Outage Probability of Selection Cooperation in the Low to Medium SNR Regime

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***Abstract*— Selection has been shown to be an effective and practical method of implementing cooperative diversity. However, the available analysis has largely focused on diversity order and the asymptotically high signal to noise ratio (SNR) regimes. We approximate the outage probability of selection cooperation for all SNR levels and arbitrary channel distributions. The approximations are significantly better than the available high-SNR approximations for practical values of outage probability.**

***Index Terms*— Outage probability, cooperative diversity, selection.**

I. Introduction

SELECTION cooperation, recently proposed in [1], [2], has been shown to be an attractive alternative to cooperative diversity schemes based on maximum ratio combining (MRC). By selecting only one cooperative partner, selection cooperation avoids the known problem of bandwidth expansion of a pure MRC system (the pre-log term), as well as the synchronization and power-splitting problems of distributed space-time codes.

In [2], we obtain the outage probabilities of selection cooperation in the high signal to noise (SNR) regime, i.e., where $\frac{2^{2R}-1}{SNR} \ll 1$, where R is the target rate. Although interesting for diversity analysis, the results may not be valid for practical systems where rate adaptation can render this condition unlikely. This problem was first identified by Laneman et al. in [3]. The authors used the above assumption to provide decode-and-forward and amplify-and-forward analysis for a three-node network, but noted that the diversity analysis for higher rate systems is still an open problem.

The validity of the high-SNR analysis is also questionable for low rates but higher diversity orders. In [2], we use the assumption to analyze selection cooperation in a single-source and multi-source network. In both cases, the higher the diversity order, the lower the outage probabilities for which the analytical and simulated values are close. Even for a small network (and corresponding diversity order) of 5, this convergence occurs below outage probability of 10^{-5}. However, it is likely that system design will be at the more practical outage probability values of $10^{-2} - 10^{-3}$.

The authors of [4], while avoiding this high-SNR assumption, preclude the existence of a source-destination channel. This approach, while valid for some scenarios, cannot be generalized to systems where cooperation is used to increase the quality of a communication along an existing link. As shown by Gunduz and Erkip, opportunistic cooperation - using cooperation only when the quality of the source destination link falls below a certain threshold - provides significant benefits in performance [5]. It is thus important to include the source-destination channel in the outage analysis.

In this paper, we develop outage probability approximations for decode-and-forward selection cooperation employing arbitrary SNR and channel distributions. The approximations are based on the Taylor series and third-order approximations of the exponential function. These closed-form expressions can be used in various stages of system design: to predict or verify outage probabilities, for example, or to maximize rates given a target outage probability. They also serve as a basis for further work in multi-hop settings. We also note that although the approach developed here is used specifically for decode-and-forward selection cooperation, it can also be applied to any cooperative scheme, such as MRC, whose outage probability involves a product of exponentials.

In a related work, Zhao and Adve provide outage probability results at arbitrary SNR in a cooperative diversity, amplify-and-forward system incorporating maximum ratio combining [6]. The results apply to independent and identically distributed (i.i.d.) channels; although approximations to the non-i.i.d. case are also developed, they cannot be used for selection cooperation.

This paper is structured as follows. Section II describes the system model. The approximations are developed in Section III and verified in Section IV. Section V concludes the work.

II. System Model

We consider the model of [2], where a source s communicates at a rate R with its destination d while $m-1$ cooperating nodes help in this transmission using decode-and-forward and selection cooperation. The channel a_{ij} between nodes i and j is modelled as a flat and slowly-fading Rayleigh channel with variance $1/\lambda_{i,j}$, i.e., $|a_{ij}|^2 \sim \lambda_{i,j} \exp\left[-\lambda_{i,j}|a_{ij}|^2\right]$. $1/\lambda_{r,d}$, $1/\lambda_{s,r}$ and $1/\lambda_{s,d}$ denote the average relay-destination, source-relay, and source-destination channel power. The channel between any two nodes is assumed independent of all other channels.

Due to the half-duplex constraint, communication is performed in two time slots. The source distributes its data in the first time slot. The destination and each of the $(m-1)$ relays attempt to decode this information. From the decoding set $\mathcal{D}(s)$ - the set of relays that decoded the source information correctly - the relay with the best instantaneous relay-destination channel to the destination forwards the information in the second time slot. The destination combines the data

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from the source and the relay using maximal ratio combining (MRC).

