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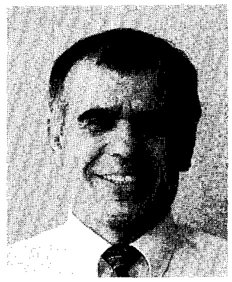

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Space-Time Adaptive Processing Using Circular Arrays

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Abstract

A direct data-domain (D^3) least-squares space-time adaptive-processing (STAP) approach is presented for adaptively enhancing signals in a non-homogeneous environment of jammers, clutter, and thermal noise, utilizing a circular antenna array. The non-homogeneous environment may consist of non-stationary clutter. The D^3 approach is applied directly to the data collected by a circular antenna array (utilizing space), and in time (Doppler) diversity. Conventional STAP generally utilizes statistical methodologies, based on estimating a covariance matrix of the interference, using the data from various range cells of the circular array and assuming that it is a uniform linear array. However, for highly transient and inhomogeneous environments, the conventional statistical methodology may be difficult to apply. Moreover, the error in forming the covariance matrix by assuming that the data collected by the circular array is assumed to be a uniform linear array is highly problem dependent. Hence the D^3 method is presented, as it analyzes the data in space and time over each range cell separately. However, it treats the antenna array as circular, i.e., it deals with the antenna structure in its proper form. Limited examples are presented to illustrate the application of this approach.

Keywords: Adaptive arrays; adaptive signal processing; adaptive radar; space-time adaptive processing; circular arrays; direct data domain method; least squares methods

1. Historical Background

For airborne radars, it is necessary to detect targets in the presence of clutter, jammers, and thermal noise. The airborne-radar scenario has been described in [1-3], and is summarized here for completeness. This scenario is depicted in Figure 1. It is necessary to suppress the levels of the undesired interferers well below the small, desired signal returns. The problem is complicated due to the motion of the platform, as the ground clutter received by an airborne radar is spread out in range, spatial angle, and also over Doppler. One way to detect small signals of interest in such a noisy environment is to have a high-gain array, providing sufficient power and a large-enough aperture to achieve narrow beams. In addition, the array must have extremely low sidelobes, simultaneously, on transmit and receive. This may sometimes be very difficult and expensive to achieve, in practice.

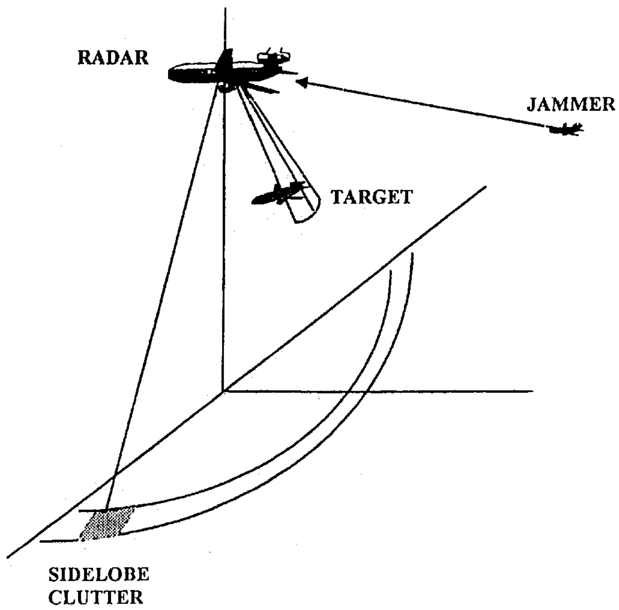

Figure 1. The scenario of an airborne radar.

