Channel Prediction and Feedback in Multiuser Broadcast Channels

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***Abstract*—Multiuser linear precoding requires channel state information (CSI) at the transmitter. In the absence of channel reciprocity between the uplink and downlink, a feedback mechanism must be designed to communicate CSI estimates from the mobile receivers to the transmitter. Limiting the total feedback rate is an important design goal for multiuser multiple-input, multiple-output systems, as the feedback overhead can potentially consume a large percentage of system resources, especially when the total number of antennas is large. In this paper, we focus on the challenges of feedback delay and reducing feedback rate; we predict N-frames-ahead, based on the one-step Kalman predictor, and derive a theoretical expression for the prediction mean squared error (MSE). We present simulation results that illustrate a tradeoff between prediction MSE and computational complexity, and also demonstrate situations where adaptive delta modulation (ADM) can be used to exploit temporal redundancy and reduce the required feedback rate.**

I. Introduction

Standards proposals for fourth generation wireless systems (e.g., WiMAX [1] and 3GPP LTE [2]) encourage the use of multiple antennas at the transmitter and/or receiver. These multiple-input multiple-output (MIMO) systems use the additional spatial degrees of freedom provided by antenna arrays to improve reliability, increase data rates, and multiplex multiple users. The greatest performance gains in MIMO systems are achieved when channel state information (CSI) is available at the transmitter allowing for, e.g., beamforming or precoding [3]. This paper focuses on linear precoding in the downlink of *multiuser* MIMO (MU-MIMO) systems [4]–[6].

In a frequency division duplex (FDD) system, different frequency bands are allocated to uplink and downlink communication, so downlink CSI must be estimated at the receiver and provided to the transmitter using an uplink feedback mechanism. It has recently been demonstrated this may also be required in broadband time division duplex (TDD) systems [7]. Thus, in this work, we consider a feedback system that can be used in either FDD or TDD systems. Such a feedback system must provide the transmitter accurate CSI in a timely fashion while minimizing the overhead due to feedback.

Precoding performance suffers when time-varying fading combines with propagation and computational delays, as the channel coefficients at the time of feedback reception may be drastically different from those used to design the precoder matrix. Kalman filtering has been proposed in [8] in a system using TDD reciprocity; however, the authors only consider one-frame-ahead prediction. When the total delay is taken into account in the proposed system (i.e., block decoding of the pilot and feedback symbols, processing of the channel estimates, optimization of the new precoder, and propagation delays in both directions), a more realistic assumption is to consider a delay of 3 data frames or more. In this work, we propose an N-frames-ahead predictor based on the Kalman filter, and evaluate its performance for $N = 3$.

The reduction of feedback overhead is a significant challenge in multiuser systems. There is a fundamental tradeoff between the feedback rate and the accuracy of the CSI at the transmitter. Since the required feedback rate scales multiplicatively in the number of transmit antennas and in the number of users and receive antennas, feedback efficiency is an important design criterion. In this paper, we propose the use of adaptive delta modulation (ADM) [9], [10] to reduce the feedback rate requirements by exploiting the temporal correlation of CSI.

The remainder of this paper is organized as follows. Section II illustrates the system model being used and provides details of both the channel model and the one-step Kalman predictor model. Section III proposes the N-step-ahead predictor, and Section IV describes the ADM scheme used for quantization. We present our simulation results in Section V and conclusions in Section VI.

Notation: Lower case italics, e.g., x, represent scalars while lower case boldface type is used for vectors (e.g., $\mathbf{x}$). Upper case italics, e.g., N, are used for constants and upper case boldface represents matrices, e.g., $\mathbf{X}$. $\mathrm{diag}(\mathbf{x})$ represents the diagonal matrix formed using the entries in vector $\mathbf{x}$. The superscripts $[\cdot]^T$, $[\cdot]^*$, and $[\cdot]^H$ denote the transpose, complex conjugate, and Hermitian operators, and $\mathbb{E}[\cdot]$ represents the statistical expectation operator. $\mathbf{I}_N$ and $\mathbf{0}_{M\times N}$ are the $N \times N$ identity matrix and $M \times N$ all-zeroes matrix respectively. $\mathcal{CN}(\mathbf{m}, \mathbf{Q})$ denotes the complex Gaussian probability distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{Q}$.

