we are searching for

For a linear array with look direction ϕ

$$\mathbf{s} = [1, e^{jkd\sin(\phi)}, e^{j2kd\sin(\phi)}, \ldots, e^{j(N-1)kd\sin(\phi)}]^T$$

Phased Arrays (7)

- Received signal

$$\mathbf{x} = \alpha_t \mathbf{s} + \mathbf{n}$$

- Now, $\mathbf{n}$ includes both interference and noise
- Key difference from noise-only case

$$\text{Noise only: } \mathbf{R} = E\left[\mathbf{n}\mathbf{n}^H\right] = \sigma^2 \mathbf{I}$$

$$\text{With interference: } \mathbf{R} = E\left[\mathbf{n}\mathbf{n}^H\right] \neq \sigma^2 \mathbf{I}$$

- Interference is now “coloured”

Phased Arrays (8)

- With interference, optimal weights require both amplitude and phase control

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{s}$$

- $\mathbf{s}$: the steering vector corresponding to the look direction
  - this may not be the target direction
  - target is discrete interference when looking elsewhere!

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Phased Arrays (9)

- With these weights

$$\begin{aligned} \text{Output} = y &= \mathbf{w}^H \mathbf{x} = \mathbf{s}^H \mathbf{R}^{-1} \mathbf{x} \\ &= \left[\mathbf{s}^H \mathbf{R}^{-1/2}\right] \left[\mathbf{R}^{-1/2} \mathbf{x}\right] \\ &= \tilde{\mathbf{s}}^H \tilde{\mathbf{x}} \end{aligned}$$

- Also, $\tilde{\mathbf{x}} = \mathbf{R}^{-1/2}\mathbf{x} = \alpha_t \tilde{\mathbf{s}} + \tilde{\mathbf{n}}$

$$\begin{aligned} \tilde{\mathbf{n}} &= \mathbf{R}^{-1/2} \mathbf{n} \\ \Rightarrow E\left[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\right] &= \mathbf{R}^{-1/2} E\left[\mathbf{n}\mathbf{n}^H\right] \mathbf{R}^{-1/2} \\ &= \mathbf{R}^{-1/2} \mathbf{R} \mathbf{R}^{-1/2} = \mathbf{I} \end{aligned}$$

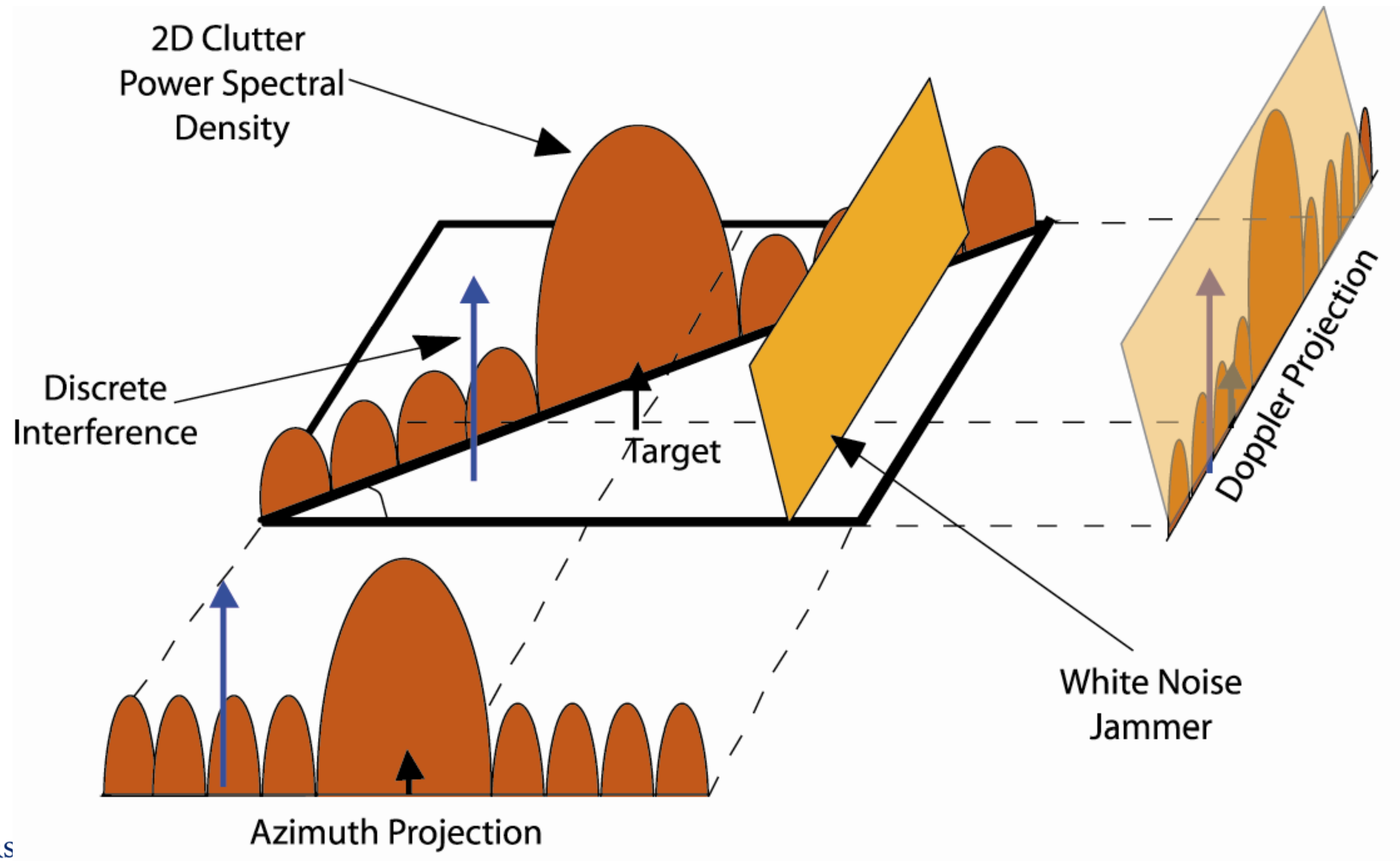
- This is the whiten-then-match filter

Phased Arrays (10)

- The most important problem: the matrix $\mathbf{R}$ is unknown a priori
  - must be estimated using training data samples
  - need at least $2N$ samples
  - these samples must
    - not contain any target
    - be "homogeneous", i.e., statistically independent and identically distributed in relation to the interference
  - usually, for each look range (the primary range cell) choose training data from range cells close by
    - also called secondary data

I.3 : Space-Time Adaptive Processing

- Can extend this to both space and time
  - because target may not be seen in 1D only



Space-Time Adaptive Processing (2)

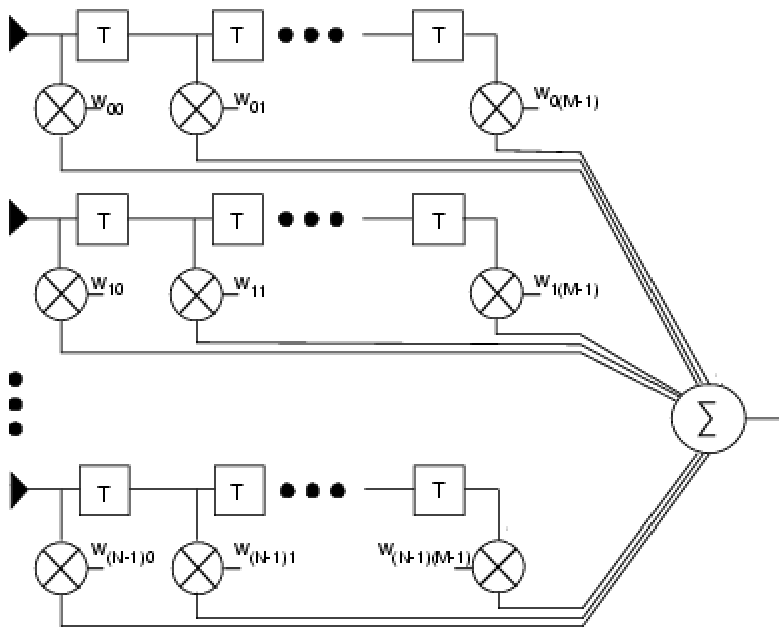
- N elements (spatial channels), M pulses in a coherent pulse interval (CPI)
  - Use of multiple pulses provides Doppler resolution
  - f_r : pulse repetition rate

$$\begin{aligned}
\mathbf{s} &= \mathbf{s}_t(f_d) \otimes \mathbf{s}_s(\phi) \\
\mathbf{s}_s(\phi) &= \left[1, e^{jkd\sin(\phi)}, e^{j2kd\sin(\phi)}, \ldots, e^{j(N-1)kd\sin(\phi)}\right]^T \\
\mathbf{s}_t(f_d) &= \left[1, e^{j(2\pi f_d/f_r)}, e^{j2(2\pi f_d/f_r)}, \ldots, e^{j(M-1)(2\pi f_d/f_r)}\right]^T
\end{aligned}$$

  - f_d : the look Doppler frequency

Space-Time Adaptive Processing (3)

- Again, $\mathbf{w} = \mathbf{R}^{-1}\mathbf{s}$, however, now…



- …and $\mathbf{R}$ is an $NM \times NM$ matrix, making estimating this matrix very hard

Space-Time Adaptive Processing (4)

- To deal with estimation issues, usually one reduces the adaptive degrees of freedom (DoF)
  - joint domain localized processing
    - processing in a small region around look angle/Doppler
  - ΣΔ—STAP
    - Use sum (Σ) and difference (Δ) channels only
  - parametric adaptive matched filter
    - parametrize the matched filter
  - fast fully adaptive processing
    - break large problem into a series of small problems
  - many others

STAP (5): Reduced Rank STAP

- Several reduced rank methods can be described as

$$\begin{aligned} \tilde{\mathbf{x}} &= \mathbf{T}^H \mathbf{x} \\ \tilde{\mathbf{s}} &= \mathbf{T}^H \mathbf{s} \\ \tilde{\mathbf{R}} &= \mathbf{T}^H \mathbf{R} \mathbf{T} \end{aligned}$$

$\mathbf{T} = (NM \times D)$ transformation matrix

$$\tilde{\mathbf{w}} = \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{s}}$$

- Computational load is reduced by a factor of $(NM/D)^3$
- Required sample support $2NM \rightarrow 2D$
  - in practice, sample support is the fundamental problem

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Space-Time Adaptive Processing (6)

- In applying STAP in the real world, a non-homogeneity detector (NHD) is important
  - identifies samples within the secondary data set that are statistically inconsistent
    - these samples are discarded
- Several types of NHD in the literature
  - all search for some kind of discriminant
- Reducing DoF coupled with NHD makes it possible to implement STAP

I.4 : Target Models

- Target amplitude a function of its radar cross section
  - for complex targets, a sum of returns from different parts making the amplitude a random variable
- Swerling models:
  - Type I: Amplitude Gaussian, independent scan-to-scan
  - Type II: like type-I, independent pulse-to-pulse
  - Type III: One dominant, other smaller surfaces: constant plus Gaussian independent scan-to-scan
  - Type IV: like type-III, independent pulse-to-pulse
  - Type V: constant throughout
    - best case

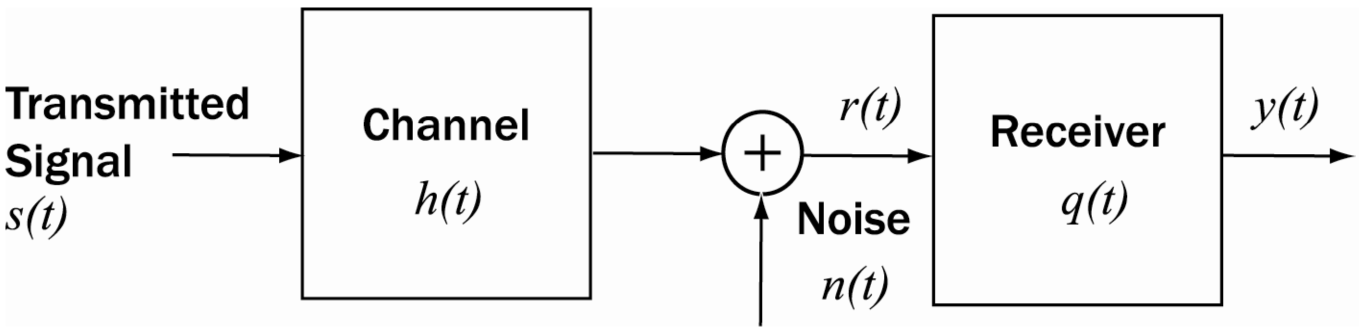
I.5 : Waveform Diversity

- Broad term covering waveform design and adapting waveforms in real-time to better improve detection/localization
  - generally try to improve signal-to-interference plus noise ratio
- "Diversity" implies having a choice of multiple waveforms to achieve a specific purpose
- Start with waveform design…
  - …followed by MIMO radar…
  - …followed by joint consideration of MIMO radar and waveforms

A Brief History

- Waveform Diversity
  - first discussions in late 1990s at AFRL-Rome
  - had spent 90s working on STAP and knowledge-based processing
  - some work on joint design of waveforms and processing
  - renewed interest in distributed apertures
- Work at AFRL and other place culminated in the 1st Annual Waveform Diversity Workshop in Feb. 2003
  - stayed "annual" until 2005 or so…
  - was expanded into the series of Waveform Diversity and Design conferences

An Early Waveform Design Problem



- Received signal: $r(t) = s(t) \star h(t) + n(t) = p(t) + n(t)$
- Output signal: $y(t) = p(t) \star q(t) + n(t) \star q(t)$
- We want max-SNR in output signal – sampled at appropriate time τ_0
- Use the fact that noise is white: $S_n(f) = N_0/2$

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Waveform Design (2)

$$\mathrm{SNR}_{t=\tau_0} = \frac{\left|\int_{-\infty}^{\infty} q(\tau)p(\tau_0-\tau)\right|^2 d\tau}{(N_0/2)\int_{-\infty}^{\infty} |q(\tau)|^2 d\tau}$$

