Varying FM Rates in Adaptive Processing for Distributed Radar Apertures

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***Abstract*— Previous work in waveform diversity for distributed apertures for target detection has focused largely on orthogonal transmissions. This paper will investigate an alternative approach; implementing waveform diversity based on differing slopes of the linear FM pulse to the application of target detection for a distributed radar aperture system in the presence of noise and clutter. This paper will add develop the required signal model corresponding to the proposed system, accounting for the cross-coupling between the linearly FM pulses. This paper will determine whether applying this type of waveform diversity will result in improved performance in the discrimination of the target from noise and interfering sources and compare the performance whether this method is a feasible solution. A crucial step is the optimization of the FM rates using sequential quadratic programming.**

I. Introduction

Recent works in the area of adaptive processing using waveform diversity in different radar applications has shown promising performance improvements. In particular, the work in [1] proposed the use of frequency diversity for a system of distributed radar apertures. By choosing a different transmit frequency at each element of the array this work showed that frequency diversity significantly reduces the grating lobes resulting from the distributed network. Waveform diversity was also studied in [2] for the application of target tracking in the presence of clutter. In that work, a general FM structure was selected and the waveform parameters, such as the FM waveform type, the FM rate and the wave duration, are selected to minimize a cost function involving the actual target position and the estimated target position. The simulations from [2] showed clear benefits in adapting the waveform to the scenario at hand as the MSE of the target tracking was reduced for the case involving waveform diversity. These works set the stage for and motivate further research in other means of implementing waveform diversity to the important problem of weak target detection in interference.

The system under consideration is a very sparse array of sub-apertures placed thousands of wavelengths apart. Each sub-aperture of the array transmits a linear-FM waveform with its own frequency slope. Unlike in our companion paper [3], the transmissions overlap and hence interact with each other at each receiver. Each aperture also receives and processes all the transmitted signals. Due to the fact that Waveform diversity is achieved using multiple signals characterized by different frequency slopes.

In [2], the authors introduce waveform diversity based on differing slopes of linear FM pulses for a target tracking application. This paper will investigate in the implementation of a similar approach to the application of target detection for a distributed radar aperture system in the presence of noise and clutter. Crucially, the authors of [2] ignored the important issue of grating lobes created by the widely distributed apertures, an issue of importance in target detection. This paper will add to the signal model given by [4], which was then extended to allow for the implementation of frequency diversity [1]. However, frequency diversity raises the issue of phase coherence over widely spaced frequencies. Using differing slopes in FM pulses avoids this issue.

This paper is organized as follows: Section II presents the signal model for the system under consideration. Section III presents results of simulations of adaptive processing based on our signal model. Finally, Section IV wraps up this paper, drawing some conclusions.

II. Signal Model

Consider a distributed radar system with N elements spread in the $x - y$ plane at locations (x_n, y_n) where $n \in 1, \ldots, N$. Each element transmits a linear-FM pulse, parameterized by FM slope β. All elements transmit simultaneously. Each receiver matches the received signal to each of the N transmitted signals resulting in N outputs per receiver. As a result, with M pulses in a coherent pulse interval (CPI), the output signal is a length-N^2M vector.

The transmitted signal is a train of linear FM pulses:

$$s(t) = u(t)e^{j(2\pi f_c t + \psi)};\ u(t) = \sum_{m=0}^{M-1} u_p(t - mT_r), \quad (1)$$

$$u_p(t) = \frac{1}{\sqrt{T_p}}\text{rect}\left(\frac{t}{T_p}\right)e^{j\pi b t^2}, \quad (2)$$

where f_c is the carrier frequency of the waveform, ψ is a random phase, T_r is the pulse repetition interval, T_p is the pulse width and b corresponds to the slope rate of the linear FM pulse and varies from element to element.

In a distributed radar, to focus on a single look point (X, Y, Z) in space, each element delays this signal by [1]

$$\Delta T_n = \frac{max\{D_n\} - D_n}{c}, \quad (3)$$

where D_n is the distance from (X, Y, Z) to the n^{th} element.

