Combined CDMA and Matrix Pencil Direction of Arrival Estimation

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***Abstract*— This paper extends the low complexity, Matrix Pencil direction of arrival (DOA) estimation algorithm to the CDMA case. The CDMA processing gain is used to suppress interfering signals, allowing Matrix Pencil to accurately estimate the DOA of the signal of interest. It is shown here that this CDMA/MP algorithm is able to estimate an angle of arrival (AOA) with a low mean square error using only a single snapshot. The popular Root-MUSIC algorithm which takes twice the execution time has a significantly higher MSE.**

***Index Terms*— CDMA, SDMA, Matrix Pencil, DOA, AOA**

I. Introduction

High resolution DOA estimation algorithm have many important applications in wireless communications, including E911 and spatial beamforming. The class of noise subspace algorithms, such as MUSIC, are some of the most popular algorithms proposed for DOA estimation [1]. These algorithms separate the noise and signal subspaces based on an eigenvalue-decomposition of the spatial covariance matrix. This matrix is estimated by averaging over several snapshots. In the case of Root-MUSIC, the eigenvectors of the noise subspace are used to form a complex polynomial whose roots correspond to the signal DOA.

Given enough snapshots, MUSIC type algorithms yield fairly accurate results. However, two shortcomings limit their use in real time applications. First, the computation complexity of noise subspace algorithms is usually very high since covariance matrix and root estimation are very expensive operations. Sophisticated hardware is required to carry these operations which increases the cost of manufacture and decreases the ability to run in real time. Second, several snapshots are required for an accurate estimate of the noise subspace algorithms.

In radar and other signal processing applications, the Matrix Pencil (MP) algorithm has been shown to provide accurate DOA estimates with a single snapshot [2] and extremely low computation burden. However, a major limitation in applying this technique to the CDMA case is that, given an array of N_w elements, MP can generate an accurate estimate only when

$$M_s \leq \frac{N_w + 1}{2}, \tag{1}$$

where M_s is the number of source presented. We propose here to overcome this limitation by using the CDMA spreading gain to suppress interfering signals and multipaths while increasing the strength of the signal of interest. The CDMA spreading gain increases the SINR. The despread signal, in theory, only has the DOA information of a single signal path. The constraint (1) can now be met easily.

The advantage of using MP is that the computation complexity is much lower than that of noise subspace algorithms because MP does not have to estimate any spatial covariance matrix or to find any roots of a polynomial. Besides, MP works well even with a single snapshot which makes it very attractive to real time applications.

In this paper, the theory of MP as applied to the CDMA case is investigated. It is followed by an numerical example to demonstrate the effectiveness of this CDMA/MP algorithm in terms of accuracy and speed.

II. Theory

A. Matrix Pencil Algorithm

Consider a linear antenna array with N_w of elements which receives M_s signals. In the absence of noise, the signal received by k^{th} element is

$$\begin{aligned} y_k &= \sum_{i=1}^{M_s} A_i \exp[j2\pi s_i k d], \\ &= \sum_{i=1}^{M_s} A_i z_i^k, \end{aligned} \tag{2}$$

where,

$$\begin{aligned} z_i &= e^{j2\pi s_i d}, & (3)\\ s_i &= \cos\theta_i, & (4)\\ k &= 0, \ldots, N_w - 1, \end{aligned}$$

where d is the antenna element spacing in wavelengths, θ is the angle from the endfire direction, and A_i is the complex amplitude of signal i.

For DOA estimation, the goal is to estimate z_i and M_s and A_i is ignored. Consider the following matrix, (the following explanation is mainly from [3],)

$$\mathbf{Y} = \begin{bmatrix} y_0 & \cdots & y_L \\ y_1 & \cdots & y_{L+1} \\ \vdots & \ddots & \vdots \\ y_{N_w-L-1} & \cdots & y_{N_w-1} \end{bmatrix}_{(N_w-L)\times(L+1)}, \tag{5}$$

where L is called the pencil parameter. In MATLAB notation, define the following two matrices