- By Cauchy-Schwarz, $q(t) = p^*(\tau_0 - t)$, the matched filter

$$\mathrm{SNR} \propto \int_{-\infty}^{\infty} |p(t)|^2 dt = \int_{-\infty}^{\infty} |P(f)|^2 df = \int_{-\infty}^{\infty} |S(f)H(f)|^2 df$$

- In turn, if the transmitter knows $h(t)$ what is the optimal $s(t)$?
  - the one that maximizes the SNR

Waveform Design (3)

- This leads to an eigenvalue equation

$$\lambda s(t) = \int_{-\infty}^{\infty} s(\tau) K(t-\tau) d\tau$$

where

$$K(t) = \int_{-\infty}^{\infty} h(^{*}(\tau) h(t-\tau) d\tau$$

is the kernel of the channel $K(t) = \mathcal{F}^{-1}\left[|H(f)|^2\right]$

- This arises because of the magnitude squared term in the SNR
- Choose eigenfunction corresponding to largest λ
- If noise is coloured, whiten it first

Waveform Design (4)

- Need to normalize the energy to ensure the transmitter meets its power constraint
- So far, no limit on bandwidth
  - incorporate bandwidth constraints by limiting the kernel function in the frequency domain
- Special case: flat channel
  - leads to prolate spheroidal wave functions

II : MIMO Radar

- Multiple Input Multiple Output radar systems
  - exploits multiple transmitters, multiple receivers, multiple waveforms
    - i.e., all available degrees of freedom
  - a generalization of multistatic radar
  - let's agree that MIMO radar research did not start in 2004
    - called "multistatic radar", "distributed apertures", "waveform diversity", "netted radar"....
    - it is important to emphasize that MIMO radar research builds on previous works in this area

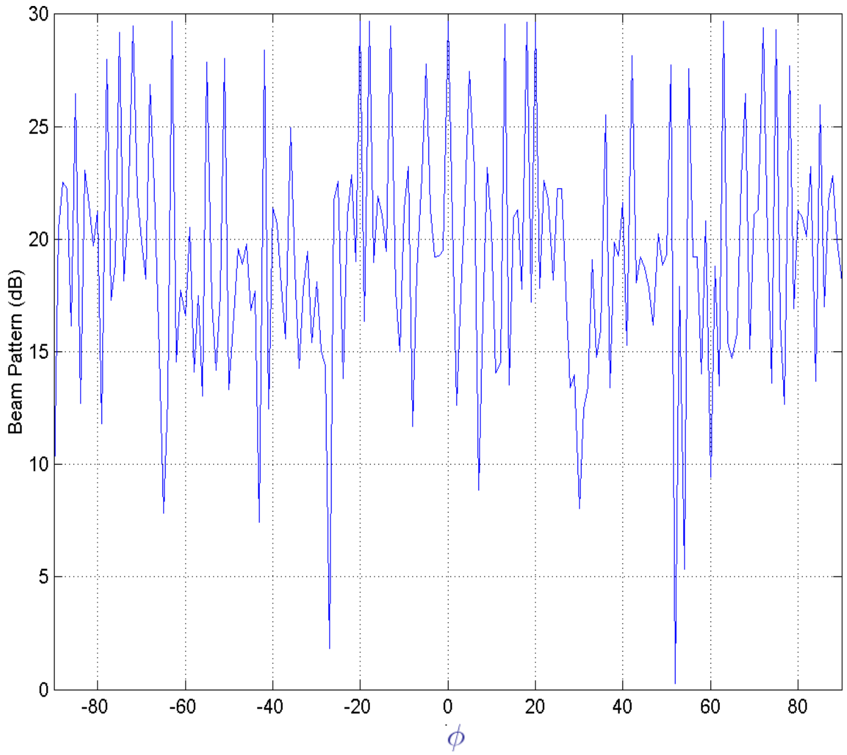
MIMO Radar : Introduction (2)

- Statistical MIMO radar:
  - often widely spaced apertures
    - conceivably acts as one big aperture
  - the target response for each transmit-receive pair is statistically independent
    - possibly due to different look angles or different frequencies
- Coherent MIMO radar
  - closely spaced apertures operating on the same frequency, e.g., French RIAS system (1984)
  - same target response to all tx-rx pairs

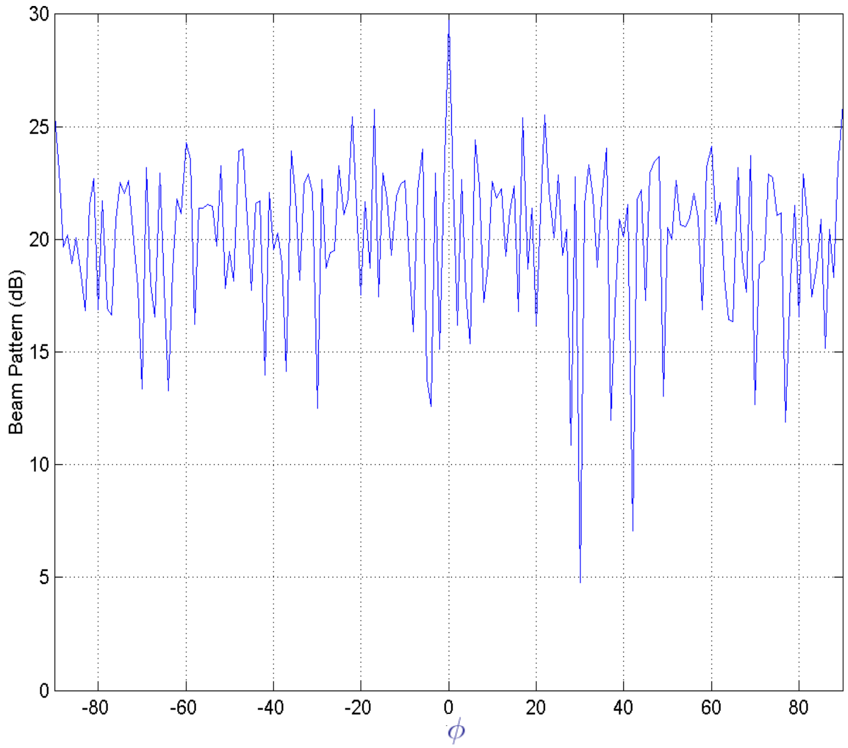
II.1 : Sample Result

Beampatterns

Without Frequency Offset (No Diversity)



With Frequency Offset (Waveform Diversity)



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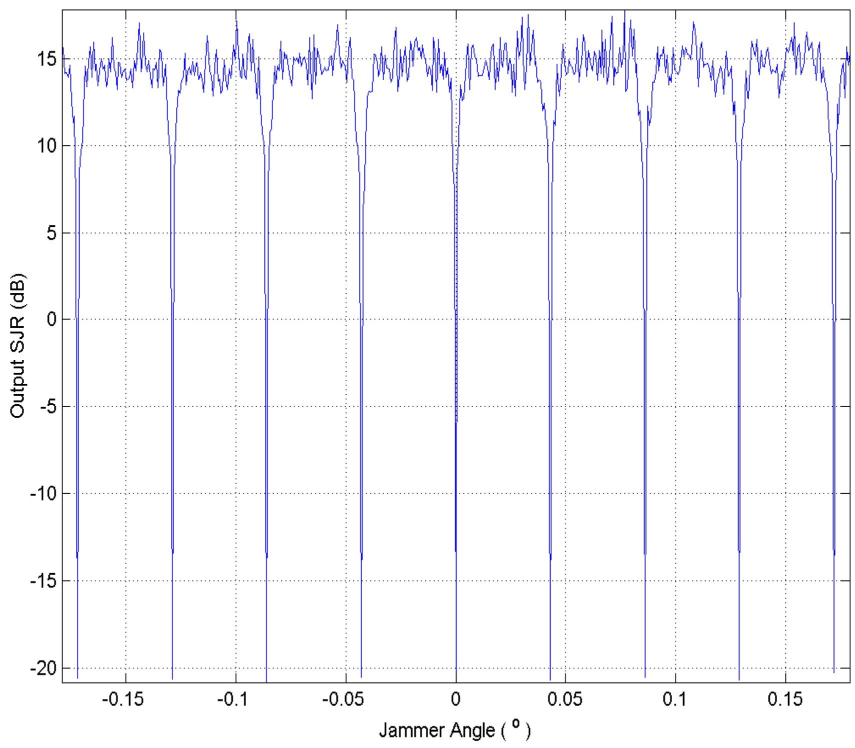
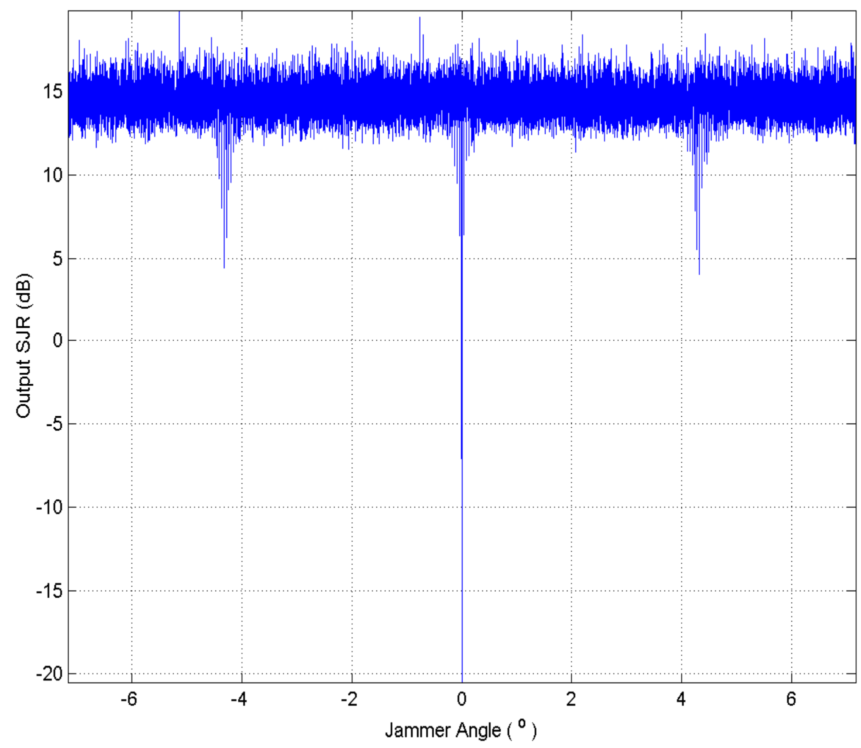
Sample Results : Uniform Spacing

Ability to distinguish signal from interference

Without Frequency Offset (No Diversity)



With Frequency Offset (Waveform Diversity)



Signal-to-jammer ratio

II.2 Virtual Array Representation

- Consider N_t transmit and N_r receive antennas
  - transmitters at $\mathbf{p}_n, n = 0, \ldots, N_t - 1$
  - receivers at $\mathbf{q}_m, m = 0, \ldots, N_r - 1$
  - assume that the transmitters transmit N_t orthogonal coherent waveforms
    - e.g., using orthogonal codes
    - the receiver can distinguish each waveform without error
    - after matching to the n^{th} transmit signal at the m^{th} receiver the received signal from target is

$$x_{nm} = \alpha_t h_{nm} + \mathrm{noise}$$

Virtual Array Representation (2)

- where

$$h_{nm} = e^{j\mathbf{k}\cdot(\mathbf{p}_n+\mathbf{q_m})}$$

- This is equivalent to a receive aperture at locations $(\mathbf{p}_n + \mathbf{q_m})$
- Another interpretation: each transmitter has its own receive aperture
  - the positions of the virtual array are a convolution of the transmitter and receiver positions

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Virtual Array Representation (3)

- Consider the transmitters and receivers on a grid
  - usually grid of $\lambda/2$ spacing
  - for now consider a 1-D line
- Example: 3 transmitter antennas at [1 1 1]

  3 receive antennas at [1 1 1] (co-located)
- Equivalent to 5 receive antennas with relative weighting of [1 2 3 2 1]
  - acts a virtual array of 5 elements
  - some elements have excess weighting because they are sampled repeatedly
    - e.g., Tx1-to-Rx3 is the same as Tx3-to-Rx1

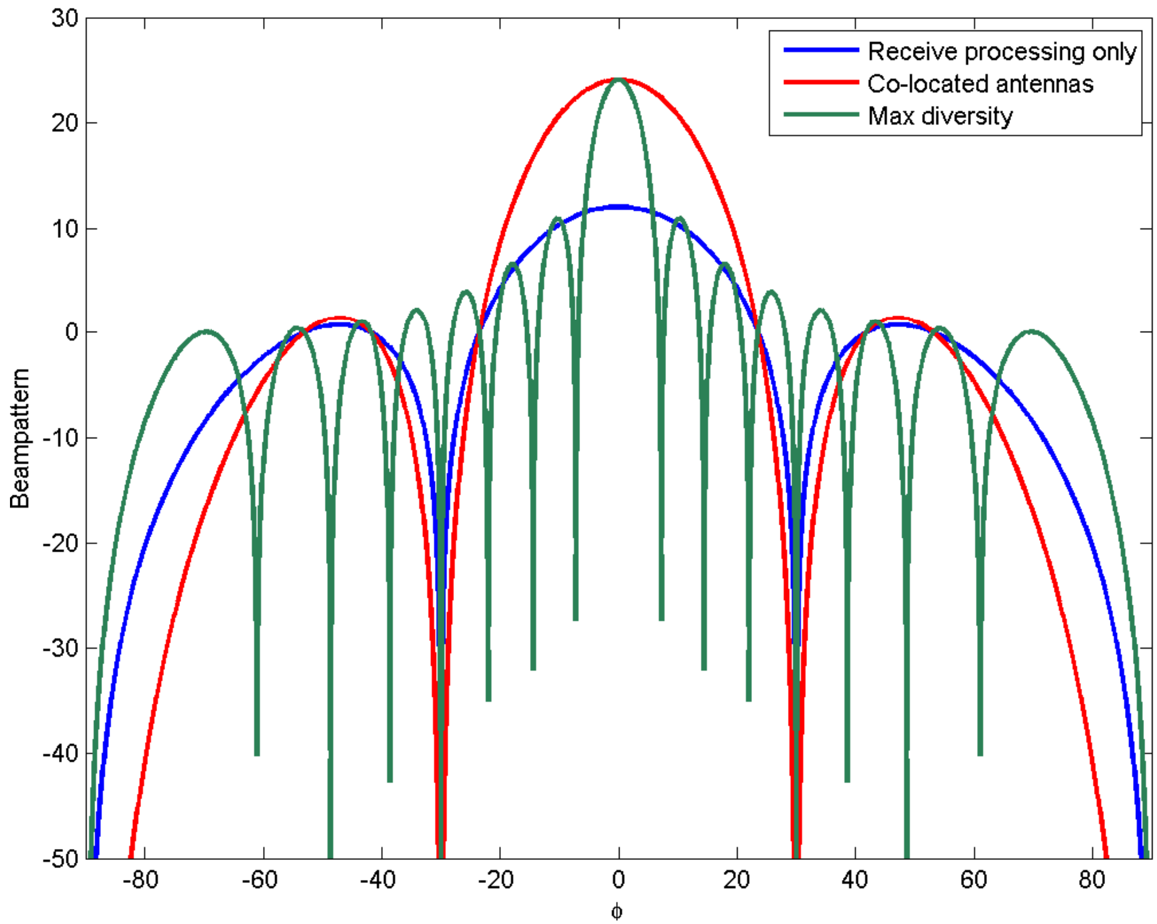
Virtual Array Representation (4)