II. System Model

In practice, a CSI feedback system requires three basic stages: channel estimation, channel prediction, and feedback

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TABLE I
CHANNEL PARAMETERS

Parameter	Value (units)
Carrier frequency (f_c)	2.3 GHz
Channel sampling rate (f_s)	200 Hz
Frame duration (T_{fr})	5 ms
Max. feedback + processing delay	15 ms (3 frames)
Mobile velocity (v)	3 km/h
Maximum Doppler frequency (f_D)	6.4 Hz

quantization. In this paper, we focus only on prediction and quantization, and assume that perfect channel estimates are available to each mobile receiver.

A. Channel Model

We consider the prediction and feedback of spatially uncorrelated slow-fading Rayleigh channels generated by a modified version of Jakes' fading model [11]. Channel parameters (specified in Table I) are selected to represent typical values for a WiMAX system based on the IEEE 802.16e standard [1] and the ITU Pedestrian-A model.

The discrete time fading channel sequence $h(n)$ models a channel with maximum Doppler frequency f_D and sampling period T_{fr}, which has autocorrelation $R(k)$ at lag k,

$$R(k) \quad = \quad \mathbb{E}\left[h(n)h(n-k)^*\right] = J_0(2\pi f_D k T_{fr}), \qquad (1)$$

where $J_0(\cdot)$ is the order-zero Bessel function of the first kind.

We assume that the Rayleigh fading channels can be modeled by an order U autoregressive (AR) process; i.e.,

$$h(n) \quad = \quad \sum_{k=1}^{U} a_k h(n-k) + v(n) \qquad (2)$$

and the process noise $v(n) \sim \mathcal{CN}(0, \sigma_p^2)$. The AR coefficients a_k are determined by solving the set of $U \times U$ Yule-Walker equations [12], $\mathbf{a} = \mathbf{R}^{-1}\mathbf{v}$, where

$$\begin{aligned}
\mathbf{a} &= \begin{bmatrix} a_1 & a_2 & \cdots & a_U \end{bmatrix}^T, \\
\mathbf{R} &= \begin{bmatrix} R(0) & R(-1) & \cdots & R(-U+1) \\ R(1) & R(0) & \cdots & R(-U+2) \\ \vdots & \vdots & \ddots & \vdots \\ R(U-1) & R(U-2) & \cdots & R(0)] \end{bmatrix}, \\
\mathbf{v} &= \begin{bmatrix} R(1) & R(2) & \cdots & R(U) \end{bmatrix}^T.
\end{aligned}$$

B. Kalman Filter

In order to predict the channel, we first formulate the one-step-ahead Kalman predictor [13]. The system uses the following model to describe the evolution of the state $\mathbf{x}_n$ of a single scalar channel coefficient (as described in (2)), and the corresponding measurement z_n,

$$\begin{aligned}
\mathbf{x}_n &= \mathbf{A}\mathbf{x}_{n-1} + \mathbf{v}_n \qquad (3) \\
z_n &= \mathbf{C}\mathbf{x}_n + w_n,
\end{aligned}$$

with state transition matrix $\mathbf{A}$ and measurement matrix $\mathbf{C}$:

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix} a_1 & a_2 & \cdots & a_{U-1} & a_U \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \\
\mathbf{C} &= \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}. \qquad (4)
\end{aligned}$$

The temporally uncorrelated process and measurement noise are distributed as $\mathbf{v}_n \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_p)$ and $w_n \sim \mathcal{CN}(0, \sigma_o^2)$, respectively, with $\mathbf{Q}_p = \text{diag}\left(\left[\sigma_p^2 \quad \mathbf{0}_{1\times(U-1)}\right]\right)$. Here, due to the assumption of perfect channel estimation, $\sigma_o^2 = 0$; extensions of this work being considered include channel estimation error with non-zero σ_o^2. The time-update (prediction) equations and measurement (correction) equations are defined as follows, where the predicted and corrected forms of the state vector $\mathbf{x}$ and state covariance matrix $\mathbf{P}$ are indicated by the subscripts $[\cdot]_{(n|n-1)}$ and $[\cdot]_{(n|n)}$, respectively:

$$\begin{aligned}
\mathbf{x}_{(n|n-1)} &= \mathbf{A}\mathbf{x}_{(n-1|n-1)} \qquad (5) \\
\mathbf{P}_{(n|n-1)} &= \mathbf{A}\mathbf{P}_{(n-1|n-1)}\mathbf{A}^H + \mathbf{Q}_p \\
\mathbf{K}_n &= \mathbf{P}_{(n|n-1)}\mathbf{C}^H\left(\mathbf{C}\mathbf{P}_{(n|n-1)}\mathbf{C}^H + \sigma_o^2\right)^{-1} \\
\mathbf{x}_{(n|n)} &= \mathbf{x}_{(n|n-1)} + \mathbf{K}_n\left(z_n - \mathbf{C}\mathbf{x}_{(n|n-1)}\right) \\
\mathbf{P}_{(n|n)} &= \left(\mathbf{I} - \mathbf{K}_n\mathbf{C}\right)\mathbf{P}_{(n|n-1)}.
\end{aligned}$$

We initialize the state vector and error covariance matrix as $\mathbf{x}_{0|0} = \mathbf{0}_{U\times 1}$ and $\mathbf{P}_{0|0} = \mathbf{I}_U$.

III. N-STEP AHEAD PREDICTION

A. Generating the Channel Prediction

By expanding the linear dynamic model in (3) for N subsequent time intervals, the state at time $n+N$ can be written as

$$\mathbf{x}_{n+N} \quad = \quad \mathbf{A}^N\mathbf{x}_n + \sum_{k=1}^{N}\mathbf{A}^{N-k}\mathbf{v}_{n+k}. \qquad (6)$$

We form our N-step-ahead prediction of the Kalman state vector $\hat{\mathbf{x}}_{(n+N|n)}$ based on the measurement corrected state $\mathbf{x}_{(n|n)}$ at time n as

$$\hat{\mathbf{x}}_{(n+N|n)} \quad = \quad \mathbb{E}\left[\mathbf{x}_{n+N}\right] = \mathbf{A}^N\mathbf{x}_{(n|n)}, \qquad (7)$$

and find the predicted channel coefficient $\hat{h}(n+N) = \mathbf{C}\mathbf{x}_{(n+N|n)}$. Obtaining the estimate in this manner is equivalent to performing N iterations of the Kalman filter by using only the time-update (prediction) steps without performing any of the measurement update (correction) steps.

B. Prediction Error Analysis

Given the assumption of error-free observations z_n of the channel at time n, it follows that $\mathbf{x}_{(n|n)} = \mathbf{x}_n$; that is, the measurement corrected state is equal to the true state at time n. Thus, the N-step prediction error $\mathbf{e}_N$ can be written as

$$\mathbf{e}_N = \mathbf{x}_{n+N} - \hat{\mathbf{x}}_{(n+N|n)} = \sum_{k=1}^{N}\mathbf{A}^{N-k}\mathbf{v}_{n+k}. \qquad (8)$$

It follows that the N-step prediction error covariance matrix $\mathbf{P}_{(n+N|n)}$ is

$$\mathbf{P}_{(n+N|n)} = \mathbb{E}\left[\mathbf{e}_N \mathbf{e}_N^H\right] = \sum_{k=1}^{N} \mathbf{A}^{N-k} \mathbf{Q}_p \left(\mathbf{A}^{N-k}\right)^H, \quad (9)$$

since the noise vectors $\mathbf{v}_k$ are temporally uncorrelated. The N-step prediction MSE for the estimate $\hat{h}(n+N)$ is the first (top-left) entry in this matrix.

IV. Adaptive Delta Modulation

Delta modulation can be employed effectively in systems using oversampling; in the slow-fading system under investigation, the oversampling rate is equivalent to the inverse of the normalized Doppler rate, $1/(f_D \cdot T_{fr})$. Slow fading occurs at the proposed pedestrian velocities (e.g., $v \simeq 3$ km/h), resulting in an oversampling factor of approximately 30.