For a reflecting artifact l at (X_l, Y_l, Z_l), $(l \in 1, \ldots, L)$, the signal sent by element n and reflected by artifact l to a receiving element i have a total round trip time

$$\tau_{inl} = \frac{1}{c}\Big[\sqrt{(x_n - X_l)^2 + (y_n - Y_l)^2 + Z_l^2} + \sqrt{(x_i - X_l)^2 + (y_i - Y_l)^2 + Z_l^2}\Big], \quad (4)$$

As a result, the signal received at the i^{th} element (reflected by the l^{th} artifact) sent by element n is

$$r_{inl}(t) = A_l u(t - \tau_{inl}) e^{j2\pi(f_c + f_{dl})(t - \tau_{inl})},$$

where A_l is the associated amplitude and f_{dl} the Doppler shift.

At each receiver, the signal is again delayed in order to focus on a look point. This delay may be applied before or after down conversion of the received signal. The received signal after the delay and down conversion is given by

$$\begin{aligned}\hat{r}_{inl}(t) &= e^{-j2\pi(f_c - \Delta T_i)} r_{inl}(t - \Delta T_i) \\ &= A_l u(t - \tau_{inl} - \Delta T_i) e^{-j2\pi f_c \tau_{inl}} e^{j2\pi f_{dl}(t - \tau_{inl} - \Delta T_i)}\end{aligned} \quad (5)$$

By match filtering this signal received according to each of the FM rates for the N transmissions, the output signal at the n-th element corresponding to the i-th transmission is

$$\begin{aligned}x_{inl}(t) &= \int_{-\infty}^{\infty} \hat{r}_{inl}(t) u_{p_n}^*(\tau - t) d\tau, \\ &= A_l e^{-j2\pi f_c \tau_{inl}} \int_{-\infty}^{\infty} \Big[e^{j2\pi f_{dl}(t - \tau_{inl} - \Delta T_i)} \\ &\quad \times \sum_{m=0}^{M-1} u_{p_i}(\tau - mT_r - \tau_{inl} - \Delta T_i) u_{p_n}^*(\tau - t) \Big] d\tau, \\ &= A_l e^{-j2\pi f_c \tau_{inl}} \sum_{m=0}^{M-1} \int_{-\infty}^{\infty} \Big[e^{j2\pi f_{dl}(t - \tau_{inl} - \Delta T_i)} \\ &\quad \times u_{p_i}(\tau - mT_r - \tau_{inl} - \Delta T_i) u_{p_n}^*(\tau - t) \Big] d\tau.\end{aligned} \quad (6)$$

Let $\tau' = \tau - mT_r - \tau_{inl} - \Delta T_i$ which results in

$$\begin{aligned}x_{inl}(t) &= A_l e^{-j2\pi f_c \tau_{inl}} \sum_{m=0}^{M-1} e^{j2\pi f_{dl} m T_r} \int_{-\infty}^{\infty} \Big[e^{j2\pi f_{dl} \tau'} \\ &\quad \times u_{p_i}(\tau') u_{p_n}^*(\tau' - (t - mT_r - \tau_{inl} - \Delta T_i)) \Big] d\tau'\end{aligned}$$

Finally, the received signal is

$$\begin{aligned}x_{inl}(t) &= A_l e^{-j2\pi f_c \tau_{inl}} \textstyle\sum_{m=0}^{M-1} \big[e^{j2\pi f_{dl} m T_r} \\ &\quad \times \chi_{in}(t - mT_r - \tau_{inl} - \Delta T_i, f_{dl}) \big],\end{aligned} \quad (7)$$

where $\chi_{in}(\tau)$ is the *cross ambiguity function* between the transmitted linear-FM signal at rate b_n and the receive filter matched to rate b_i.

A. The Covariance Matrix

The adaptive process is based on the covariance matrix of the interference artifacts [4]. This section details two approaches to developing this covariance matrix.

1) Optimal Covariance Case: In the case for the optimal covariance matrix statistics, data is collected from the look point range gate. The sampling time of the look point range gate for the m^{th} pulse is

$$t_s = mT_r + \tau_{Lin} + \Delta T_i \quad (8)$$

where τ_{Lin} is the total travel time for the signal sent from the n^{th} transmitter to the look point and received by the i^{th} receiver. It is noted that t_s is not a function of the receiving element as

$$\begin{aligned}\tau_{Lin} + \Delta T_i &= \frac{D_i}{c} + \frac{D_n}{c} + \frac{max\{D_n\}}{c} - \frac{D_n}{c} \\ &= \frac{\max\{D_n\} + D_n}{c},\end{aligned}$$

allowing all receiving elements to use the same range gate sample.