III. Outage Probability

From [2], [7], the probability of outage of selection cooperation can be written as

$$\Pr[I_{sel} < R] = \sum_{\mathcal{D}(s)} \Pr[\mathcal{D}(s)] \Pr\left[I_{sel} < R | \mathcal{D}(s)\right], \quad (1)$$

where $\mathcal{D}(s)$ is the decoding set of source s. Without resorting to high-SNR approximation, the probability of a decoding set is simply [2], [7]

$$\Pr[\mathcal{D}(s)] = \prod_{r\in\mathcal{D}(s)} e^{-\lambda_{s,r}a} \times \prod_{r\notin\mathcal{D}(s)} \left(1 - e^{-\lambda_{s,r}a}\right), \quad (2)$$

where $a = (2^{2R} - 1)/\text{SNR}$. Next, the outage probability of selection cooperation given a decoding set $\mathcal{D}(s)$ is shown [2]:

$$\Pr\left[I_{sel} < R|\mathcal{D}(s)\right] = \lambda_{s,d} \int_0^a \left[\prod_{r=1}^{|\mathcal{D}(s)|} e^{-\lambda_{r,d}a} \left(e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y}\right)\right] e^{-\lambda_{s,d}y} dy \quad (3)$$

The evaluation of this integral is difficult due to the product of the exponential. We thus make two approximations:

1) Taylor Series expansion as shown in (4) on the next page. Because $\lambda_{s,d}$ is always positive, the expression on the right side of (4) is a decreasing function of y. This is readily shown by examining its derivative with respect to y:

$$\frac{d}{dy}\left(1 - \lambda_{s,d}y + \frac{1}{2}\lambda_{s,d}^2 y^2 - \frac{1}{6}\lambda_{s,d}^3 y^3\right) = -\frac{1}{2}\lambda_{s,d}^3 \left(y - \frac{1}{\lambda_{s,d}^2}\right)^2 - \frac{1}{2}\lambda_{s,d} < 0, \quad \forall y, \lambda_{s,d} \geq 0. \quad (5)$$

y_0 is the point where this expression becomes negative, i.e., $(1 - \lambda_{s,d}y + \frac{1}{2}\lambda_{s,d}^2 y^2 - \frac{1}{6}\lambda_{s,d}^3 y^3) = 0$.

$$\Rightarrow y_0 = \frac{(1+\sqrt{2})^{\frac{2}{3}} - 1 + (1+\sqrt{2})^{\frac{1}{3}}}{\lambda_{s,d}(1+\sqrt{2})^{\frac{1}{3}}}. \quad (6)$$

2) Third-order approximation of

$$e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y} \approx f_r(a^3 - y^3), \quad (7)$$

where f_r is determined below.

The second term could also be expanded using Taylor series; the product of the terms, however, would significantly increase the number of terms in the approximation. In this paper, we choose order-three approximations which, as we show in Section IV, yield good results. Clearly, the accuracy of the approximations could be further increased by increasing the approximation order.

Using the approximation in (4), the upper limit of the integral in (3) is L, where $L = \min(y_0, a)$. The parameter f_r is then obtained by minimizing $E(f_r)$, the total squared error $es(y, f_r) = |(e^{\lambda_{r,d}a} - e^{\lambda_{r,d}y}) - f_r(a^3 - y^3)|^2$ over the range $(0, L)$, as shown in (8) on the next page. To minimize $E(f_r)$, we set the derivative of this expression with respect to f_r to zero and obtain the value of f_r as shown in (9) on the next page. Using Binomial Expansion, the resulting outage probability approximation can thus be written as shown in (10) on the next page.

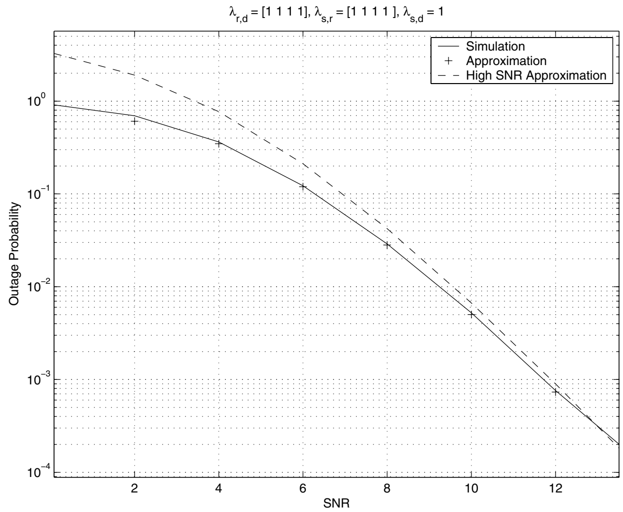

Fig. 1. Outage Probabilities vs. SNR. $R = 1$ b/s/Hz. $\lambda_{r,d} = [1, 1, 1, 1]$, $\lambda_{s,r} = [1, 1, 1, 1]$, and $\lambda_{s,d} = 1$.