In the proposed system, we have considered the use of constant factor adaptive delta modulation (CF-ADM) with one-bit memory, as was first described in [9] and has been recently used in [10]. This method of delta modulation uses an individual delta modulator to track each of the real and imaginary parts of each channel coefficient; thus, two bits of feedback per antenna path are transmitted per feedback period. The step size of the delta modulator is adapted according to a simple heuristic that depends only on the values of the current feedback bit $b(n)$ and its previous value $b(n-1)$. When both bits are identical, the step size is increased by a multiplicative factor α; if they differ, the step size is decreased by the inverse factor $1/\alpha$. This method for scaling step sizes allows for both rapid tracking when the channel is changing relatively quickly and for convergence to the true value when the channel response is changing slowly.

V. Simulation Results

In this section, we present simulation results to illustrate the performance of the proposed prediction and quantization mechanisms for channel feedback in the system with an $N = 3$ frame feedback delay. For each value of user velocity v and AR order U, the process noise variance σ_p^2 is estimated based on the comparison of 2000 channel samples to their corresponding AR models. Simulation results are averaged over 1000 realizations of a slow-fading Rayleigh channel, which is generated according to the modified Jakes' model of [11]; we discard the first 100 predictions to allow the Kalman filter to converge to a steady-state solution. The simulated data points are then generated by averaging over the subsequent 400 predictions.

Figure 1 compares the simulated prediction MSE to the theoretical expression derived in (9). We are able to draw several conclusions from this figure. First, we see that the theoretical expression of (9) is not an exact match to the simulated prediction MSE; this difference may be attributed to the fact that the AR process does not accurately model the bandlimited power spectral density of the Rayleigh fading channel [14]. Nonetheless, the approximation is good enough to allow for effective prediction and the consistent difference

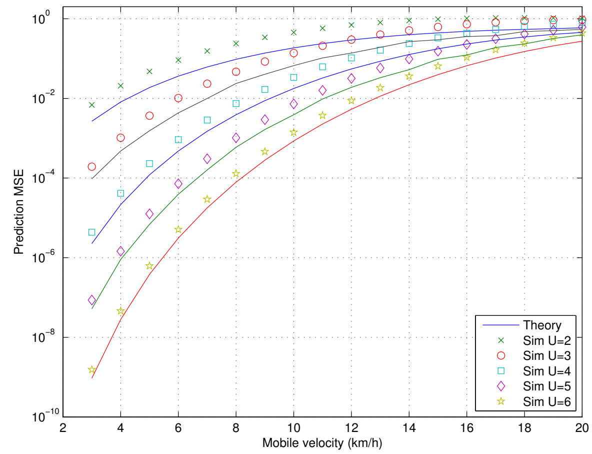

Fig. 1. Prediction MSE vs. velocity v and AR order U

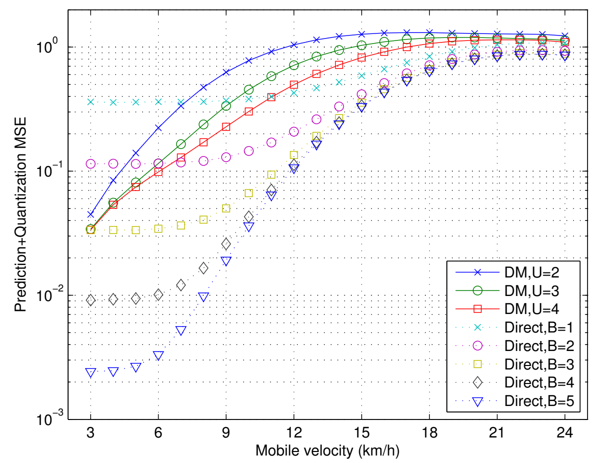

Fig. 2. Delta modulation vs. direct quantization

between the theoretical and simulated MSE suggests that the theoretical MSE expression can be used for system design. It is evident that the use of an order $U = 2$ AR model (as proposed in [8]) may be insufficient when $N > 1$ and/or at higher velocities. Furthermore, this figure suggests that Kalman prediction is effective at the target ITU Pedestrian A velocity of 3 km/h, and demonstrates a tradeoff between prediction MSE and computational complexity (i.e., AR order). Finally, this figure illustrates that increasing the AR model order does not decrease prediction MSE above a threshold velocity (approximately 20 km/h, or a normalized Doppler rate of 0.21). At these higher velocities, such feedback schemes may not be feasible, and system designers may wish to revert to open loop techniques that are able to operate effectively in the absence of CSI.