- Can be further improved by using a thinned array
- Example: 3 antenna elements at [1 1 0 1] and co-located receive antennas results in a virtual array of [1 2 1 2 2 0 1] (a 6-element virtual array)
- Transmitter and receiver not necessarily co-located
- Example: 3 transmitter antennas at [1 1 1]

  3 receive antennas at [1 0 0 1 0 0 1 0 0]

  results in [1 1 1 1 1 1 1 1 1] (9 elements)
- Here, there are no repeated paths and each transmit-receive pair is unique

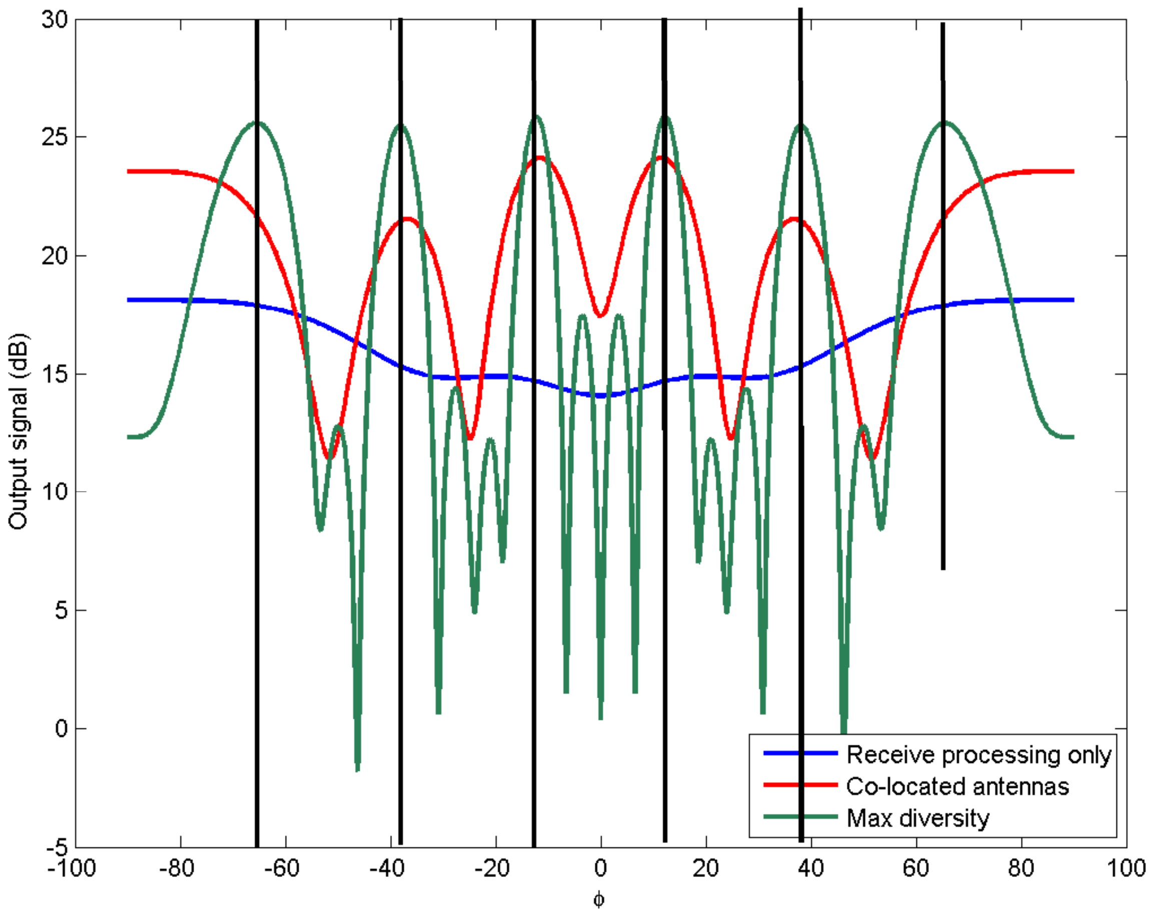
Impact on Resolvability



$$N_t = N_r = 4$$

- Co-located antennas provides only a little improvement in resolvability
  - improves antenna due to transmit and receive
- Max diversity creates the largest virtual array
  - leads to improvements in both gain and resolvability

Impact on Resolvability (2)



- 6 targets equally spaced in angle
- Vertical lines indicate locations of targets
- The max-diversity array can detect all 6
- The co-located antennas case detects 4 of the 6 reliably
  - small errors visible
- Receive only processing cannot detect any
- Depends on orthogonal transmissions

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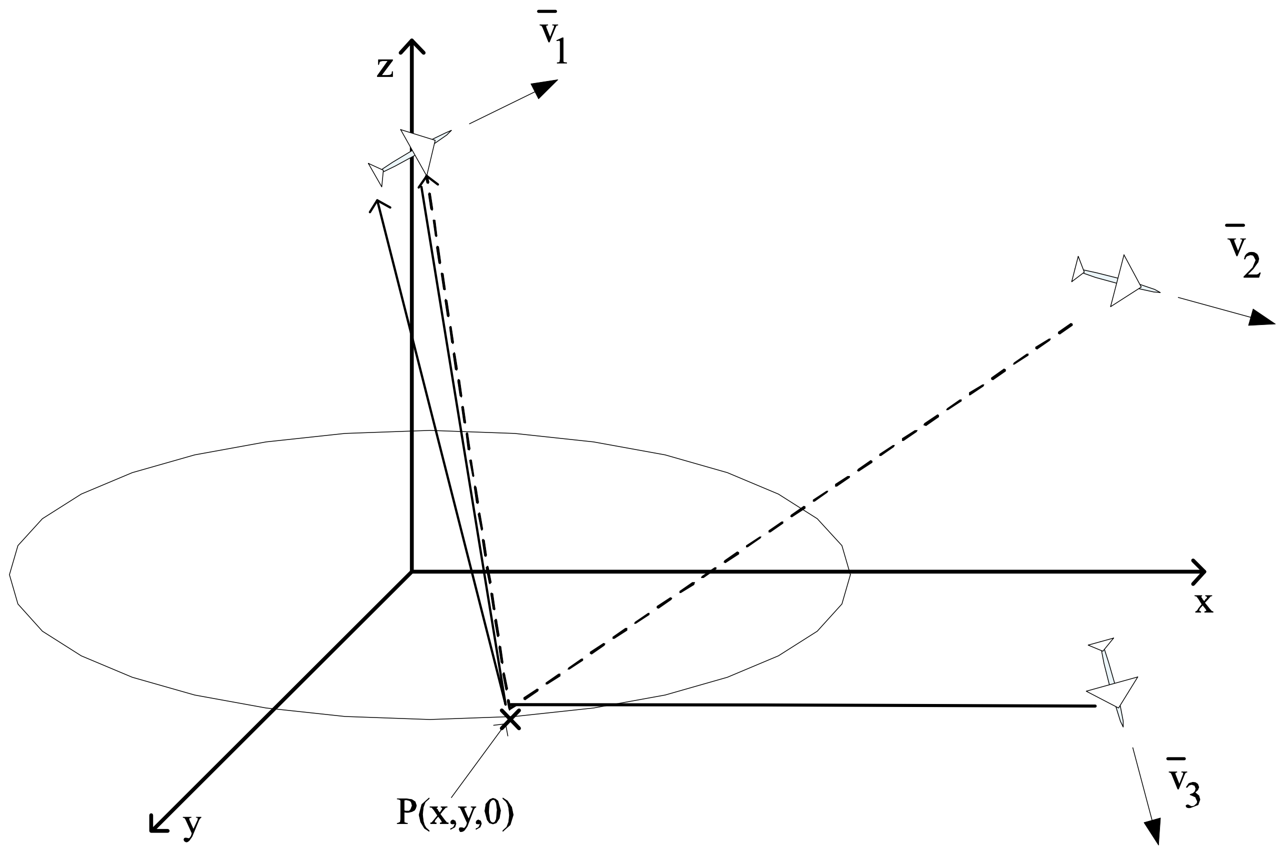
Parameter Estimation

- With a receive phased array: N_r elements in array
  - can estimate up to $\left\lceil \frac{2N_r}{3} \right\rceil$ parameters
- With co-located tx/rx MIMO array: $N_t + N_r - 1$ in virtual array
- Max # elements in virtual MIMO array: $N_t N_r$
  - can estimate up to between

$$\left\lceil \frac{2(N_t + N_r - 1)}{3} \right\rceil \quad \text{and} \quad \left\lceil \frac{2N_t N_r}{3} \right\rceil$$

parameters

II.3 : Theoretical Analysis



How do we characterize such a system?

II.3.1 : Target Models

- In the case of co-located antennas, the target response is the same to all antennas
- N_t transmit antennas, N_r receive antennas, M pulses
  - transmit antennas at $\mathbf{p}_i, i = 0, \ldots, N_t - 1$
  - receiver antennas at $\mathbf{q}_j, j = 0, \ldots, N_r - 1$
  - parameter vector for transmitter i , receiver j : $\mathbf{\Theta}_{i,j}$
  - target at location: $\mathbf{p}$, velocity $\mathbf{v}$, parameters $\mathbf{\Theta} = (\mathbf{p}, \mathbf{v})$

$$\tau_{ij} = \tau_i(\mathbf{p}) + \tau_j(\mathbf{p}) \qquad f_{ij} = f_i(\mathbf{\Theta}) + f_j(\mathbf{\Theta})$$

  - τ_{ij} relative delay, f_{ij} relative Doppler, $\mathbf{\Theta}_{i,j} = (\tau_{ij}, f_{ij})$

Target Models (2)

- Signal transmitted by antenna i : $s_i(t)$
- Signal received by antenna j :

$$x_j(t) = \sum_{i=1}^{N_t} \alpha_{ij} s_i(t - \tau_{ij}) e^{j2\pi f_{ij} t} + n_j(t)$$

$\sqrt{-1}$ not antenna index!

- α_{ij} : target amplitude seen at receiver j due to signal from transmitter i
- Next step: matched filtering and sampling
  - possibly matching to N_t pulse shapes
  - M pulses in a CPI

Target Models (3)

- Writing the signal over M pulses into a vector

$$\mathbf{x}_j = \mathbf{S}_j(\mathbf{\Theta})\boldsymbol{\alpha}_j + \mathbf{n}_j$$

- $\mathbf{S}_j(\mathbf{\Theta}) : M \times N_t$: signal matrix, $\boldsymbol{\alpha}_j : N_t \times 1$ amplitude vector
- Now, combining all N_r receive antennas

$$\mathbf{x} = \mathbf{S}(\mathbf{\Theta})\boldsymbol{\alpha} + \mathbf{n}$$

$$\boldsymbol{\alpha} = \left[\boldsymbol{\alpha}_1^T, \boldsymbol{\alpha}_2^T, \ldots, \boldsymbol{\alpha}_{N_r}^T\right]^T$$

$\mathbf{n}$: noise vector

Target Models (4)

- Also,

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_1(\boldsymbol{\Theta}) & 0 & \dots & 0 \\ 0 & \mathbf{S}_2(\boldsymbol{\Theta}) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{S}_{N_r}(\boldsymbol{\Theta}) \end{bmatrix}$$

- The data covariance matrix given by

$$\mathbf{R} = E\left[\mathbf{x}\mathbf{x}^H\right] = \mathbf{S}\boldsymbol{\Sigma}\mathbf{S}^H + \sigma^2\mathbf{I}$$

$$\boldsymbol{\Sigma} = E\left[\boldsymbol{\alpha}\boldsymbol{\alpha}^H\right]$$

Target Models (5)

- Rank of $\boldsymbol{\Sigma}$ is the key : an $N_r N_t \times N_r N_t$ matrix

$$\boldsymbol{\Sigma} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$$

- If rank-1, $\boldsymbol{\Sigma} = \lambda \mathbf{u}\mathbf{u}^H$ and the target signal is coherent across the $N_r N_t$ transmit-receive pairs
  - a coherent target, for example with co-located MIMO radar
- If rank = $N_r N_t$ (maximum possible), the target returns are non-coherent across the $N_r N_t$ transmit-receive pairs
  - a non-coherent target and maximum diversity
- Could be somewhere in between too

II.3.2 : Diversity Order

- In wireless communications, diversity order measures the number of independent paths in multi-antenna systems
  - slope of the BER v/s SNR curve at high SNR
  - usually achieved at moderate SNR regime
- Can we use this idea in MIMO radar systems?
- A few concerns:
  - High SNR is irrelevant in radar systems
  - Probabilities of detection/miss only make sense if false alarm is kept constant
  - The rising part of the P_D curve is of most interest