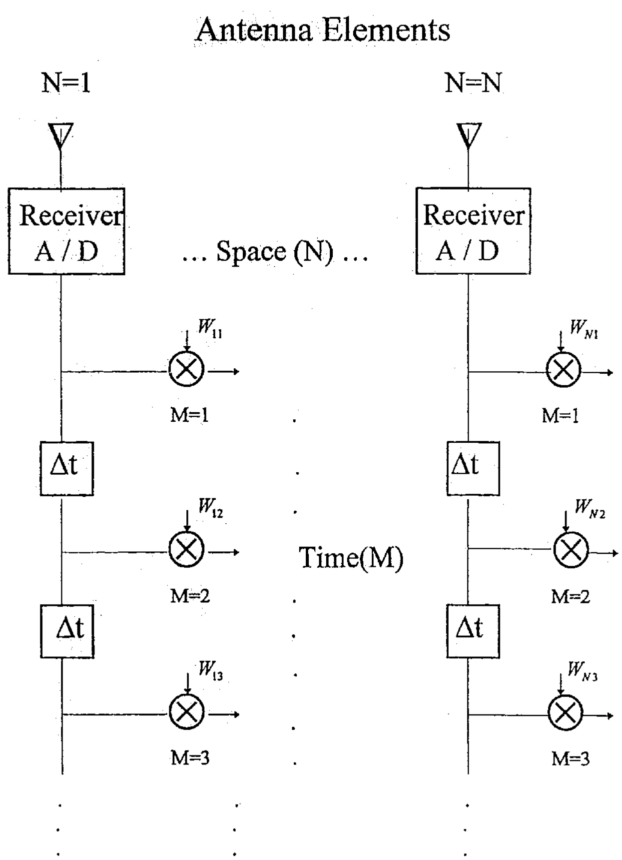

Figure 2. The data consisting of antenna elements, time samples, and information for each range cell.

An alternate way to perhaps achieve the same goal is through Space-Time Adaptive Processing (STAP). STAP is carried out by performing two-dimensional filtering on signals, which are collected by simultaneously combining signals from the elements of an antenna array (the spatial domain), as well as from the multiple pulses from a coherent radar (the temporal domain). The data-collection mechanism is shown in Figure 2. The temporal domain thus consists of multiple pulse-repetition periods of a coherent processing interval. By performing simultaneous multidimensional filtering in space and time, the goal is not only to eliminate clutter that arrives at the same spatial angle as the target, but to also remove clutter that comes from other spatial angles, but has similar Doppler frequencies as the target. Hence, STAP provides the necessary mechanism to detect low observables from an airborne radar.

In this paper, we consider a pulsed Doppler radar, consisting of a circular phased array situated on an airborne platform, which is moving at a constant velocity. The radar consists of an antenna array, where each element has its own independent receiver channel.

The circular array consists of a total of E elements, equally distributed in the azimuth angle, as shown in Figure 3. The angular separation, θ_e, between each of the elements is

$$\theta_e = \frac{2\pi}{E}. \tag{1}$$

Let the spatial coordinate of the nth element be (x_n, y_n), and this is oriented along θ_n with respect to the x axis. The angle is given by