The optimal covariance matrix includes the received signals over all interfering artifacts. The sample for the n^{th} transmitted signals, i^{th} applied matched filter, m^{th} pulse and l^{th} interfering artifact when $x_{inl}(t)$ is sampled at t_s is

$$x_{(c)inml} = A_l e^{-j2\pi f_c \tau_{inl}} e^{j2\pi f_{dl} m T_r} \chi_{in}(\tau_{Lin} - \tau_{inl}, f_{dl}). \quad (9)$$

By summing over all the interfering artifacts, the final signal sample at each of the receiver is

$$x_{(c)inm} = \sum_l A_l e^{-j2\pi f_c \tau_{inl}} e^{j2\pi f_{dl} m T_r} \chi_{in}(\tau_{Lin} - \tau_{inl}, f_{dl}). \quad (10)$$

Therefore, the space-time snapshot consists of the samples of (10) for each of the i matched filters applied to the n transmitted signals with each having a different FM rate and m pulses. The snapshot of length N^2M has the form

$$\mathbf{x} = [x_{111} \ \cdots \ x_{N11} \ x_{121} \ \cdots \ x_{NN1} \ x_{112} \ \cdots \ x_{NNM}]^T. \quad (11)$$

Using (10), the $N^2M \times N^2M$ covariance matrix $\mathbf{R}_c$ may be defined according to the elements in the matrix,

$$\begin{aligned}\{\mathbf{R}_c\}_{pq} &= E\{x_{(c)inm} x_{(c)\alpha\beta r}^*\} \\ &= E\{\sum_{l,k} A_l A_k^* e^{-j2\pi f_c(\tau_{inl} - \tau_{\alpha\beta k})} e^{j2\pi T_r(m f_{dl} - r f_{dk})} \\ &\quad \times \chi_{in}(\tau_{Lin} - \tau_{inl}, f_{dl}) \chi_{\alpha\beta}^*(\tau_{L\alpha\beta} - \tau_{\alpha\beta k}, f_{dk})\}, \\ &= \sum_{l,k} E\left\{A_l A_k^*\right\} e^{-j2\pi f_c(\tau_{inl} - \tau_{\alpha\beta k})} e^{j2\pi T_r(m f_{dl} - r f_{dk})} \\ &\quad \times \chi_{in}(\tau_{Lin} - \tau_{inl}, f_{dl}) \chi_{\alpha\beta}^*(\tau_{L\alpha\beta} - \tau_{\alpha\beta k}, f_{dk})\}.\end{aligned}$$

Since the random phases of A_l and A_k are uncorrelated, thus, we will have $E\{A_l A_k^*\} = 0$ for $l \neq k$ and thus, the double sum can be simplified to a single sum. In the above derivation, the p^{th} element of $\mathbf{R}_c$ corresponding to matched filtering with the i^{th} FM rate on the signal sent from the n^{th} transmitting element for pulse m while the q^{th} element refers to FM rate α on the receiving matched filter, transmitted by element β of pulse r.

Therefore, the covariance matrix of the interference plus noise return is given by

$$\mathbf{R}_u = \mathbf{R}_c + \sigma^2\mathbf{I}, \tag{12}$$

where σ^2 is the average noise power (set to unit power in simulations). It should be noted that the target return is not included in the optimal covariance matrix of unwanted signals even at look-points where the target is located because the optimal covariance matrix is sampled at the look-point range gate. If the target return were to be included into the optimal covariance matrix $\mathbf{R}_u$, then the weighted vector $\mathbf{w}$, which will be discussed in a later section, will be able to nullify all the effects of the grating lobes that is caused by the unwanted detection of the target at all look-points.

On the other hand, in the estimated covariances case, since the covariance matrix $\hat{\mathbf{R}}_u$ is created through the average of the signal return samples in the space surrounding the look-point, therefore, the target is a part of the signal return sample as it will be seen in the following section.