Background

- A regular radar is characterized by its probability of detection P_D for a fixed probability of false alarm P_{FA}
- We wish to analyze the impact of using multiple independent (K) platforms
- Let's start with a single platform; the received signal vector is given by

$$\mathbf{z}_k = \begin{cases} \alpha_k \mathbf{s}_k + \mathbf{n}_k, & \text{if target is present} \\ \mathbf{n}_k, & \text{if target is absent} \end{cases}$$

Background (2)

- This signal is processed using the weights

$$\mathbf{w}_k = \mathbf{s}_k$$

leading to the statistic

$$\eta = |\mathbf{w}^H \mathbf{x}|^2$$

- Neyman-Pearson test uses the likelihood ratio:

$$\frac{f(\eta|H_1)}{f(\eta|H_0)} \gtrless_{H_0}^{H_1} t_0$$

H_1: the target present hypothesis

H_0 : the target absent hypothesis

t_0 : threshold that determines P_{FA}

Background (3)

- Under H_0, η is exponentially distributed with mean λ_0 and

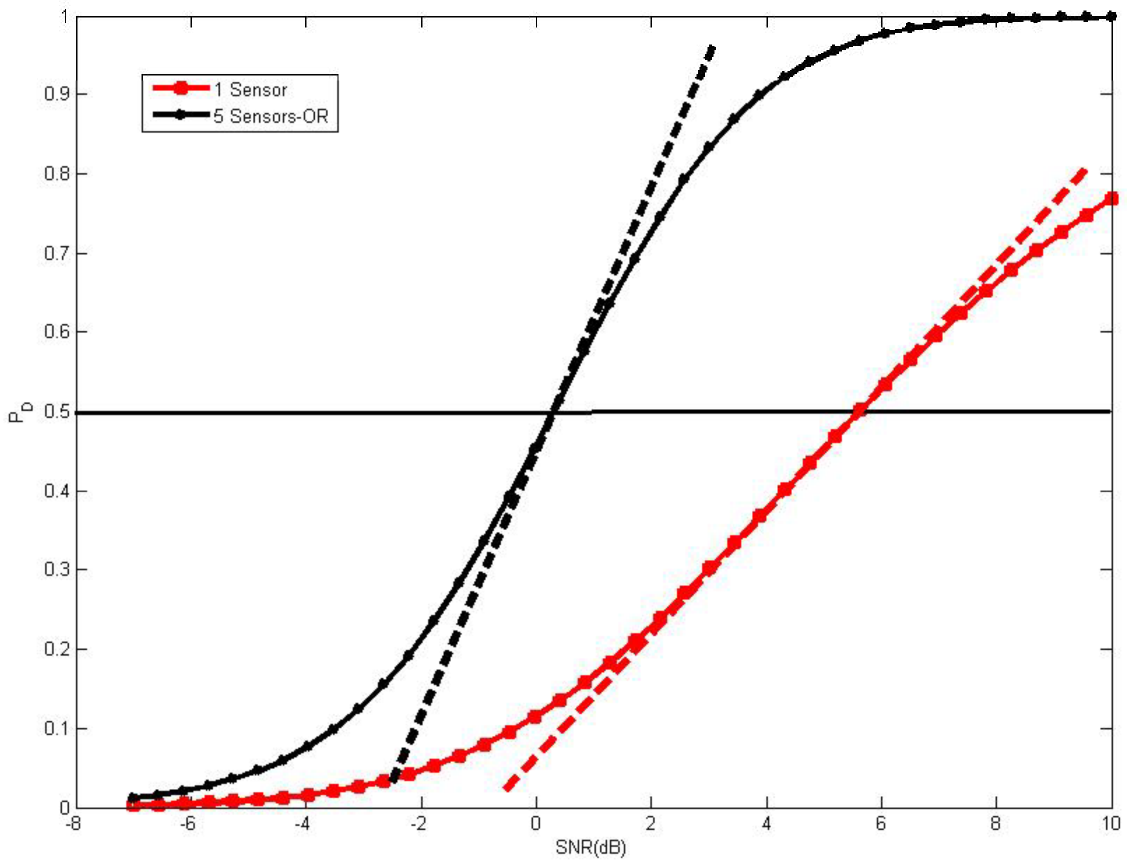
$$P_{FA} = e^{-t_0/\lambda_0}$$

- Similarly, under H_1, η is exponentially distributed with mean λ_1 and

$$P_D = e^{-t_0/\lambda_1}$$

- Note that reducing the threshold (increasing sensitivity) increases both P_D and P_{FA}
- As we use a MIMO radar, the analysis must account for this increase in both measures

Illustrating Diversity Order in MIMO



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Diversity Order : Definition

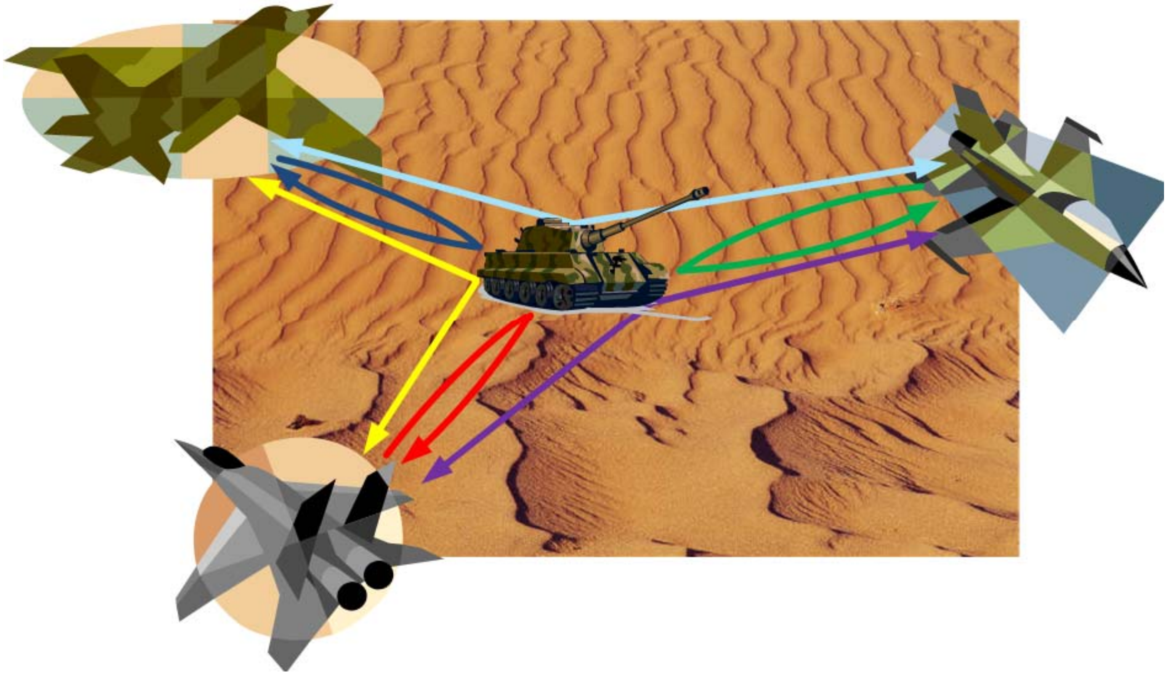
- Our definition also uses a slope:

The *diversity order* of a radar system is the slope in a linear scale of the probability of detection versus SNR curve at $P_D = 0.5$ *for a fixed probability of false alarm*.

- Definition captures
  - the SNR range of interest
  - is valid only for a fixed false alarm rate
  - the interaction of the spatial degrees of freedom and processing scheme

System Model

- K sensors, N antennas each
- Noise limited
- Uses the Swerling-II model for the target
- Signal received at sensor :



$$\mathbf{x}_k = \begin{cases} \alpha_k \mathbf{s}_k + \mathbf{n}_k, & \text{if target is present} \\ \mathbf{n}_k, & \text{if target is absent} \end{cases}$$

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Single Sensor

- For a single sensor

$$\lambda_0 = \frac{N\gamma}{1 + N\gamma} \quad \lambda_1 = N\gamma$$

  and diversity order is N

- For co-located antennas that see the same target amplitude (not really MIMO) diversity order is KN

Joint Detection

- Each sensor transmits its exact likelihood ratio to a fusion center
  - the fusion center combines the LR from all K sensors
    - the LR are proportional to signal power
    - similar to maximal ratio combining in communications
  - each sensor contributes an exponential RV
    - the sum follows a gamma distribution

$$f(x; K, \theta) = x^{K-1} \frac{e^{-x/\theta}}{\theta^K \Gamma(K)}$$

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Joint Detection (2)

- PDF under H_0 provides the threshold by finding the false alarm rate (P_{FA})
- PDF under H_1 then finds P_D
- For large K, the diversity order proportional to $N\sqrt{K}$
- The improved detection probability is partially offset by increased false alarm rate

Distributed Detection

- The Neyman-Pearson test is optimal under CFAR
  - Each sensor reports a local decision (u_k) to the fusion center
- The fusion center combines the decisions into a final decision
- Optimal combiner needs knowledge of statistics at the sensors:

$$\Lambda(\mathbf{u}) = \sum_{i=1}^{n_{H_1}} \log \frac{\Pr(1_i|H_1)}{\Pr(1_i|H_0)} + \sum_{i=1}^{n_{H_0}} \log \frac{\Pr(0_i|H_1)}{\Pr(0_i|H_0)},$$

  - compare $\Lambda(\mathbf{u})$ to a threshold that sets P_{FA}
  - Again, the diversity order is proportional to $N\sqrt{K}$

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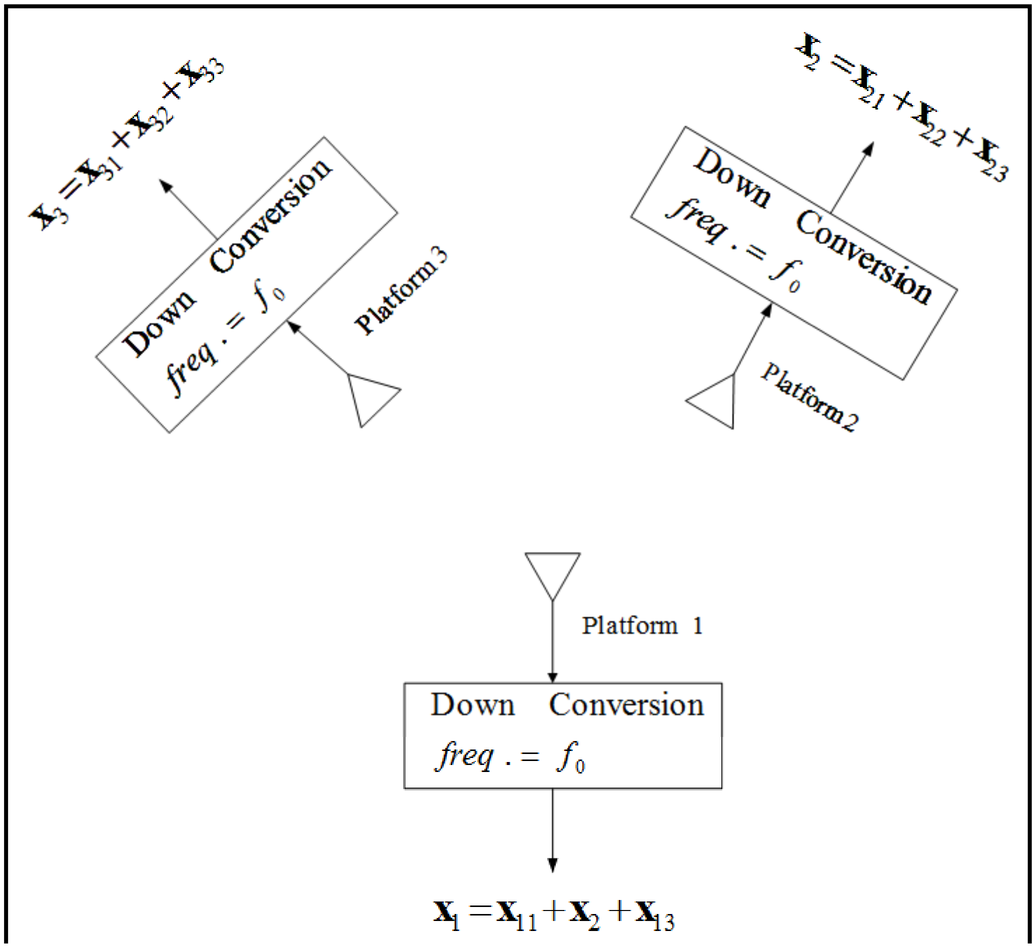
Distributed Detection (2)

- More practically,
  - OR, AND, MAJ rules
- OR rule: the diversity order is proportional to $N \ln K$
- AND rule: the diversity order is proportional to N
  - *there is no gain due to distributed sensors*
  - the reduced detection probability is offset exactly by the reduced false alarm rate
- MAJ rule: somewhere in between

II.4 : STAP with Distributed Sensors

- Non-Frequency Diversity (NFD) Case : K platforms use same frequency (overlapping waveforms)