$$\theta_n = \frac{2\pi}{E}(n-1). \tag{2}$$

If R is the radius of the circular array, then

$$x_n = \mathrm{R}\cos\theta_n, \tag{3}$$

$$y_n = \mathrm{R}\sin\theta_n. \tag{4}$$

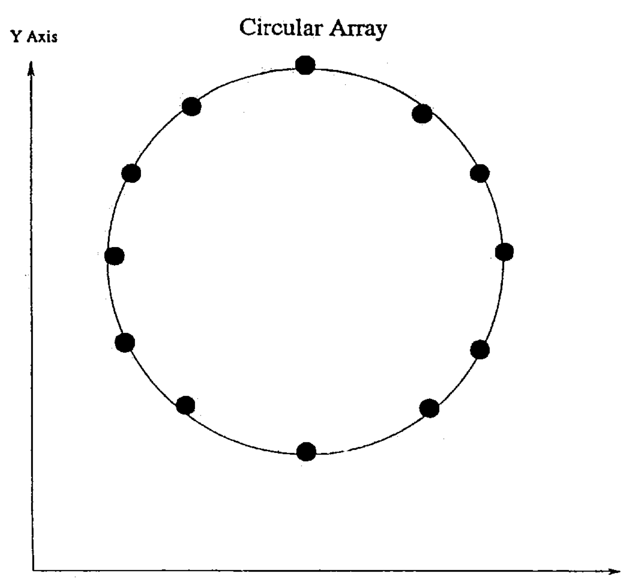

Figure 3. A configuration of a circular array

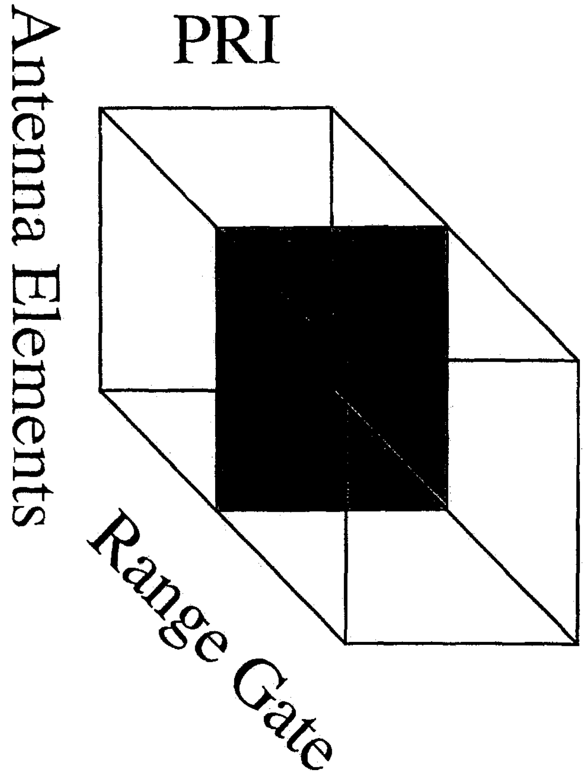

Figure 4. The generation of the data cube from space (antenna) and time (Doppler) samples.

Here, it is assumed that the elements of the array are omnidirectional point radiators. However, it is quite straightforward to take into account the mutual coupling between the elements, and even the electromagnetic coupling between the phased array and the airborne platform.

Let us assume that at any time, only N of the E elements are active. The radar transmits a coherent burst of M pulses, at a constant pulse-repetition frequency. The time interval over which the received pulses are collected in the array is the coherent processing interval. The pulse-repetition interval is the inverse of the pulse-repetition frequency. A pulsed waveform of a finite duration (and an approximately finite bandwidth) is transmitted. On receive, at any of the given N elements, matched filtering is done, where the receiver bandwidth is equal to the transmitting bandwidth. Matched filtering is carried out separately on each pulse return, after which the signals are digitized and stored. So, for each pulse-repetition interval, R time samples are collected to cover the desired range interval. Hence, we term R as the number of range cells. Therefore, with M pulses and N antenna elements, each having its own independent receiver channel, the received data for a coherent processing interval consists of RMN complex baseband samples. These samples are often referred to as the data cube, consisting of $R \times M \times N$ complex baseband samples of the received pulses. The data cube then represents the voltages defined by $V(m;n;r)$ for $m = 1,\ldots,M$; $n = 1,\ldots,N$, and $r = 1,\ldots,R$, as shown in Figure 4. These measured voltages contain the signal of interest (SOI), jammers, and clutter, including thermal noise. A space-time snapshot then is referred to as MN samples for a fixed range-gate value of r. In the D³ procedure to be described, the adaptive weights are applied to the space-time snapshot for the range cell r. Here, a two-dimensional array of weights, numbering $N_a N_t$, is used to extract the signal of interest for the range cell r. N_a is always taken to be $N-1$. Hence, the weights are defined by $w(p;q;r)$ for $p = 1,\ldots,N_t < M$ and $q = 1,\ldots,N_a = N-1$, and these are used to extract the signal of interest at the range cell r. Note that in this system, the number of time samples, M, must be greater than $N_a N_t$. In this procedure, we essentially perform a high-resolution filtering in two dimensions for each range cell.

Conventional STAP processing, as available in the published literature, deals with the statistical treatment of clutter, and this involves estimating a covariance matrix of the interference, using data over the range cells [4-10]. The statistical procedures thus require secondary data for processing, and this may be in short supply for a non-stationary environment. In addition, the formation of the covariance matrix and the computation of its inverse are not only computationally intensive, but also break down under a highly non-stationary environment. This is particularly true when the clutter scenario changes from land to urban to sea clutter, and when there are blinking or barrage jammers (which is also called hot clutter).