$$\mathbf{Y_1} = \mathbf{Y}(:, 2 : L + 1), \quad (6)$$

$$\mathbf{Y_2} = \mathbf{Y}(:, 1 : L). \quad (7)$$

These two matrices can be written as

$$\mathbf{Y_1} = \mathbf{Z_1 R Z_0 Z_2}, \quad (8)$$

$$\mathbf{Y_2} = \mathbf{Z_1 R Z_2}, \quad (9)$$

where,

$$\mathbf{Z_1} = \begin{bmatrix} 1 & \cdots & 1 \\ z_1 & \cdots & z_{M_s} \\ \vdots & \ddots & \vdots \\ z_1^{(N_w-L-1)} & \cdots & z_{M_s}^{(N_w-L-1)} \end{bmatrix}_{(N_w-L)\times M_s} \quad (10)$$

$$\mathbf{Z_2} = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ 1 & z_2 & \cdots & z_2^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_{M_s} & \cdots & z_{M_s}^{L-1} \end{bmatrix}_{M_s\times L}, \quad (11)$$

$$\mathbf{Z_0} = \mathrm{diag}\begin{bmatrix} z_1 & z_2 & \cdots & z_{M_s} \end{bmatrix}, \quad (12)$$

$$\mathbf{R} = \mathrm{diag}\begin{bmatrix} R_1 & R_2 & \cdots & R_{M_s} \end{bmatrix}. \quad (13)$$

Note that if the entries of the matrix $\mathbf{Z_0}$ can be estimated, an estimate of the DOA may be obtained using Eqns. (3) and (4). Consider the following matrix pencil,

$$\mathbf{Y_1} - \lambda\mathbf{Y_2} = \mathbf{Z_1 R}\{\mathbf{Z_0} - \lambda\mathbf{I}\}\mathbf{Z_2}. \quad (14)$$

The estimates for z_i are, therefore, the generalized eigenvalues of the matrix pair $\mathbf{Y_1}, \mathbf{Y_2}$. The angles of arrival are then estimated to be

$$\theta_i = \cos^{-1}\left[\frac{\Im[\ln(z_i)]}{d}\right], \quad (15)$$

where $\Im[z_i]$ is the imaginary part of z_i.

For noisy data, Total Least Squares Matrix Pencil (TLSMP) can be used. The method starts with a singular value decomposition of $\mathbf{Y}$. i. e.

$$\mathbf{Y} = \mathbf{U\Sigma V}^H. \quad (16)$$

We estimate the number of signals M_s to be the number of dominant singular values. Let us consider the following four matrices

$$\mathbf{U}' = \mathbf{U}(:, 1 : M_s), \quad (17)$$

$$\mathbf{V}' = \mathbf{V}(:, 1 : M_s), \quad (18)$$

$$\mathbf{\Sigma}' = \mathbf{\Sigma}(1 : M_s, 1 : M_s), \quad (19)$$

$$\mathbf{Y}' = \mathbf{U}'\mathbf{\Sigma}'\mathbf{V}'^T. \quad (20)$$

And form $\mathbf{Y_1}$ and $\mathbf{Y_2}$ as follows

$$\mathbf{Y_1'} = \mathbf{Y}'(:, 2 : L + 1), \quad (21)$$

$$\mathbf{Y_2'} = \mathbf{Y}'(:, 1 : L). \quad (22)$$

The estimates for z_i are then the generalized eigenvalues of the matrix pair $\mathbf{Y_1'}, \mathbf{Y_2'}$ and the angles estimated using Eqn. (15).

B. CDMA/MP DOA Estimation Algorithm

In the case of a CDMA signal, the amplitude A_i can be written as a spread signal, i. e. we can rewrite eqn. (2) as

$$y_k = \sum_{i=1}^{M_s} K_i b_i(t) c_i(t) e^{j\phi_i} z_i^k, \quad (23)$$

where K_i is the amplitude, $b_i(t)$ is the bitstream, $c_i(t)$ is the chipstream, and $e^{j\phi_i}$ is the random phase of signal i.

After the match filter of user j,

$$y_k = K_j b_j e^{j\phi_j} + \frac{1}{G}\sum_{i\neq j} K_i b_i e^{j\phi_i} z_i^k, \quad (24)$$

where G is the CDMA processing gain. As one can see from eqn. (24), the strengths of all interfering signals are reduced by a factor of G. Therefore, only the DOA information of a single user is left after the CDMA despreading.

III. Numerical Example

The scenario in this example is an asynchronous CDMA[1] (uplink) environment. Each source is modulated using BPSK. Each multipath has an uniformly distributed random phase and a constant amplitude according to the SNR. Table I and II summarizes all the parameters in this example.