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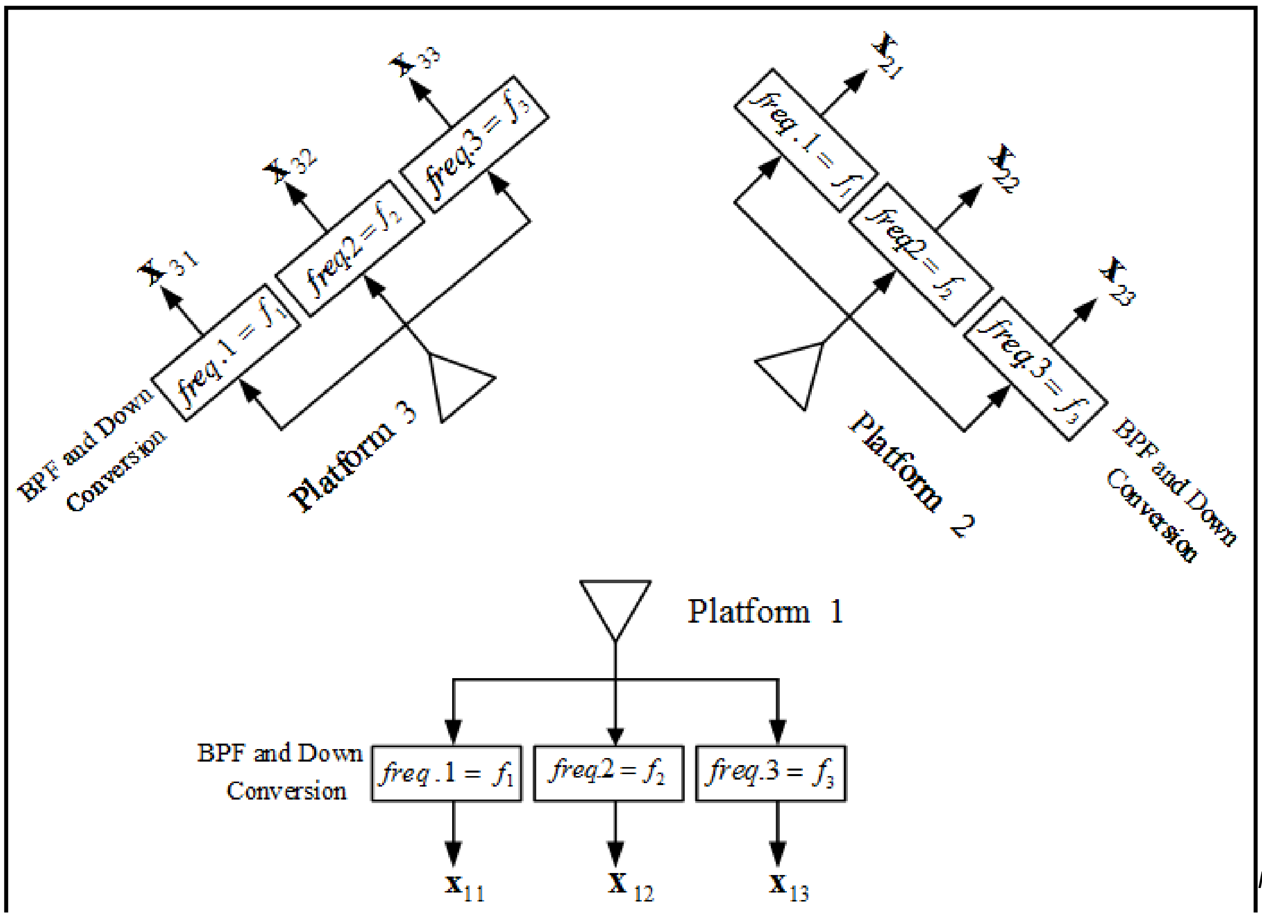
STAP with Distributed Sensors (2)

- Frequency Diversity (FD)/orthogonal waveforms case
  - Each platform uses a different frequency



System Model

- **Received signal**
  - **NFD case**

$$H_0: \quad \mathbf{x}_p = \sum_{q=1}^{K} \mathbf{x}_{pq} = \sum_{q=1}^{K} \mathbf{c}_{pq} + \mathbf{n}_p$$

$$H_1: \quad \mathbf{x}_p = \sum_{q=1}^{K} \mathbf{x}_{pq} = \sum_{q=1}^{K} [\alpha_{pq}\mathbf{s}_{pq} + \mathbf{c}_{pq}] + \mathbf{n}_p$$

$\mathbf{c}_{pq}$: clutter signal

α_{pq} : target amplitude

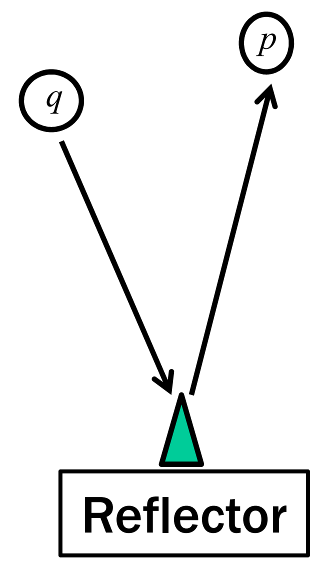
$\mathbf{s}_{pq}$: target steering vector

  - **FD case**

$$H_0: \quad \mathbf{x}_{pq} = \mathbf{c}_{pq} + \mathbf{n}_p$$

$$H_1: \quad \mathbf{x}_{pq} = [\alpha_{pq}\mathbf{s}_{pq} + \mathbf{c}_{pq}] + \mathbf{n}_p$$

$$\Rightarrow \quad \mathbf{x}_p = \begin{bmatrix} \mathbf{x}_{p1} \\ \mathbf{x}_{p2} \\ \vdots \\ \mathbf{x}_{pK} \end{bmatrix}$$



Interference Covariance Matrix

- Define

$$\mathbf{R}_p = E\left[\mathbf{x}_p \mathbf{x}_p^H\right] \qquad \mathbf{R}_{pq} = E\left[\mathbf{x}_{pq} \mathbf{x}_{pq}^H\right]$$

- NFD case

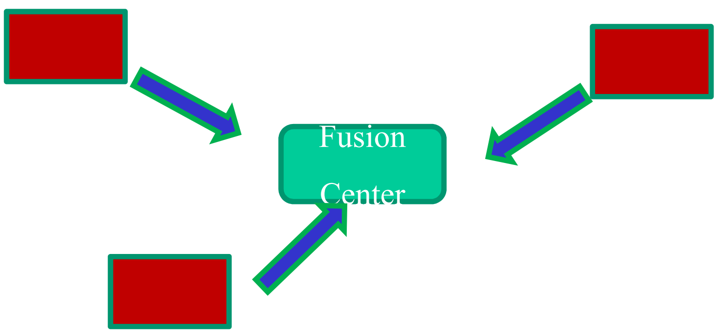
$$\mathbf{R}_p = \mathbf{R}_{p1} + \mathbf{R}_{p1} + \cdots + \mathbf{R}_{pK}$$

- FD case

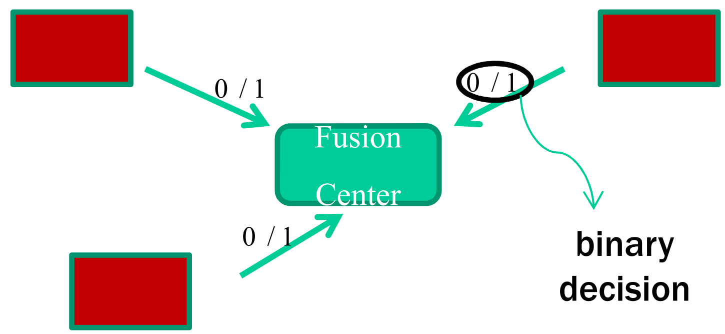
$$\mathbf{R}_p = \begin{bmatrix} \mathbf{R}_{p1} & 0 & \ldots & 0 \\ 0 & \mathbf{R}_{p2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \mathbf{R}_{pK} \end{bmatrix}$$

(De)Centralized? (Sub)Optimum?

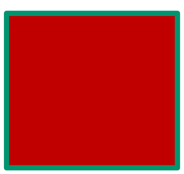
- Centralized Algorithm



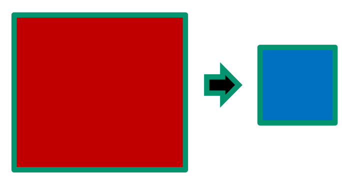
Decentrailzed Algorithm



- Optimum Algorithm



Suboptimum (Reduced Rank) Algorithm



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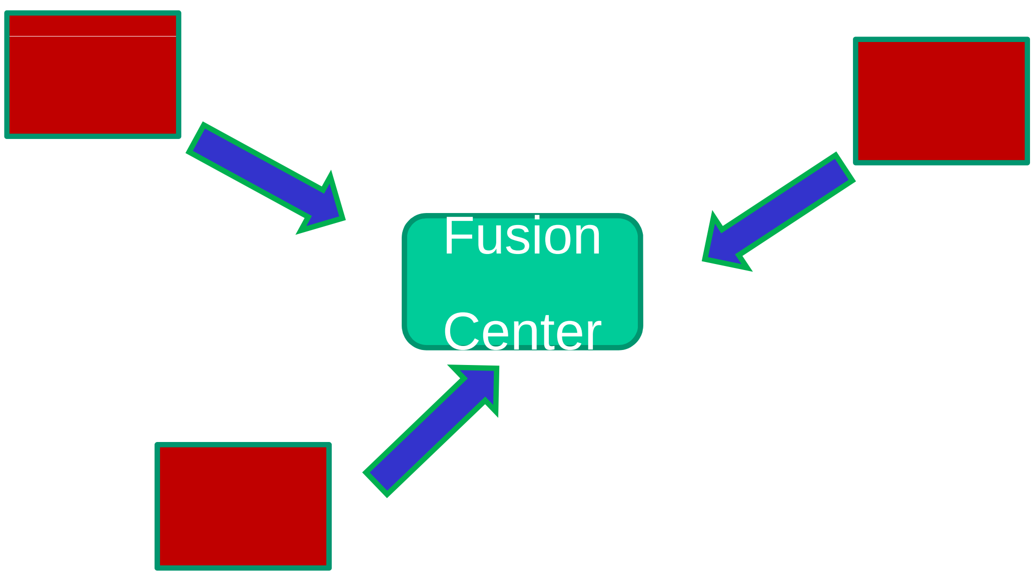
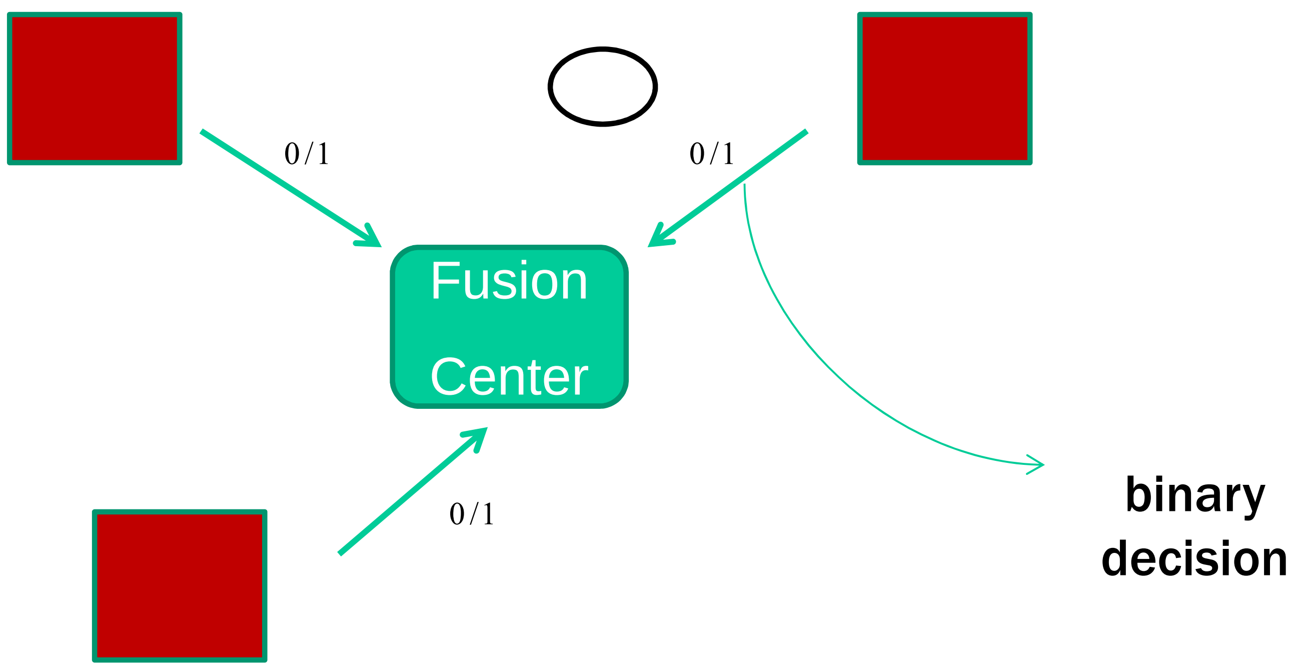
Decentralized Reduced Rank Algorithm

- Optimum Centralized Algorithm
  - – Computation Load
  - – Sample Support
  - – Large Communication BW
  - + Optimum Performance