Initially, the D³ method was developed to deal with adaptive problems in the spatial domain [11-12, 14]. Next, it was extended to deal with non-uniformly spaced non-planar arrays [13]. The circular antenna array has also been treated in [13], for space-only applications. Finally, this technique has been extended to deal with the two-dimensional filtering problem of space-time adaptive processing [15]. The methodology has been applied to the MACARM (multi-channel Airborne Radar Measurements) database to detect a Saberliner in the presence of urban, land, and sea clutter, by sensing with a phased array mounted near the nose of a BAC1-11 aircraft [15]. There, the array was a uniform linear array. In this paper, the same methodology is extended to deal with circular arrays.

Here, we use the D³ approach to deal with STAP for circular arrays. In this alternate approach, the joint space-time (multi-dimensional) filtering is carried out for each range cell separately, and, hence, we process the data dealing with each space-time snapshot individually. No secondary data are required in this methodology. The STAP procedure for a circular array is described next.

2. Direct Data Domain (D³) Least-Squares STAP

From the data cube shown in Figure 4, we focus our attention to the range cell r, and consider the space-time snapshot of MN data for this range cell.

We assume that the signal of interest for this range cell r is incident on the circular array from an azimuth angle θ_s from the x axis, and is at Doppler frequency f_d . Our goal is to estimate its amplitude, given θ_s and f_d . Let us define $S(p;q)$ to be the complex voltage received at the qth antenna element, corresponding to the pth time instance, for some range cell r. We further stipulate that the known voltages $S(p;q)$ are due to a signal of unity magnitude, incident on the array from the azimuth angle θ_s corresponding to Doppler frequency f_d . Hence, the signal-induced voltages under the assumed array geometry and a narrow-band signal are a complex sinusoid, given by

$$S(p;q) = \exp\left[j2\pi\left(\frac{x_q \cos\theta_s + y_q \sin\theta_s}{\lambda} + f_d p\Delta t \right) \right] \quad (5)$$

$$\text{for } q = 1,\ldots, N;\ p = 1,\ldots, M,$$

where

λ = wavelength of the RF radar signal,

Δt = pulse-repetition interval.

Let $X(p;q)$ be the actual measured complex voltages that are in the data cube of Figure 4, for the range cell r. The actual voltages, X, will contain the signal of interest of amplitude α (α is a complex quantity); jammers, which may be due to coherent multipaths, both in the main lobe and in the sidelobe; and clutter, which is the reflected electromagnetic energy from the ground, and which will compete with the signal of interest at the Doppler frequency of interest. There is also a contribution to the measured voltage for the range cell r from receiver thermal noise. Hence, the actual measured voltages, $X(p;q)$, are

$$X(p;q) = \alpha \exp\left[j2\pi\left(\frac{x_q \cos\theta_s + y_q \sin\theta_s}{\lambda} + f_d p\Delta t \right) \right] \quad (6)$$

$$+ \text{Clutter} + \text{Jammer} + \text{Thermal noise} = V(p;q;r).$$

Now, if one forms the following difference of the signals from range cell r, then the elements of the matrix pencil

$$[X]_{N_a - N_t} - \alpha[S]_{N_a - N_t} \quad (7)$$

represents the contribution due to the unwanted signal multipaths, jammers, unwanted signals at the same Doppler, and receiver thermal noise. In the adaptive processing, the goal is to take a weighted sum of the matrix elements defined by Equation (7), and to extract the signal of interest for the range cell r. The total number of degrees of freedom then represent the total number of weights. This is the product $N_a N_t = N_t(N-1)$, where N_a is the number of spatial degrees of freedom (and is always equal to $N-1$, in this case), and N_t is the number of temporal degrees of freedom. In this formulation, it is necessary that the total number of time samples, M, be greater than $(N-1)N_t$, i.e.,

$$M \ge (N-1)N_t. \quad (8)$$

It is next illustrated how a Direct Data Domain (D^3) least-squares approach is taken for the extraction of the signal of interest.

The development of the least-squares procedure for one dimension is available in [11, 12]. Here, the same least-squares procedure is extended to two dimensions. Consider the two following matrices: C_1 and C_2. The elements of C_1 and C_2 are formed by

$$C_1(x;y) = S(g+h-1;d+e-1), \quad (9)$$

$$C_2(x;y) = X(g+h-1;d+e-1), \quad (10)$$

where

$$x = e + (g-1)N_a, \quad (11)$$

$$y = d + (h-1)N_a, \quad (12)$$

$$1 \le d \le N_a = N-1, \quad (13)$$

$$1 \le e \le N_a = N-1, \quad (14)$$

$$1 \le g \le N_t, \quad (15)$$

$$1 \le h \le N_t, \quad (16)$$

so that $1 \le x; y \le N_a N_t$. Here, N_a and N_t represent the number of degrees of freedom in space and time, respectively.