TABLE I
PARAMETER SUMMARY

Parameters	Symbols	Values
no. of element	M	7
processing gain	G	128
PN period	PN	$2^{15} - 1$
element spacing	d/λ	0.5
sample per chip	s/c	4
pencil parameter	L	4
given no. of source	S	1
trial	i	1000
chip	c	

TABLE II
MOBILE USER PROPERTIES

User	Path	SNR (dB)	AOA	delay(c)
1	1	-20	85°	0
	2	-27	80°	8
	3	-23	90°	20
2	1	-20	50°	24
	2	-20	100°	4
	3	-20	120°	28

In this example, multipath 1 of user 1 is demodulated.

[1] it also applies to synchronous CDMA

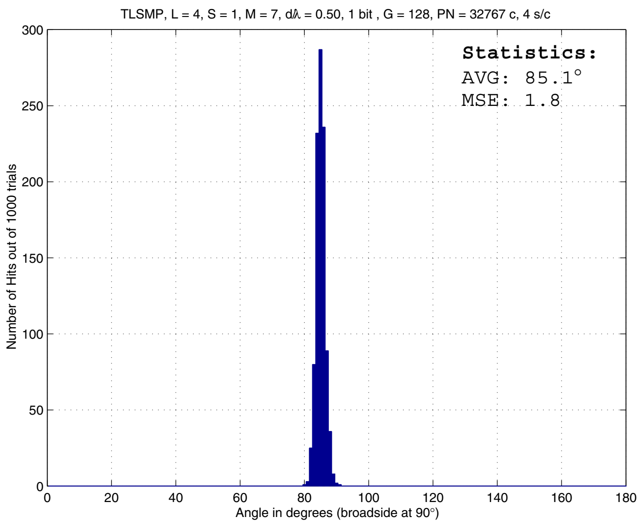

Fig. 1. TLSMP, $L = 4$, 1 bit

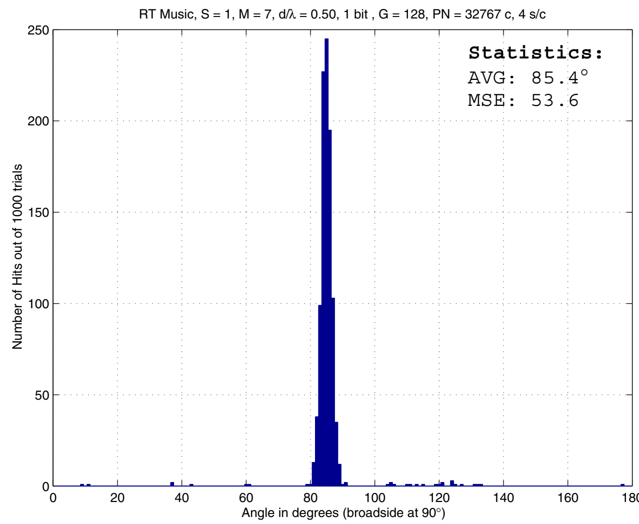

Fig. 2. Root-MUSIC, 1 bit

A. Accuracy

Figs. 1 and 2 are histograms of 1000 estimates generated by TLSMP and Root-MUSIC respectively with 1 snapshot. Fig. 1 illustrates the effectiveness of the CDMA/TLSMP algorithm. It is able to generate a very accurate estimate with a mean of $85.1°$ and an MSE of 1.8. CDMA/Root-MUSIC, on the other hand, fails miserably with an MSE of 53.6 as shown in Fig. 2. For spatial beamforming, the MSE of 1.8 achieved by CDMA/TLSMP is good enough for most practical purposes as the antenna beamwidth is seldom less than $2°$.

For improved performance, a number of snapshots may be used. The performance of CDMA/Root-Music starts to catch up in performance as the number of snapshot increases. Figs. 3, 4, 5 and 6 shows that TLSMP and Root-MUSIC have similar performance at 5 and 10 snapshots. In fact, the performances are very similar even with 35 snapshots. (not shown)

For Root-MUSIC, the trade off between number of snapshots and performance is because Root-MUSIC algorithm requires at least N_w snapshots to obtain an estimate of the spatial covariance matrix. CDMA/TLSMP, on the other hand, requires only a single snapshot to generate a good estimate. As the number of snapshot increases, the estimated covariance matrix of Root-MUSIC is no longer singular. The estimate becomes more accurate as shown in Figs. 4 and 6.