2) Estimated Covariance Case: In the case of estimating the covariance matrix, K snapshots of the signal return $x_{inl}(t)$ are sampled at range gates that are surrounding the look point range gate. The corresponding sampling times are

$$t_k = mT_r + \tau_{Lin} + \Delta T_i + kT_s, \tag{13}$$

where T_s is the sampling time and it is chosen that $k \in [-\frac{K}{2}, \frac{K}{2}]$. With the information that the signal returns are sampled at t_k and the resulting signal return has the form

$$\hat{x}_{(c)inm} = \sum_{l=1}^{L} A_l e^{-j2\pi f_c \tau_{inl}} e^{j2\pi f_{dl} mT_r} \chi(\tau_{Lin} - \tau_{inl} + kT_s, f_{dl}). \tag{14}$$

The signal return for the estimated covariance case are stacked into a vector, $\hat{\mathbf{x}}_k$, in the same way as the optimal covariance case according to (11). It should be noted that the vector $\hat{\mathbf{x}}_k$ consists of the total clutter return, the target return (if the look point is not the target point), and the noise return.

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{c(k)} + \hat{\mathbf{x}}_{t(k)} + \hat{\mathbf{x}}_{n(k)}$$

Using the K snapshots of the return signal, the estimated covariance matrix is formed according to

$$\hat{\mathbf{R}}_u = \frac{1}{K}\sum_{k=1}^{K} \hat{\mathbf{x}}_k \hat{\mathbf{x}}_k^H \tag{15}$$

where $\hat{\mathbf{x}}_k$ is the signal return sample of the k^{th} snapshot.

B. Cross-Ambiguity Function

As mentioned in the previous section, the cross-ambiguity function defines the output of the matched filter with the i^{th} FM rate on the signal that is sent from the n^{th} transmitting element for a particular time delay and doppler frequency value. The cross-ambiguity function is defined mathematically by

$$\chi_{in}(\tau, f) = \int_{-\infty}^{\infty} u_{p_i}(t) u^*_{p_n}(t-\tau) e^{j2\pi f t} dt$$

Substituting the complex envelope of the FM pulse in (2) into the definition of the cross-ambiguity function we get

$$\begin{aligned}
&\chi_{in}(\tau, f) \\
&= \frac{1}{T_p}\int_{-\infty}^{\infty}\Big[\text{rect}\left(\frac{t}{T_p}\right)\text{rect}\left(\frac{t-\tau}{T_p}\right) \\
&\qquad \times e^{j\pi b_i t^2} e^{j\pi b_n (t-\tau)^2} e^{j2\pi f t}\Big] dt, \\
&= \frac{1}{T_p} e^{-j\pi b_n \tau^2} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}-\tau} e^{j\pi(b_i - b_n)t^2} e^{j2\pi(b_n\tau + f)t} dt, \\
&= \frac{1}{T_p} e^{-j\pi b_n \tau^2} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}-\tau} \Big[e^{-j\pi(b_i-b_n)\left(\frac{b_n\tau+f}{b_i-b_n}\right)^2} \\
&\qquad \times e^{j\pi(b_i-b_n)\left\{t^2 + \frac{2(b_n\tau+f)t}{(b_i-b_n)} + \left(\frac{b_n\tau+f}{b_i-b_n}\right)^2\right\}}\Big] dt, \\
&= \frac{1}{T_p} e^{-j\pi b_n \tau^2} e^{-j\pi\frac{(b_n\tau+f)^2}{b_i-b_n}} \\
&\qquad \times \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}-\tau} e^{j\pi(b_i-b_n)\left(t+\frac{b_n\tau+f}{b_i-b_n}\right)^2} dt.
\end{aligned} \tag{16}$$

In order to simplify the expression and solve the integral, some variables are defined as follows

$$\begin{aligned}
C_1 &= \frac{1}{T_p} e^{-j\pi b_n \tau^2} e^{-j\pi\frac{(b_n\tau+f)^2}{b_i-b_n}} \\
C_2 &= -\frac{b_n\tau + f}{b_i - b_n} \\
\xi &= b_i - b_n
\end{aligned}$$