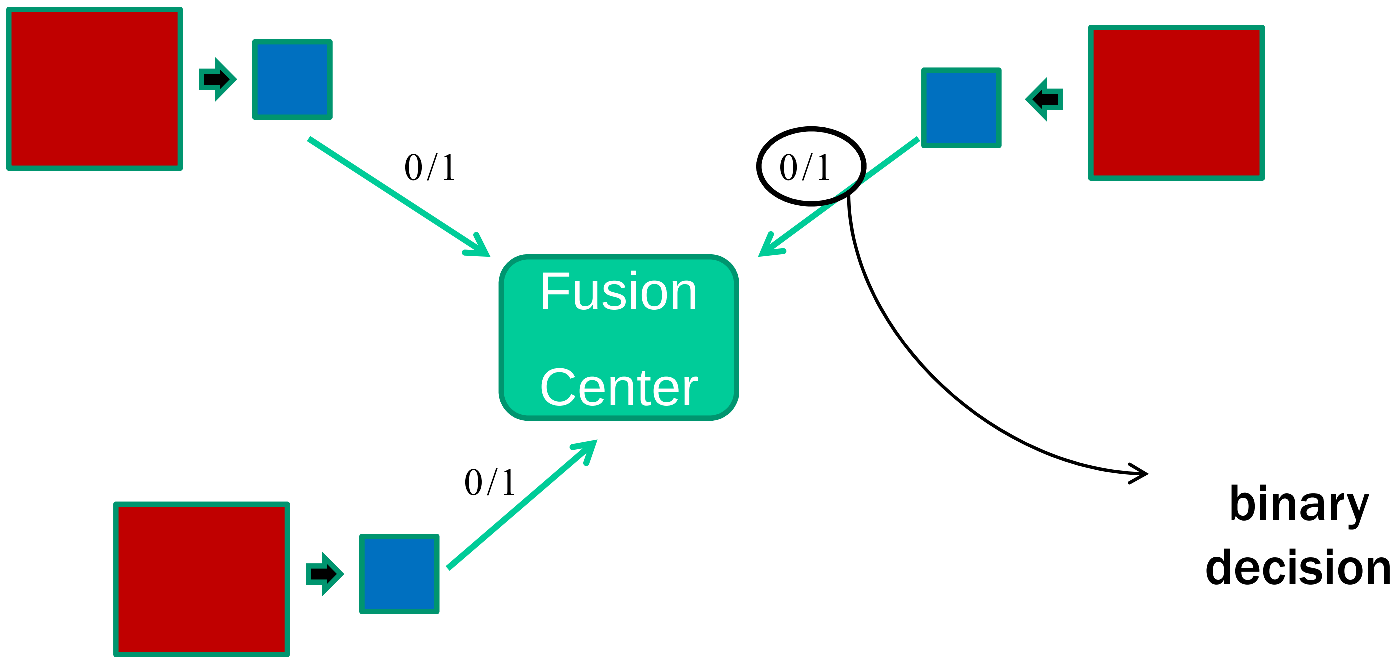
Decentralized Reduced Rank Algorithm

- Optimum Decentralized Algorithm
  - – Still have computation load & sample support problem
  - + Reduced Communication BW



Decentralized Reduced Rank Algorithm

- Sub-Optimum Decentralized Algorithm
  - – Performance Degradation
  - + Reduced Computation Load
  - + Reasonable Sample Support



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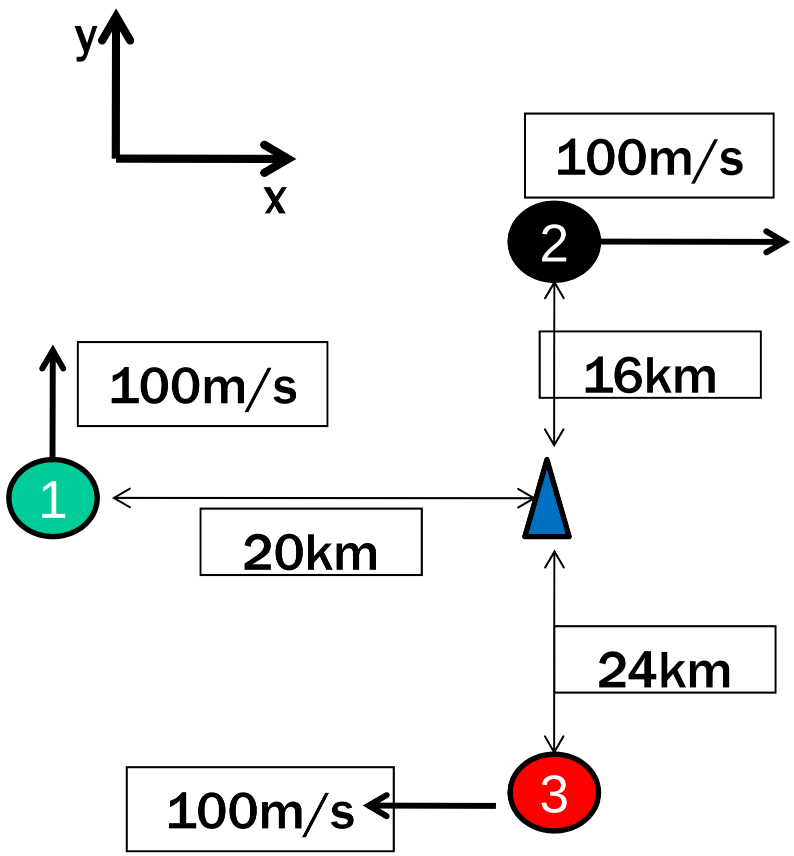
Simulation Results

- Simulation Scenario



Carrier freq.

NFD

f0=450MHz

FD

f1 = 450MHz

f2 = 430MHz

f3 = 410MHz

Target speed : 10m/s

PRF : 1KHz

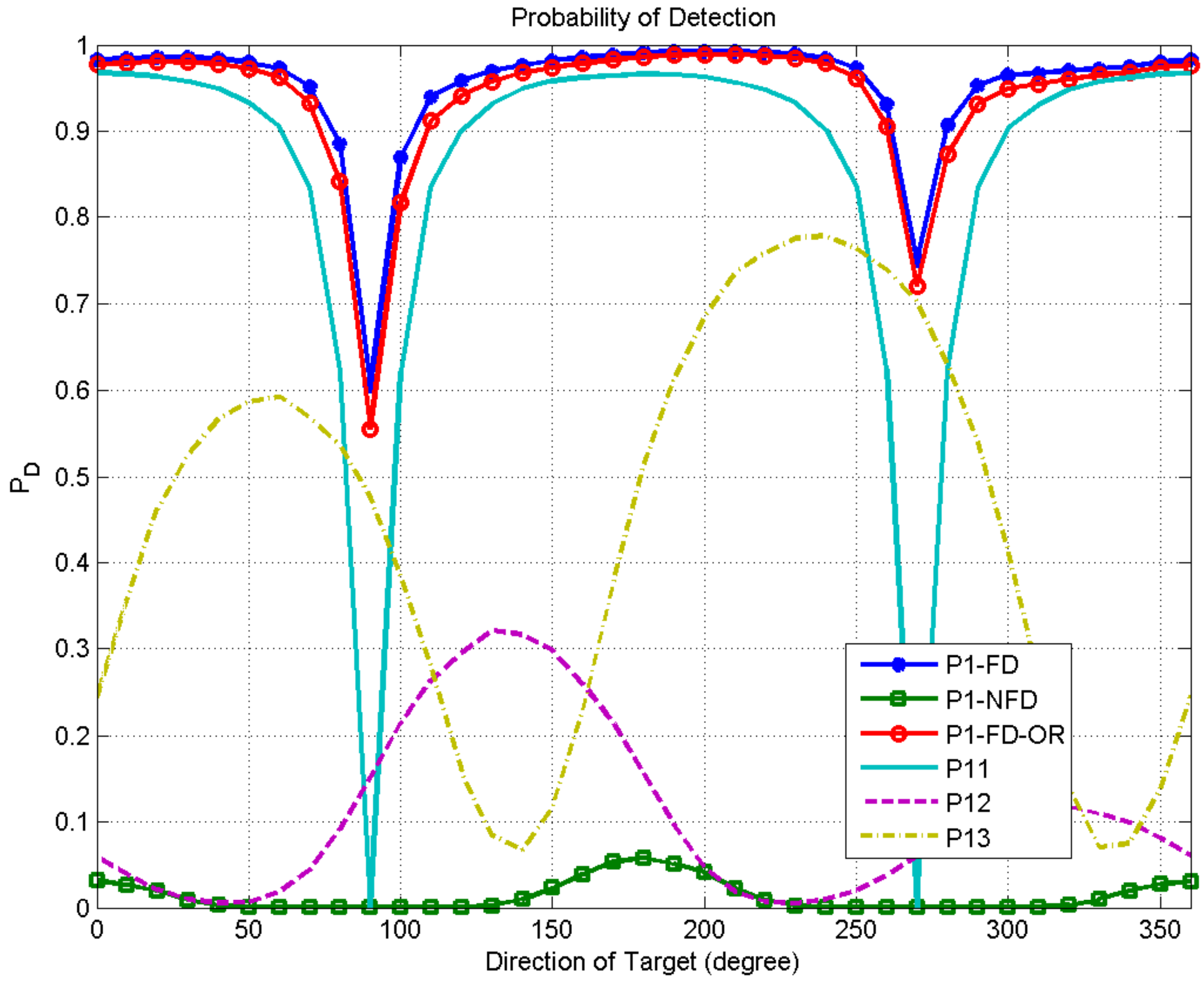
# of array elements : 10

# of pluses : 10

# of platforms : 3

# of clutter patches : 360

Simulation Results

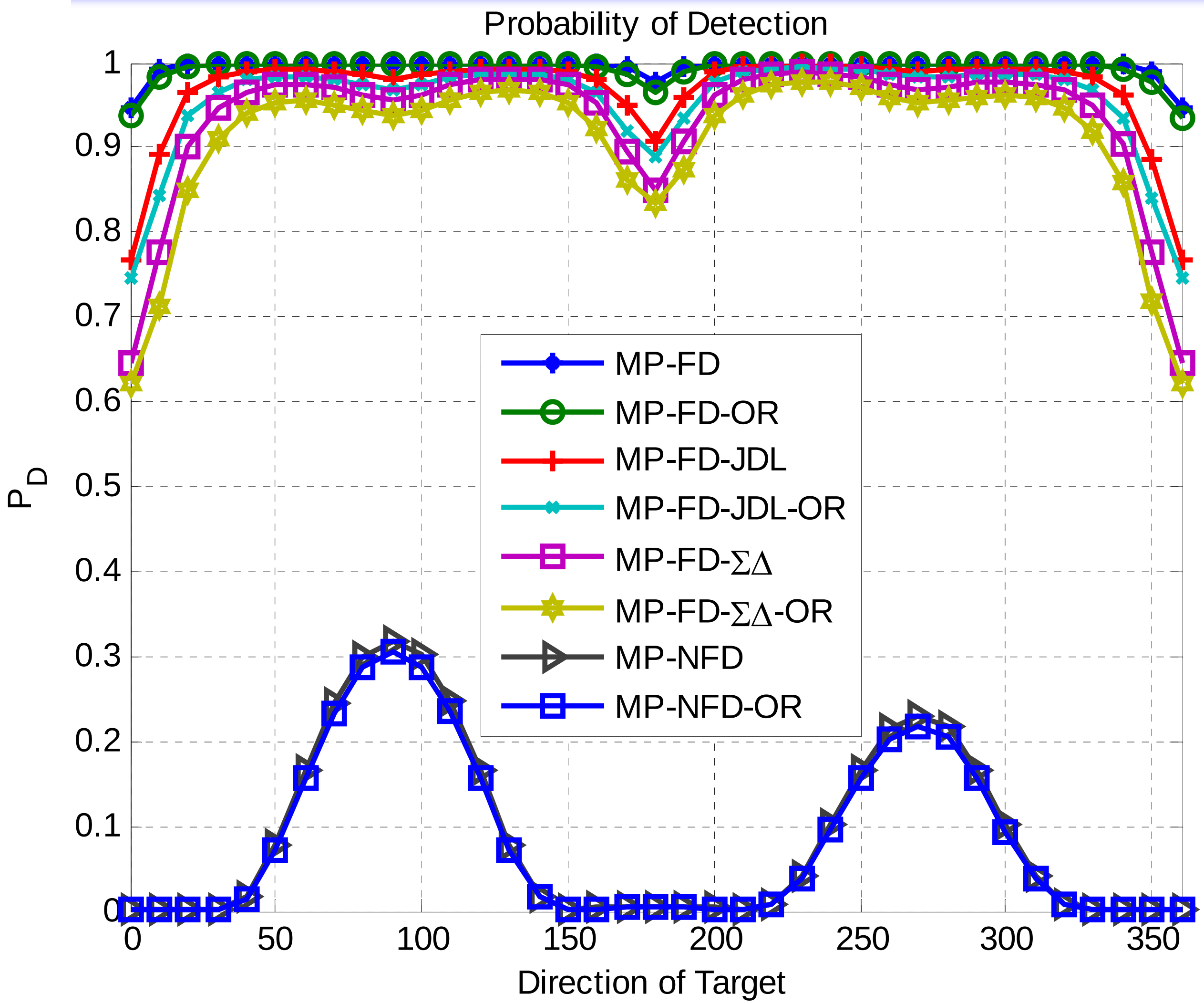


Prob. of detection for individual platforms

Note both the poor performance and the sensitivity to target direction

SNR=15dB

Simulation Results : Multiple Platforms



SNR=10dB

Prob. of detection for MIMO case platforms

Note both the poor performance for the NFD case and the robustness in the FD case