B. Computation Speed

Fig. 7 is a plot of the average time to finish 1000 estimates. It illustrates the crucial advantage of using the CDMA/TLSMP for direction finding. This algorithm is about 1.7 times[2] faster than Root-MUSIC algorithm initially. The improvement in execution time increases as the number of snapshots increases. This increase is directly related to the computational expense of estimating a covariance matrix.

[2]Base on MATLAB Profile function

Fig. 8, is a plot of Root-MUSIC computation time breakdown, shows a clear picture that as the number of snapshot increases, the covariance matrix estimation becomes more dominant in terms of computation time. On the other hand, TLSMP scales nicely. In fact, the total time as well as the percentage time of TLSMP stays about the same as the number of snapshot increases. It is because the computation load mostly depends on the size of the antenna array and the pencil parameter, L, rather than the number of snapshots. Therefore, while using several snapshots, the performance TLSMP and Root-MUSIC are similar, the computation load of TLSMP is significantly lower.

IV. Conclusion

A novel DOA estimation algorithm, CDMA/MP, is investigated in this paper. The power of this estimator lies on the fact that MP algorithm is very efficient and works well even with a single snapshot as demonstrated in the numerical example. It is far better than what Root-MUSIC algorithm can achieve. It becomes a ideal choice for real time applications over other algorithms as they usually trade speed for accuracy or vice versa.

References

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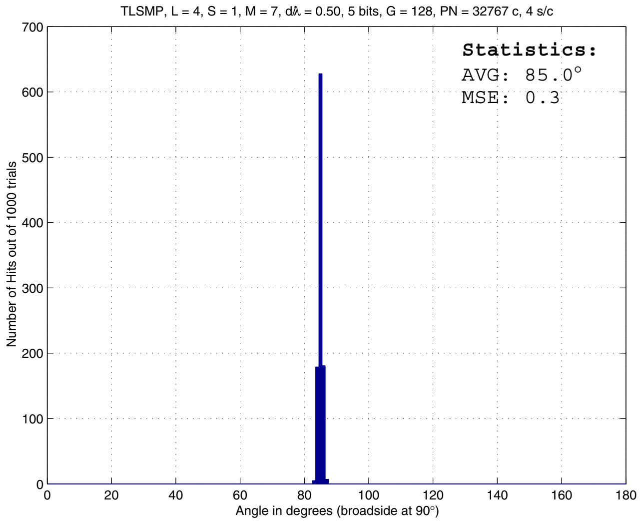