Make the substitution $t = s + C_2$ in the integral, giving $s = t - C_2$ the lower and upper bounds of the integral become $a_1 = -\frac{T_p}{2} - C_2$ and $a_2 = \frac{T_p}{2} - \tau - C_2$ respectively. As a result, the integral of the cross-ambiguity expression becomes

$$\chi_{in}(\tau, f) = C_1 \int_{a_1}^{a_2} e^{j\pi\xi s^2} ds$$

. With a minor transformation, the integral can be changed to a form that can be solved. To do so, let $s = t/\sqrt{2\xi}$. To account for both cases where ξ takes on a positive or negative value, take the absolute value of ξ and the exponent is positive for positive ξ and negative otherwise. The cross-ambiguity integral thus becomes

$$\begin{aligned}
\chi_{in}(\tau, f) &= C_1 \int_0^{a_2} e^{\pm j\pi|\xi|s^2} ds - C_1 \int_0^{a_1} e^{\pm j\pi|\xi|s^2} ds, \\
&= \frac{C_1}{\sqrt{2|\xi|}} \int_0^{(\sqrt{2|\xi|})a_2} e^{\pm j\frac{\pi t^2}{2}} dt \\
&\qquad - \frac{C_1}{\sqrt{2|\xi|}} \int_0^{(\sqrt{2|\xi|})a_1} e^{\pm j\frac{\pi t^2}{2}} dt.
\end{aligned}$$

From [5, pp. 178], the integral can be solved to an expression with sums of Fresnel Cosine and Sine Integral functions for cases where ξ is non-zero. As a result, the cross-ambiguity function is given by

$$\begin{aligned}
\chi_{in}(\tau, f) &= \frac{C_1}{\sqrt{2|\xi|}} \Big[C(\sqrt{2|\xi|}a_2) - C(\sqrt{2|\xi|}a_1) \\
&\qquad \pm jS(\sqrt{2|\xi|}a_2) \mp jS(\sqrt{2|\xi|}a_1) \Big],
\end{aligned} \tag{17}$$

where $C(x)$ and $S(x)$ are Fresnel's Cosine and Sine Integral functions respectively.

For the case where the received signal, which is sent at a particular FM rate and is matched filter with the exact same FM rate (the case where ξ is zero), the ambiguity function reduces to

$$\chi_{ii}(\tau, f) = e^{j\pi\tau f}\left(1 - \frac{|\tau|}{T_p}\right)\frac{\sin\left\{\pi T_p(b\tau + f)\left(1 - \frac{|\tau|}{T_p}\right)\right\}}{\pi T_p(b\tau + f)\left(1 - \frac{|\tau|}{T_p}\right)} \quad (18)$$

An interesting question is whether there are values for certain parameters that result in cross-ambiguity terms becoming zero. From (18), the only way the cross-ambiguity terms will be zero is when either the Fresnel Integral values are zero or when C_1 is equal to zero. By examining C_1, the only possible way to arrive at cross-ambiguity terms to be zero is if ξ is set to zero. However, ξ cannot be set to zero since waveform diversity is implemented in this paper through the varying of the FM rates. The only ways that the Fresnel integrals will have a zero value is when the parameters are zero. This indicates that either ξ must be set to zero, which cannot be done as explained above, or both a_1 and a_2 must be set to zero, which is not possible because C_2 will have to take on two different values in that scenario.

C. Processing

The significant advantage of achieving waveform diversity by varying the FM rates (as opposed to over achieving diversity by changing carrier frequency) is the possibility of coherent processing. Unlike the frequency diverse case where a random phase is introduced for each of the carrier frequencies coherence is achieved by the use of a single carrier frequency while the FM rates of the waveform is varied.

In this paper, the two STAP processing techniques that are implemented are the SMI and MSMI methods. STAP is implemented in the same fashion whether the covariance matrix is estimated, as described by (15), or optimal, which is given by (12). The equations that will be given will be in terms of the optimal covariance matrix.