$$P_{FA} = 10^{-6}$$

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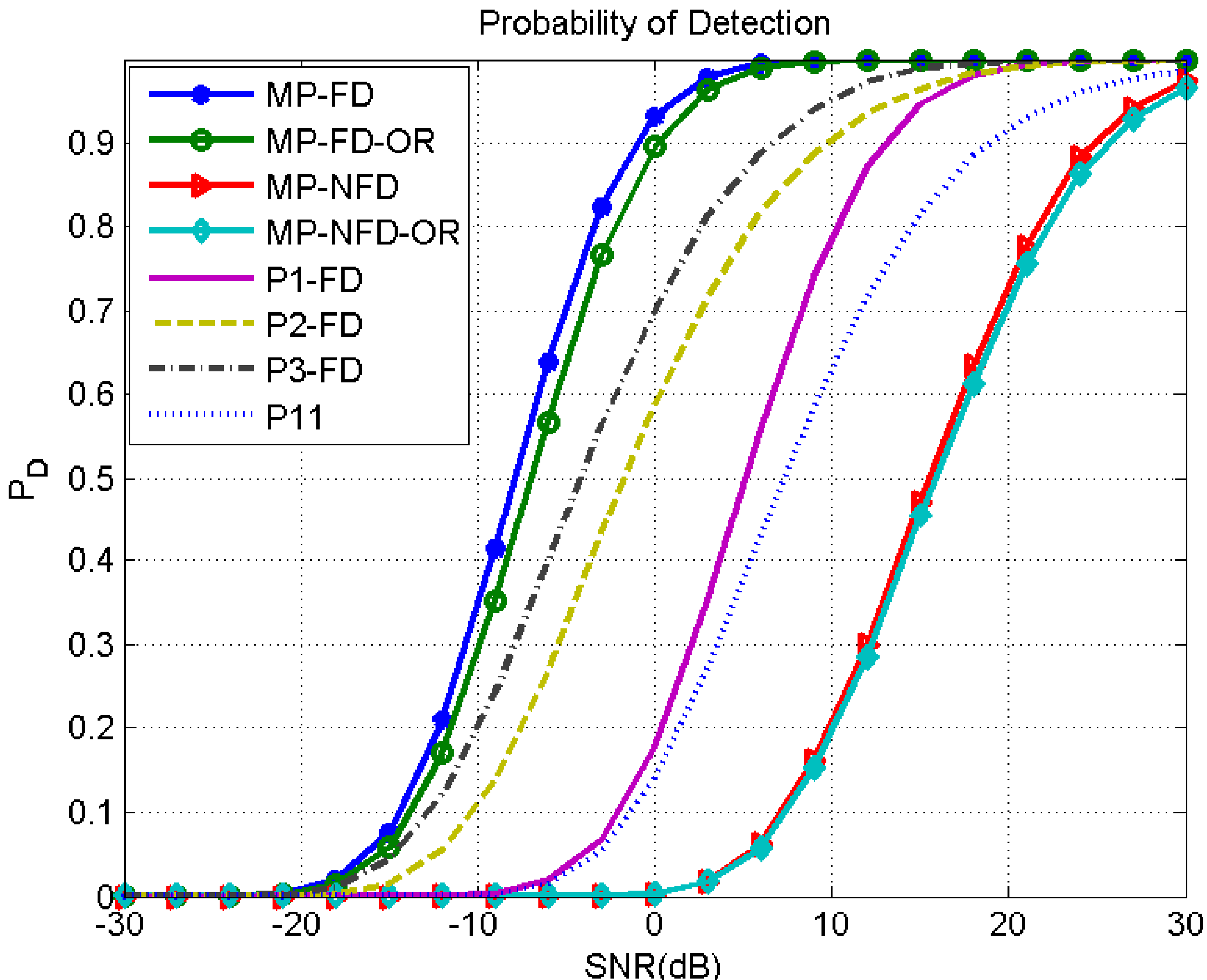
Results : Single v/s Multiple Platforms



Comparing detection probability of the single and multiple platforms

Note the difference in diversity order

$$P_{FA} = 10^{-6}$$

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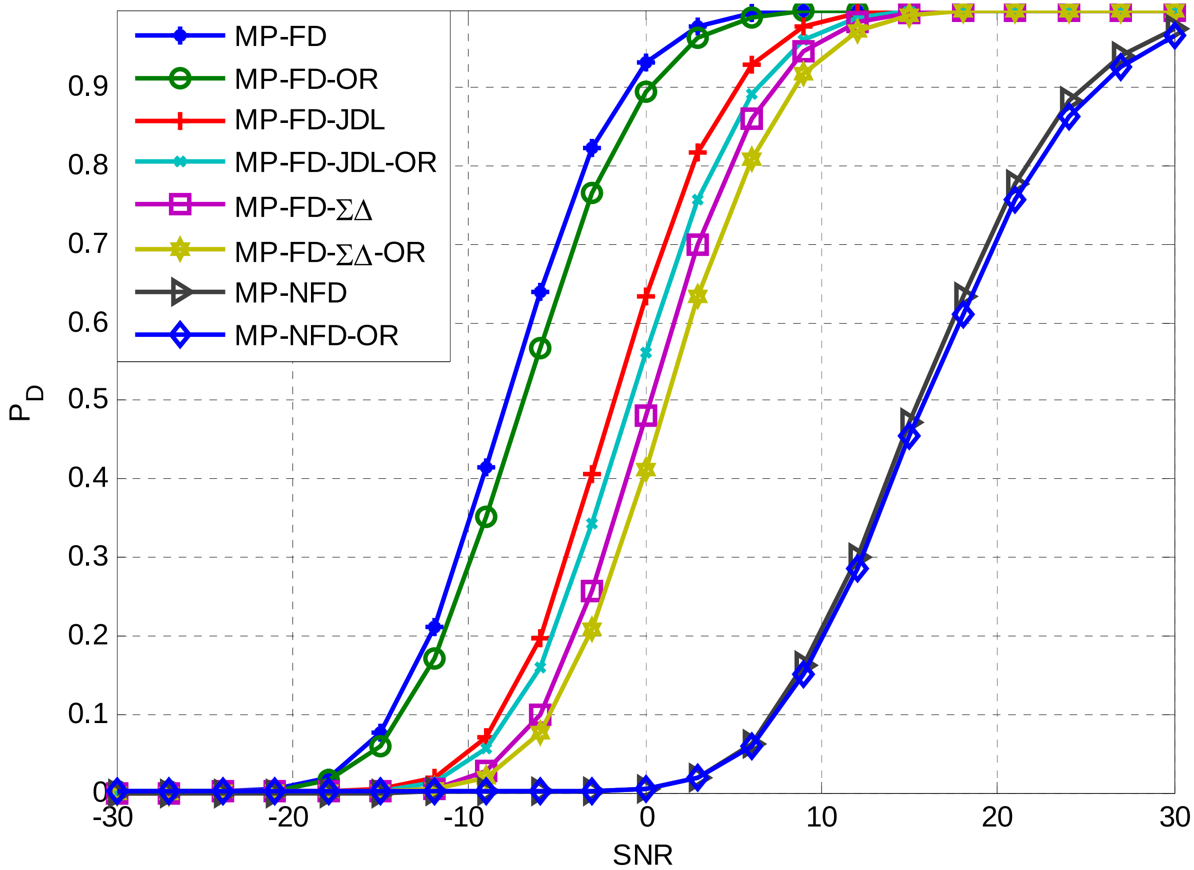
Results : Multiple Platforms



Comparing detection probability of different processing schemes

Note they all appear to have the same diversity order

$$P_{FA} = 10^{-6}$$

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Discussion : Diversity Order

- There is no loss in diversity due to using a sub-optimum STAP approach
  - the JDL and ΣΔ approaches have curves that are parallel
  - appears to have the same diversity order as the fully adaptive, centralized, scheme
    - though theoretically, asymptotically in K :
      - centralized scheme: $\sqrt{K}$;
      - distributed OR scheme: $\ln K$

- Clear loss in diversity for the NFD scheme

Discussion : Issues Not Addressed

- The results shown here – and most research in MIMO systems – are based on a key assumption
  - synchronization across platforms
  - in radar, each sample corresponds to range bin
    - in processing across multiple platforms, a key assumption is that the samples at a specific time at all platforms refers to the same range bin
      - this is crucial also for secondary data in STAP for MIMO radar
    - essentially, in these results, we have assumed true time delay

Part III : MIMO and Waveform Diversity

- So far we have considered detection and estimation using a MIMO radar
  - no discussion of the choice of waveform
  - first deal with case without constraints and then we will add some practical constraints
- MIMO ambiguity function
  - generalization of the ambiguity function to MIMO
  - interpret the ambiguity function as the cross-correlation between estimating the true target parameters $\Theta_0 = (\tau_0, f_{d0})$ and test parameters $\Theta_1 = (\tau_1, f_{d1})$

SISO Ambiguity Function Revisited

- Transmitted signal: $s(t)$
- Received signal due to target with parameters $\mathbf{\Theta}_0 = (\tau_0, f_{d0})$

$$s(t-\tau_0)e^{j2\pi f_{d0}t}$$

- Cross correlation between signal due to $\mathbf{\Theta}_0 = (\tau_0, f_{d0})$ and $\mathbf{\Theta}_1 = (\tau_1, f_{d1})$

$$\begin{aligned}\chi(\mathbf{\Theta}_0, \mathbf{\Theta}_1) &= \int_{-\infty}^{\infty} s(t-\tau_0)e^{j2\pi f_{d0}t}s^{\star}(t-\tau_1)e^{-j2\pi f_{d1}t}dt \\ &= \int_{-\infty}^{\infty} s(t)s^{\star}(t-\tau)e^{j2\pi f_d t}dt = \chi(\tau, f_d)\end{aligned}$$

$$\tau = \tau_0 - \tau_1 \qquad f_d = f_{d0} - f_{d1}$$

III.1 : MIMO Ambiguity Function

- MIMO ambiguity function a function of the waveforms, target parameters and geometry
- N_t transmit antennas, N_r receive antennas, M pulses
  - transmit antennas at $\mathbf{p}_i, i = 0, \ldots, N_t - 1$
  - receiver antennas at $\mathbf{q}_j, j = 0, \ldots, N_r - 1$
  - parameter vector for transmitter i , receiver j : $\mathbf{\Theta}_{i,j}$
  - target at location: $\mathbf{p}$, velocity $\mathbf{v}$, parameters $\mathbf{\Theta} = (\mathbf{p}, \mathbf{v})$

$$\tau_{ij} = \tau_i(\mathbf{p}) + \tau_j(\mathbf{p}) \qquad f_{ij} = f_i(\mathbf{\Theta}) + f_j(\mathbf{\Theta})$$

  - τ_{ij} relative delay, f_{ij} relative Doppler, $\mathbf{\Theta}_{i,j} = (\tau_{ij}, f_{ij})$

MIMO Ambiguity Function (2)

- To focus on the waveform, consider a unit point target
  - perfectly correlated across all transmit-receive pairs
  - set $\boldsymbol{\Sigma} = \mathbf{1}\mathbf{1}^H$ ($\mathbf{1}$ is a length-$N_r N_t$ vector of ones)

$$\begin{aligned}
\chi(\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_0) &= \int_{-\infty}^{\infty} x(t, \boldsymbol{\Theta}_1) x^\star(t, \boldsymbol{\Theta}_0) dt \\
&= \sum_{j=1}^{N_r} \sum_{i=1}^{N_t} \sum_{i'=1}^{N_t} \int_{-\infty}^{\infty} s_i(t - \tau_{ij}(\mathbf{p}_1)) s_{i'}^\star(t - \tau_{i'j}(\mathbf{p}_0)) \times \\
&\qquad e^{-2\pi f_c \tau_{ij}(\mathbf{p}_1)} e^{2\pi \tau_{i'j} f_c(\mathbf{p}_0)} e^{j2\pi(f_{ij}(\boldsymbol{\Theta}_1) - f_{i'j}(\boldsymbol{\Theta}_0))} dt
\end{aligned}$$

MIMO Ambiguity Function (3)

- This expression includes
  - $s_i(t - \tau_{ij}(\mathbf{p}_1))$: signal from element i delayed due to presumed target location $\mathbf{p}_1$
  - similar expression for true location $\mathbf{p}_0$ (note the i')
  - $e^{2\pi f_c \tau_{ij}(\mathbf{p}_1)}$, $e^{2\pi \tau_{i'j} f_c(\mathbf{p}_0)}$: phase shifts due to distance travelled
  - $e^{j2\pi(f_{ij}(\Theta_1) - f_{i'j}(\Theta_0))}$: difference in Doppler shift
  - further simplification using matrix notation

MIMO Ambiguity Function (4)

- Define a matrix associated with the transmit signals

$$\mathbf{R}_{i,i'}(\mathbf{\Theta}_1, \mathbf{\Theta}_0, j) = \int_{-\infty}^{\infty} s_i(t - \tau_{ij}(\mathbf{p}_1)) s_{i'}^{\star}(t - \tau_{i'j}(\mathbf{p}_0)) e^{j2\pi(f_{ij}(\mathbf{\Theta}_1) - f_{i'j}(\mathbf{\Theta}_0))} dt$$

- and the corresponding steering vectors,

$$\mathbf{a}_t(\mathbf{\Theta}, j) = \left[e^{j2\pi f_c \tau_{1j}}, e^{j2\pi f_c \tau_{2j}}, \ldots e^{j2\pi f_c \tau_{N_t j}} \right]^T$$

- then,

$$\chi(\mathbf{\Theta}_1, \mathbf{\Theta}_0) = \sum_{j=1}^{N_r} \mathbf{a}_t^H(\mathbf{\Theta}_1, j) \mathbf{R}(\mathbf{\Theta}_1, \mathbf{\Theta}_0, j) \mathbf{a}_t(\mathbf{\Theta}_0, j)$$

MIMO Ambiguity Function (5)

- Note that the ambiguity function is a complicated function of the geometry of the transmit-receive arrays
  - and depends on the target model
- Also, there is no free lunch
  - let $N_t = N_r$. The “clear” area that can be created in delay-Doppler space is reduced by a factor of N_t (work of Y. Abramovich and G. Frazer)

III.2 : MIMO Waveform Design

- Many different approaches
  - covariance matrix design
  - in frequency domain
  - max. mutual information/MMSE
  - with and without clutter statistics
  - IEEE search for ‘waveform design <and> MIMO radar’ results in 157 choices!