Fig. 3. TLSMP, $L = 4$, 5 bits

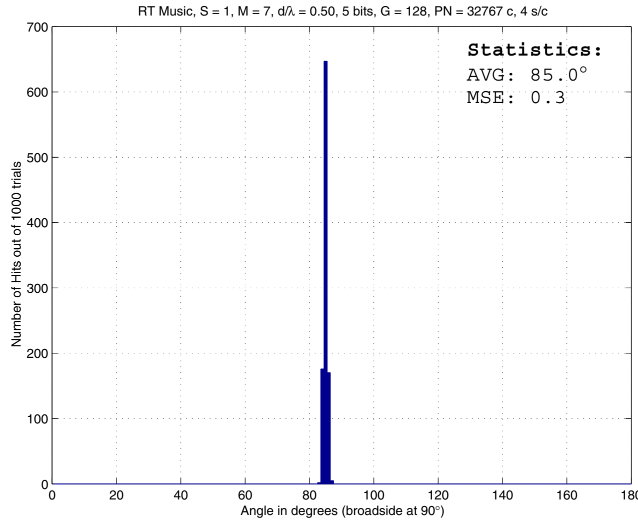

Fig. 4. Root-MUSIC, 5 bits

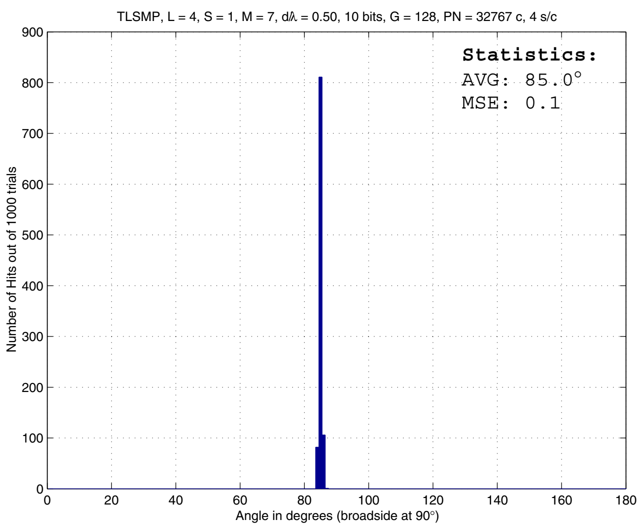

Fig. 5. TLSMP, $L = 4$, 10 bits

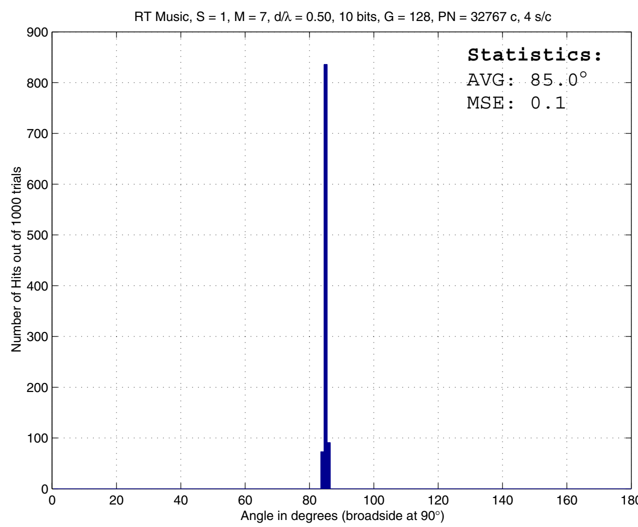

Fig. 6. Root-MUSIC, 10 bits

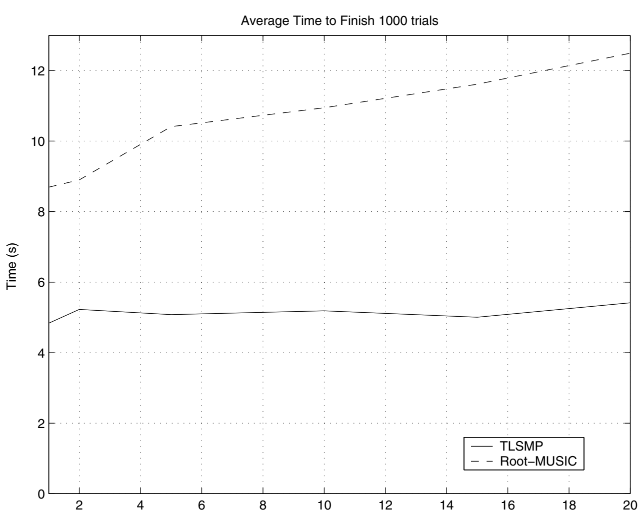

Fig. 7. Computation Speed

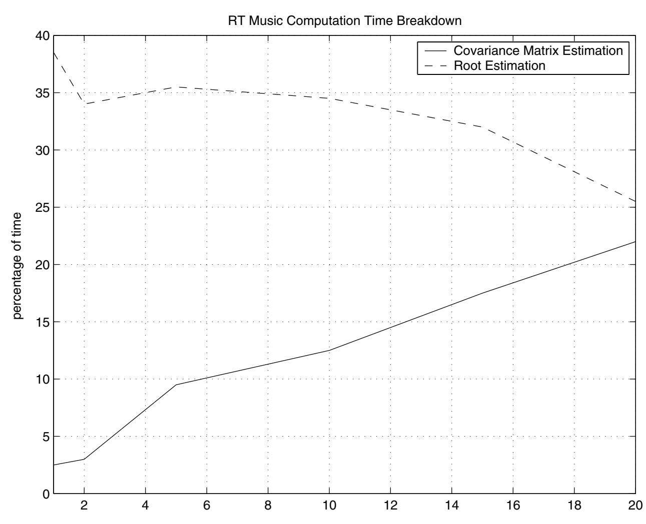

Fig. 8. Root-MUSIC computation time breakdown