1) Look-Point Steering Vector and Signal Snapshot: In this section, the look-point steering vector and signal snapshot are defined. The look-point steering vector in this application mainly defines the response from the Doppler bank of the target at the look-point, which is defined by the time delay τ_{Lin} and Doppler frequency of the target, f_{dt}. Thus, it is defined in a manner that is very similar to the signal return. The look-point steering vector has elements given as follows

$$\begin{aligned} s_{inm} &= e^{-j2\pi f_c \tau_{Lin}} e^{j2\pi f_{dt} m T_r}\chi(\tau_{Lin} - \tau_{Lin}, f_{dt}) \\ &= e^{-j2\pi f_c \tau_{Lin}} e^{j2\pi f_{dt} m T_r}\chi(0, f_{dt}) \end{aligned} \quad (19)$$

and the steering vector is formed by stacking the elements in the same way as defined by (11).

The look-point signal snapshot is defined as the sum of the target return, total clutter return, and the noise return.

$$\mathbf{x} = \mathbf{x}_c + \mathbf{x}_t + \mathbf{x}_n$$

where the elements of $\mathbf{x}_c$ is given by (9) and $\mathbf{x}_n$ is modeled by a complex Gaussian vector with zero mean and the identity matrix because in the simulation, all powers are referenced to a unit noise power level [ex. $\mathbf{x}_n \sim \mathcal{CN}(0, \mathbf{I})$]. The target return, $\mathbf{x}_t$, is defined similar to (9) and is given by

$$x_{(t)inml} = A_t e^{-j2\pi f_c \tau_{in(t)}} e^{j2\pi f_{dt} m T_r}\chi(\tau_{Lin} - \tau_{in(t)}, f_{dt})$$

where A_t is the amplitude of the target return and $\tau_{in(t)}$ is traveling time of the signal sent from the i^{th} transmitting element, reflected by the target, and received by the n^{th} receiving element.

2) Modified Sample Matrix Inversion: Adaptive processing requires the computation of the weight vector, which, in turn, requires the inverse of the optimal covariance matrix or the estimated covariance matrix. The weight vector is given by

$$\mathbf{w} = \mathbf{R}_u^{-1}\mathbf{s}, \quad (20)$$

where $\mathbf{s}$ is the look-point steering vector, which consists elements defined by (19).

The modified sample matrix inversion (MSMI) method involves a different application of the weights defined by (20). The output at a certain look-point is defined to be

$$z_{MSMI} = \frac{|\mathbf{w}^H\mathbf{x}|^2}{|\mathbf{w}^H\mathbf{s}|}. \quad (21)$$

III. Numerical Examples

In this section we present results of simulations designed to test the system described above. The system comprises a nine element radar array distributed in a $200m \times 200m$ square grid on the $x-y$ plane. The target is a point reflector located at the co-ordinates $(500m, -60m, 2km)$ with a signal-to-noise ratio (SNR) of 10dB. Clutter is modeled by a ball of interfering sources with a radius of $200m$. The chirp bandwidth of the signal is $BW = 10MHz$ and the nominal chirp duration is $T_p = 10\mu s$. The clutter-to-noise ratio (CNR), which is a measure of the total power from the interfering sources, is $50dB$.

Figure 1 shows the plot of the MSMI statistic for the system described with all the elements transmitting the same waveform where the target is scanned for in the z-direction. Given the system geometry, this closely approximates range. The clutter sources in this situation are located $800m$ away from the target in the positive z direction. It is noted that there are several ranges with high magnitudes resulting in many false alarms.

Figure 2 shows a similar the plot of the signal return for the system described with all the elements transmitting different waveforms via the changing FM rate. Once again, the target is scanned for in the z-direction. The suppression of high MSMI statistics at range cells other than the target range cell is evident. Clearly using waveform diversity significant improves target detection performance. The rates that were used to generate the plots were found from optimizing a set of rates in the simulation to maximize the difference between the main-lobe and side-lobe magnitudes. Due to