MIMO Waveform Design (2)

- N_t transmit antennas, N_r receive antennas, M pulses
  - simple case: point transmitters and receivers
  - transmitter i transmits $s_i(t)$
- In continuous time, target response between transmitter i and receiver j : $\alpha_{ij}(t)$
- In discrete time, signal component at element j

$$x_j[n] = \sum_{i=1}^{N_c} \sum_{\ell=0}^{\nu} \alpha_{ij}[\ell] s_i[n-\ell], \quad n = 0, 1, \ldots, L-1$$

- L : observation window

MIMO Waveform Design (3)

- Rewriting as matrix equation

$$\mathbf{x}_j = \sum_{i=1}^{N_t} \mathbf{S}_i \boldsymbol{\alpha}_{ij}$$

$$\mathbf{S}_i = \begin{bmatrix} s_i[0] & s_i[-1] & \ldots & s_i[-\nu] \\ s_i[1] & s_i[0] & \ldots & s_i[1-\nu] \\ \vdots & \vdots & \ddots & \vdots \\ s_i[L-1] & s_i[L-2] & \ldots & s_i[L-1-\nu] \end{bmatrix}$$

$$\boldsymbol{\alpha}_{ij} = [\alpha_{ij}[0], \alpha_{ij}[1], \ldots, \alpha_{ij}[\nu]]^T$$

MIMO Waveform Design (4)

- Rewriting as matrix equation

$$\mathbf{x}_j = \sum_{i=1}^{N_t} \mathbf{S}_i \boldsymbol{\alpha}_{ij} = \mathbf{S}\boldsymbol{\alpha}_j$$

$$\mathbf{S} = [\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{N_t}] \qquad \boldsymbol{\alpha}_j = \left[\boldsymbol{\alpha}_{1j}^T, \boldsymbol{\alpha}_{2j}^T, \ldots, \boldsymbol{\alpha}_{N_t j}^T\right]^T$$

- Combining all N_r vectors, $\mathbf{x} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_{N_r}^T\right]^T$

$$\overline{\mathbf{S}} = \mathbf{I}_{N_r} \otimes \mathbf{S} = \begin{bmatrix} \mathbf{S} & 0 & \ldots & 0 \\ 0 & \mathbf{S} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \mathbf{S} \end{bmatrix}$$

$$\mathbf{x} = \overline{\mathbf{S}}\boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \left[\boldsymbol{\alpha}_1^T, \boldsymbol{\alpha}_2^T, \ldots, \boldsymbol{\alpha}_{N_r}^T\right]^T$$

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MIMO Waveform Design (5)

- MMSE estimate : given $\mathbf{x}$ find the minimum mean squared error estimate of $\boldsymbol{\alpha}$
  - given $\overline{\mathbf{S}}$ MMSE estimate is well known

$$\hat{\boldsymbol{\alpha}} = \left(\overline{\mathbf{S}}^H\overline{\mathbf{S}} + \sigma^2\boldsymbol{\Sigma}^{-1}\right)^{-1}\overline{\mathbf{S}}^H\mathbf{x}$$

$$\mathrm{MMSE} = \mathrm{tr}\left[\left(\sigma^2\overline{\mathbf{S}}^H\overline{\mathbf{S}} + \boldsymbol{\Sigma}^{-1}\right)^{-1}\right]$$

- Optimal waveform: find the waveform ($\overline{\mathbf{S}}$) that minimizes this MMSE
  - however we must meet a power constraint

MIMO Waveform Design (6)

- The optimization problem is

$$\overline{\mathbf{S}}^* = \arg\min_{\overline{\mathbf{S}}} \operatorname{tr}\left[\left(\sigma^2 \overline{\mathbf{S}}^H \overline{\mathbf{S}} + \mathbf{\Sigma}^{-1}\right)^{-1}\right]$$
$$\text{such that } \operatorname{tr}\left[\overline{\mathbf{S}}^H \overline{\mathbf{S}}\right] \leq LP_0$$

- P_0 : energy available per time-slot over all N_r transmitters
- Generally requires an eigenvalue decomposition of $\mathbf{\Sigma}$
  – transmit on the eigenvectors of $\mathbf{\Sigma}$

MIMO Waveform Design (7)

- Eigendecompose $\boldsymbol{\Sigma} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H$
  - these are $N_r N_t M \times N_r N_t M$ matrices
- The optimal $\overline{\mathbf{S}}$ is given by

$$\overline{\mathbf{S}} = \boldsymbol{\Psi}\sqrt{\mathbf{P}}\mathbf{U}^H$$

- $\mathbf{P}$ is a power allocation matrix obtained using waterfilling
  - a result that shows up in many applications that impose a power constraint
- $\boldsymbol{\Psi}$: $LN_r \times N_r N_t M$ matrix with orthonormal columns

MIMO Waveform Design (8)

- The key is the power allocation matrix

$$\mathbf{P} = \begin{bmatrix} P_1 & 0 & \dots & 0 \\ 0 & P_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{N_t N_r M} \end{bmatrix}$$

$$= \mathrm{diag}\left[\left(\eta - \boxed{\frac{\sigma^2}{\mathbf{\Lambda}_1}}\right)^+, \left(\eta - \frac{\sigma^2}{\mathbf{\Lambda}_2}\right)^+, \dots, \left(\eta - \frac{\sigma^2}{\mathbf{\Lambda}_{N_t N_r M}}\right)^+ \right]$$

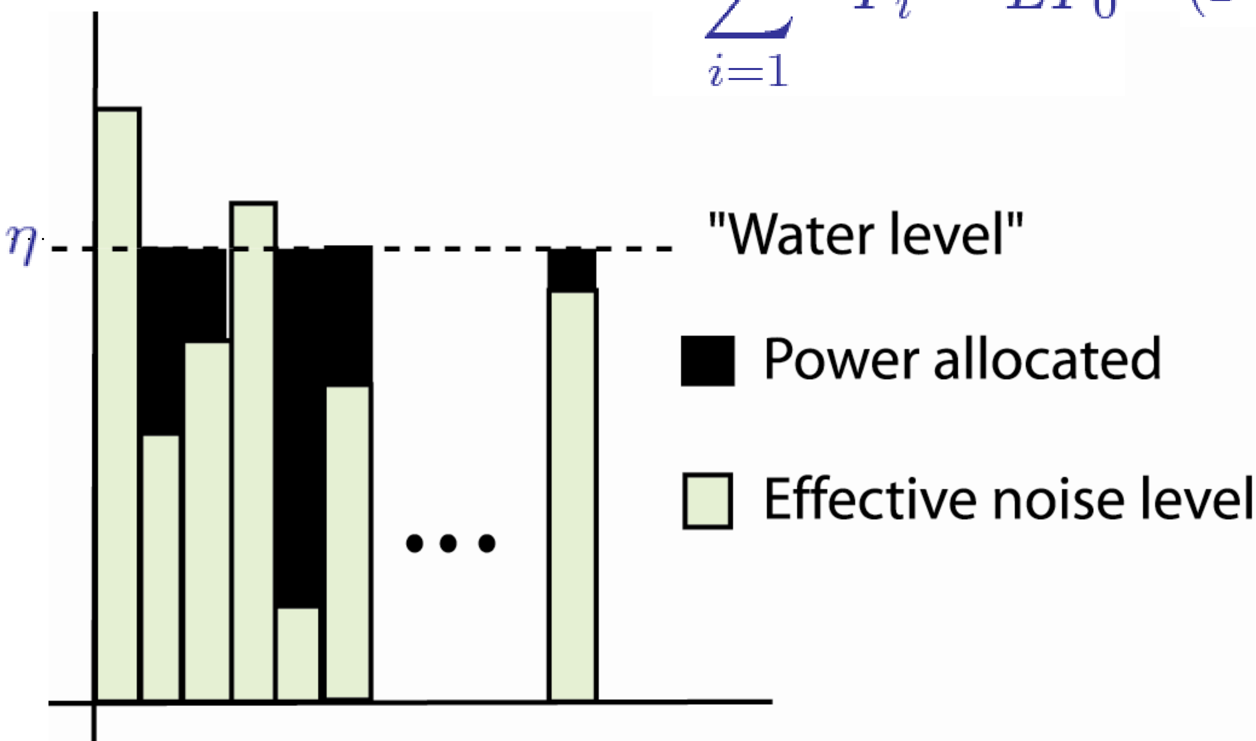
Effective noise level of Channel 1

$$(x)^+ = \max(0, x)$$

MIMO Waveform Design (9)

- η is the "water level", chosen such that

$$\sum_{i=1}^{N_t N_r M} P_i = L P_0 \quad \text{(Power constraint)}$$



Note that some "channels" are too weak to be allocated power

MIMO Waveform Design (10)

- This is a first cut at optimize waveforms
  - assumes target statistics are known
  - other than power constraint, no other constraints
  - assumes perfect synchronization
  - ignores interference
- Each of these issues has been addressed in the literature
  - constant modulus waveforms, waveforms formed by a chosen basis set, etc. etc.
  - furthermore, other optimization criteria are also considered, e.g., mutual information

III.3 : Fast-Time & Slow-Time MIMO

- MIMO waveforms still see the same total "amount" of range-Doppler space
  - fast-time versus slow-time MIMO
    - in fast-time, use time-staggered waveforms
      - reduction in PRF implies reduction in unambiguous Doppler
    - in slow-time use Doppler-shifted waveforms
      - this reduces the unambiguous Doppler
    - which is better depends on your application
  - consistent with the work of Abramovich and Frazer
- Focus here on a single-receive antenna ($N_r = 1$)
  - N_t transmit antennas, M pulses in CPI as before

Fast-Time MIMO & Non-Causal Beamforming

- Each antenna transmits an orthogonal waveform
  - time-orthogonality achieved by time-staggering waveforms
  - consider simple case of uniform linear array
  - all waveforms share the same frequency
  - transmitter i transmits $s_i(t)$
  - Received signal from target:

$$x(t) = \sum_{i=1}^{N_t} \alpha s_i(t - \tau_0) e^{j2\pi(i)\sin(\phi_{t0})} e^{j2\pi f_{d0} t}$$

ϕ_{t0} : Target direction with respect to transmit array

Non-Causal Beamforming (2)

- Similar expression for clutter and other forms of interference
- Key: on matched filtering, each waveform separates
  - in addition, there are M pulses in a CPI
  - resulting in a vector of the form

$$\mathbf{x} = \alpha\mathbf{s} + \mathbf{n}$$

  - $\mathbf{s}$: space-time steering vector
    - the spatial component comes from the angle with respect to the transmitter
  - $\mathbf{n}$: the noise and interference vector

Non-Causal Beamforming (3)

$$\mathbf{x} = \alpha\mathbf{s} + \mathbf{n}$$

- This is the exact same model as we had for STAP!
  - this is adaptive processing at a single receive antenna using the transmitted waveforms
  - can use all of what we know about STAP
  - rather dramatically called non-causal beamforming
    - though the beamforming "happens" after the transmission, not before
  - if multiple receive elements, size of problem increases, no conceptual change
  - has been applied to the Jindalee OTHR in Australia (see Frazer, Abramovich and Johnson, Radar 2008)

Slow-Time MIMO

- All transmitters transmit at the same time and use the same waveform
  - however, sub-divide the Doppler space into N_t regions
  - can be achieved by using an effective PRF that is reduced by a factor of N_t: f_r/N_t
  - signal transmitted from element i

$$s_i(t) = \sum_{m=0}^{M-1} u_p(t - mT_r)e^{j2\pi(f_c+\alpha_i mT_r)}$$

$$\alpha_i = \frac{f_r}{2}\left(N_t - 1 - 2i\right)$$

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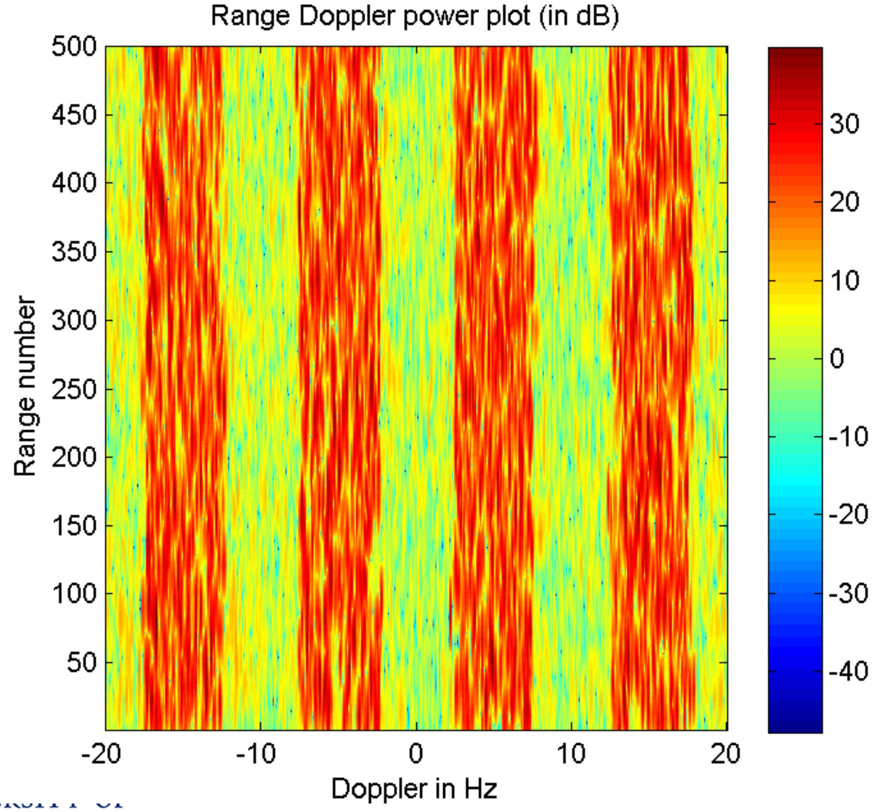
Slow-Time MIMO (2)

- This choice divides the unambiguous Doppler space of $(-f_r/2, f_r/2)$ into N_t regions
- Implemented in a new Canadian OTHR system



4-transmit antenna array

$f_r = 40$

Slow-Time MIMO (3)

- Again we end up with

$$\mathbf{x} = \alpha \mathbf{s} + \mathbf{n}$$

- However, now the steering vector is shorter
  - the spatial steering vector is the same as in the fast-time MIMO case
  - the Doppler steering vector is of length M/N_t

$$\mathbf{s} = \mathbf{s}_t(f_d) \otimes \mathbf{s}_s(\phi_{t0})$$

$$\mathbf{s}_s(\phi_{t0}) = \left[1, e^{jkd\sin(\phi_{t0})}, e^{j2kd\sin(\phi_{t0})}, \ldots, e^{j(N-1)kd\sin(\phi_{t0})}\right]^T$$

$$\mathbf{s}_t(f_d) = \left[1, e^{j(2\pi f_d/f_r)}, e^{j2(2\pi f_d/f_r)}, \ldots, e^{j(M/N_t-1)(2\pi f_d/f_r)}\right]^T$$

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Slow-Time MIMO (3)

- The problem size (with a single receive antenna) is therefore $M \times M$
  - M/N_t temporal degrees of freedom
  - N_t spatial degrees of freedom
- Again, one can apply one's favourite STAP algorithm as desired
  - note that the orthogonality here is in the Doppler domain

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