Now, in the STAP processing, the weights, W, are chosen in such a way that the contributions from the jammers, clutter, and thermal noise are made as small as possible. Hence, if we define the generalized eigenvalue problem

$$[R][W] = \{[C_2] - \alpha[C_1]\}[W] = 0, \quad (17)$$

then α, the strength of the signal, is a generalized eigenvalue, and the weights are given by the corresponding generalized eigenvector. Here, W is a column vector of length $N_a N_t$ for the range cell r. Since we have assumed that only the signal of interest is arriving from θ_s, corresponding to the Doppler f_d, the matrix $[C_1]$ is of rank one and, hence, the generalized eigenvalue equation has only one non-zero eigenvalue, which provides the complex value of the signal.

Alternately, one can view the left-hand side of Equation (17) as the total noise signal at the output of the adaptive processor, due to jammer, clutter, and thermal noise. One is therefore trying to reduce the noise voltage at the output of the adaptive processor, which is given by

$$N_{out} = [R][W] = \{[C_2] - \alpha[C_1]\}[W]. \quad (18)$$

The total output noise power then can be obtained as

$$N_{power} = [W]^H \{[C_2] - \alpha[C_1]\}^H \{[C_2] - \alpha[C_1]\}[W], \quad (19)$$

where H represents the conjugate transpose of a matrix. Our objective is to set the noise power to zero by selecting $[W]$ for a fixed signal strength α. This is done by differentiating the real quantity N_{power} with respect to the elements of $[W]$, and setting each component equation to zero. This yields Equation (17).

The number of degrees of freedom, $N_a N_t$, is determined by M and N. Clearly, if $N_a = N-1$ and $M > (N_t N_a)$, enough equations can be generated to form Equations (18). In this procedure, the number of time samples must be greater than the total number of degrees of freedom. The goal, therefore, is to extract the signal of interest at a given Doppler and angle of arrival in a given range cell r by using a two-dimensional filter of size $N_a N_t$. The filter is going to operate on the data snapshot of size NM to extract the signal of interest.

In real-time applications, it is difficult to numerically solve for the generalized eigenvalue problem in real time, particularly if the value $N_a N_t$, representing the total number of weights, is large, and the matrix $[C_2]$ is highly rank deficient. For this reason, we convert the solution of an eigenvalue problem, given by Equation (17), to the solution of a linear matrix equation.

Let

$$z_k = \exp\left[j2\pi \left(\frac{x_k \cos\theta_s + y_k \sin\theta_s}{\lambda} \right) \right], \tag{20}$$

$$\beta = \exp\left(j2\pi f_d \Delta t \right), \tag{21}$$

and we form a reduced-rank matrix $[T]$ of dimension $(N_a N_t - 1) \times (N_a N_t)$ from the elements of the matrix X, where

$$T(x;y) = \frac{X(g+h-1;k-1)}{\beta^h \bullet z_{k-1}} - \frac{X(g+h-1;k)}{\beta^h \bullet z_k}, \tag{22}$$

$$T(x+1;y) = \frac{X(g+h-1;k-1)}{\beta^h \bullet z_{k-1}} - \frac{X(g+h;k-1)}{\beta^{h+1} \bullet z_{k-1}}, \tag{23}$$

$$T(x+2;y) = \frac{X(g+h-1;k-1)}{\beta^h \bullet z_{k-1}} - \frac{X(g+h;k)}{\beta^{h+1} \bullet z_k}, \tag{24}$$

and

$$1 \le k \le N_a = N - 1, \qquad 1 \le y = N_t(k-1) + h \le N_a N_t, \tag{25}$$

for any row x [its value is between 1 and $\ge N_a N_t$], and the following variables take values between