Figure 2 illustrates the performance of the combined predictor/quantizer employing delta modulation (1 bit / real channel

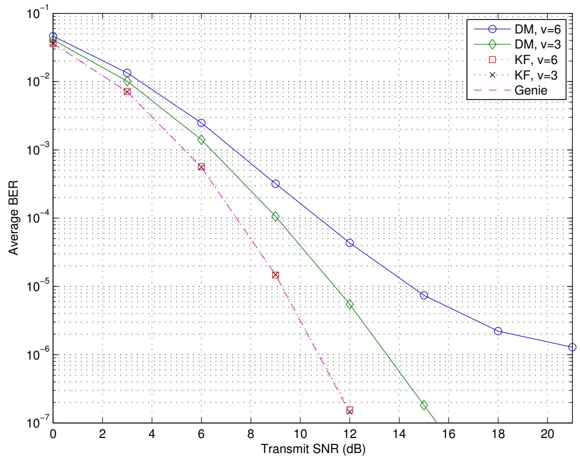

Fig. 3. BER performance under Sum-MSE linear precoding

coefficient). The MSE shown in Figure 2 is the total (prediction + quantization) MSE. Here, we have selected the value $\alpha = 1.5$ for the scaling factor by trial and error to minimize quantization error for the target velocity ($v = 3$); however, this value may be suboptimal for higher velocities. In comparing Figures 1 and 2, observe that the system performance under ADM is dominated by quantization error at low velocity, as the total MSE is several orders of magnitude larger than the prediction MSE for the corresponding values of v and U. Furthermore, negligible gains are seen when increasing $U = 3$ to $U = 4$ for the ADM based system.

In order to determine the feedback savings enabled by ADM, we compare the proposed feedback scheme to one using direct scalar quantization of the channel coefficients (i.e., using the optimal Gaussian quantization codebook as determined by the Lloyd-Max algorithm [15]), with B feedback bits per real channel coefficient, and the $U = 4$ AR model. At the target velocity of $v = 3$ km/h, the ADM-based system is able to reduce the number of feedback bits by a factor of 3; however, this advantage disappears as mobile velocity increases. Thus, while ADM can be used to reduce the required feedback rate at low velocity, higher rate feedback methods may be desired when lower total (prediction+quantization) MSE is needed. We also observe that with increasing mobile velocity, the total MSE becomes dominated by prediction error, and increasing the feedback rate no longer results in improved system performance.

Figure 3 illustrates the average bit error rate (BER) performance under linear precoding (using the sum-MSE minimizing precoder described in [5]) and using the proposed feedback scheme with AR order $U = 4$. The system employs four transmit antennas to communicate one data stream to each of two users possessing two receive antennas each; data symbols are uncoded QPSK. We see that the Kalman predictor is sufficient to provide nearly optimal performance (as compared to the genie-aided transmitter with perfect CSI). The delta modulated feedback scheme incurs only a small penalty in BER performance (approximately 1 dB and 2 dB at a target BER of 10^{-3} for $v = 3$ and $v = 6$ km/h, respectively). Finally, we observe an error floor that occurs at high SNR when using delta modulated feedback; this is evidence of the system becoming interference-limited due to the use of imperfect CSI.

VI. Conclusions

In this paper, we have proposed a scheme for low-rate predictive feedback using an N-frames-ahead predictor based on Kalman filtering and adaptive delta modulation for quantization. Simulations based on typical WiMAX parameters suggest that the proposed scheme is quite effective at pedestrian velocities. The system is shown to be limited by high-speed mobility, as performance degradation occurs when the normalized Doppler rate grows large. However, at the designed pedestrian velocities, a tradeoff allows system designers to improve prediction quality at the expense of increased computation with increasing AR model order. Simulation results suggest that delta modulated feedback can be used to reduce feedback rate requirements for the target scenario, but that strict MSE requirements or higher velocity scenarios may require the use of alternate quantization schemes.

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