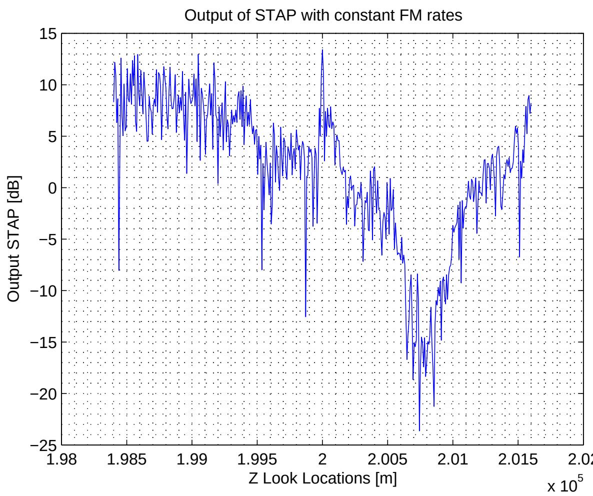

Fig. 1. MSMI statistic without diversity

the non-convex nature of the problem, sequential quadratic programming [6] was used to arrive at a numerical solution to the optimization problem. The FM rates were constrained such that the maximum rate did not exceed BW/T_p and did not fall below $BW/2T_p$.

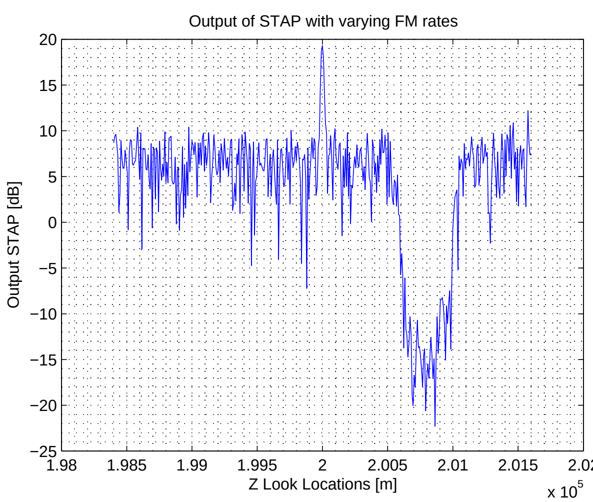

Fig. 2. MSMI statistic with varying FM rates in the waveforms transmitted

Figure 3 illustrates a significant drawback of the proposed scheme. As with Figs. 1 and 2, the plot compares a system without waveform diversity (solid line) with a system that uses waveform diversity is applied through the varying of FM rates (dashed line). It should be noted that he clutter sources in this situation are located $800m$ away from the target in the positive x direction. In this plot, the target is scanned for in the x direction. It can be seen that the grating lobes were not suppressed as in the case where the z direction was scanned. It can be concluded that the usage of diversity through the varying of FM rates will suppress grating lobes in the vertical view points and not in the planar scan points. It should be noted that minimizing the effects of grating lobes in the planar scan direction was done successfully through the usage of frequency diversity as described in [1]. It appears therefore that the ambiguity functions, by themselves, do not provide any suppression of grating lobes, and the slight differences in round-trip time do not provide adequate suppression either.

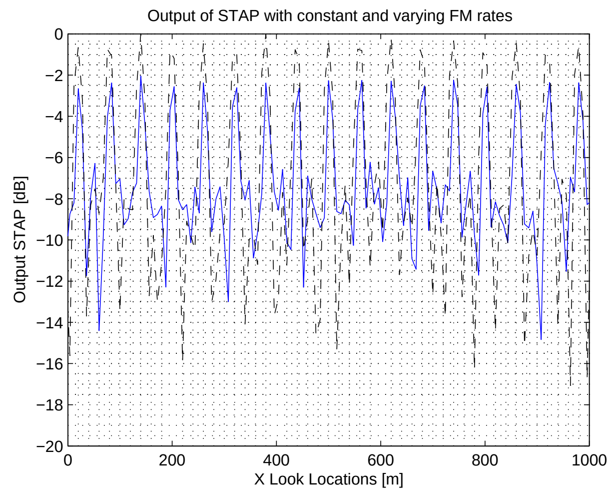

Fig. 3. Comparing MSMI statistics for varying and constant FM rates

IV. Conclusions and Future Work

This paper furthers the development of waveform diversity for distributed radar apertures. In the work of Sira et al. [2] varying FM rates were proposed as a diversity mechanism. This paper is based on this scheme, but significantly extends it to account for true time delay between elements and the use of the optimal or estimated covariance matrix. The usage of different signal structures will have an impact on the ambiguity function. Here we account for the cross-ambiguity function between the differing FM rates. An open question that arises is whether there will be a set of FM rates that will improve the detection performance and if so, whether this set of ambiguity functions translates into a particular structure of waveform.

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