$$1 \le g \le N_t N_a, \qquad 1 \le h \le N_t. \tag{26}$$

If we consider the three consecutive rows of the matrix T, we observe that they have been formed by taking a weighted difference of a two-dimensional block of the data of size $N_a N_t$. The weighted difference is taken using the elements of matrix $X(p;q)$. Therefore, $T(x;y)$ is formed by writing the $N_a N_t$ difference matrix of Equations (22) to (24) as three consecutive rows of the matrix. The weighted differences forming $N_a N_t$ elements occupy one row of the matrix. The elements of the matrix T are thus obtained by a weighted subtraction of the induced voltages from the neighboring elements (either in space or in time), so that in these elements the desired signals are cancelled out, and the elements of T contain no components of the signal corresponding to the Doppler f_d and or direction of arrival θ_s. We choose the weights $[W]$ such that

$$[T][W] = 0, \tag{27}$$

where the column matrix $[W]$ has been generated by arranging the $N_a N_t$ weights from $w(m;n;r)$ in a linear column array, corresponding to the processing of the data from range cell r. In order to restore the signal component in the adaptive processing, we fix the gain of the subarray (in both space and time) formed by fixing the first row of the matrix T. The elements of the first row are given by

$$T(1;y) = 1, \tag{28}$$

where

$$y = N_t(e-1) + h, \tag{29}$$

and

$$1 \le e \le N_a, \qquad 1 \le h \le N_t. \tag{30}$$

We set the gain of the system along the direction θ_s of the arrival of the signal, and corresponding to the Doppler frequency, f_d, at some constant, C, and so let

$$Y(1) = C. \tag{31}$$

The final equation is formed by combining Equations (22), (23), and (24), along with Equation (28), resulting in the matrix equation

$$[T][W] = [Y] = \begin{bmatrix} C \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{32}$$

Once the weight, $[W]$, is known from Equation (32), the signal strength for the range cell r is estimated from

$$\alpha = \frac{1}{C} \sum_{e=1}^{N_a} \sum_{h=1}^{N_t} W\left[N_t(e-1) + h \right] \frac{V(h;e;r)}{\beta^h \bullet z_e}. \tag{33}$$

3. Numerical Example

As an example, consider a 40-element circular array. The elements are distributed evenly along the arc of the circle, of radius seven wavelengths. Only 11 contiguous elements of the 40-element array are active at one time, so that it scans over a 45° sector. Then, the next 11 elements are excited, and so on. First, let us assume that the following 11 elements, comprising the sector from 0 to 45°, are active. The signal of interest is coming from 20°, and its level is considered to be 0 dB. The signal of interest has a Doppler of 560 Hz. In addition, we consider two other strong targets, located very close to the signal of interest. One of the targets is 37 dB stronger than the signal. It is arriving from an azimuth of 28°, and it is at a Doppler of 555 Hz. The second interference is arriving from an azimuth of 30° and is 39.5 dB stronger than the signal of interest. It has a Doppler of 565 Hz. In addition, there are two other strong interferers, located at the periphery of the active sector of 45°. One of them is arriving from 47°, and is at a Doppler of −570 Hz. It is 39 dB above the signal. The second one is arriving from 5° at a Doppler of −550 Hz, and is 38.3 dB stronger than the signal. The signals are all sampled at 1950 Hz at all the elements. In addition, there is thermal noise in all the antenna elements for all times. It is 24.7 dB below the signal level at each antenna element. A data cube over a range cell is generated for this scenario. There are 128 time samples at each antenna element, and there are 11 elements. To this data cube, we applied the Direct Data Domain Least-Squares Method. In this case, there were 10 spatial taps and 25 temporal taps. This is equivalent to filtering the data cube by a two-dimensional filter of order 250. The output signal-to-noise ratio for this scenario was estimated to be 5 dB. This results in a signal-to-interference-plus-noise enhancement from −39 dB to +5 dB.

4. Conclusion

A Direct Data Domain Least Squares approach has been presented to carry out space-time adaptive processing, using a circular array. Even though the circular array can resolve signals that are close in azimuth and Doppler, special attention must be paid to this analysis when the signals are at the same Doppler. A limited example has been presented, to illustrate the applicability of this technique. Future research needs to quantify the efficiency and the accuracy of this approach, and to observe how this new algorithm performs on real data.

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