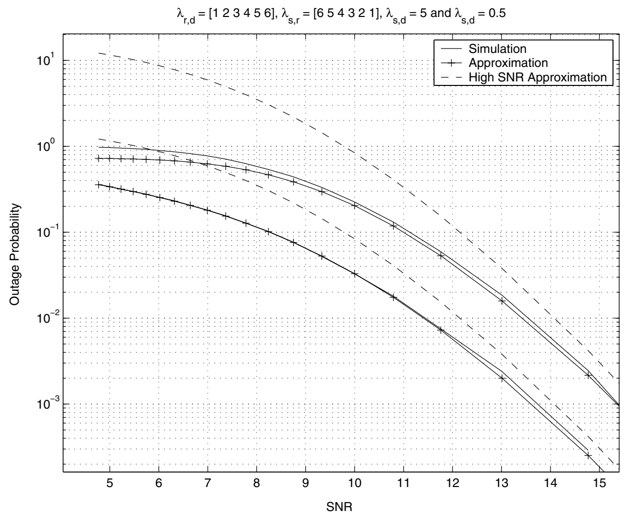

Fig. 2. Outage Probabilities vs. SNR. $R = 1$ b/s/Hz. $\lambda_{r,d} = [1, 2, 3, 4, 5, 6]$, $\lambda_{s,r} = [6, 5, 4, 3, 2, 1]$. $\lambda_{s,d} = 5$ and $\lambda_{s,d} = 0.5$.

IV. Results

In this section, we demonstrate the quality of the approximations by simulating the outage probability of selection cooperation in two scenarios. We compare the approximations developed here with the high-SNR approximations of [2]. To obtain the high-SNR approximation, we still sum over the decoding set using (2), but replace (10) with

$$\Pr\left[I_{sel} < R|\mathcal{D}(s)\right] = a^{N+1} \frac{\lambda s, d}{N+1} \prod_{r=1}^{|\mathcal{D}(s)|} \lambda_{r,d}, \quad (11)$$

obtained from [2]. Clearly, the high-SNR approximations are not meant to be used in the low-SNR regime; these plots are thus included more as a motivation for the necessity of low-SNR approximations rather than as a direct comparison. The

$$\begin{aligned} e^{-\lambda_{s,d}y} &\approx \left(1-\lambda_{s,d}y+\frac{1}{2}\lambda_{s,d}^2y^2-\frac{1}{6}\lambda_{s,d}^3y^3\right), \quad 0, \leq y \leq y_0, \\ &\approx 0 \qquad\qquad\qquad\qquad\qquad\qquad y > y_0 \end{aligned} \tag{4}$$

$$\begin{aligned} E(f_r) &= \int_0^L es(y,f_r)dy = \int_0^L |(e^{\lambda_{r,d}a}-e^{\lambda_{r,d}y})-f_r(a^3-y^3)|^2dy, \\ &= \frac{1}{14\lambda_{r,d}^4}\big[f_r\big(-28e^{\lambda_{r,d}a}a^3\lambda_{r,d}^4L+7e^{\lambda_{r,d}a}\lambda_{r,d}^4L^4-28a^3\lambda_{r,d}^3-168+28e^{\lambda_{r,d}L}a^3\lambda_{r,d}^3 \\ &-28e^{\lambda_{r,d}L}\lambda_{r,d}^3L^3+84e^{\lambda_{r,d}L}\lambda_{r,d}^2L^2-168e^{\lambda_{r,d}L}\lambda_{r,d}L+168e^{\lambda_{r,d}L}\big)+f_r^2\big(14a^6\lambda_{r,d}^4L-7a^3\lambda_{r,d}^4L^4+2\lambda_{r,d}^4L^7\big) \\ &+28e^{\lambda_{r,d}a}\lambda_{r,d}^3-7\lambda_{r,d}^3+14e^{2\lambda_{r,d}a}\lambda_{r,d}^4L-28e^{\lambda_{r,d}a}e^{\lambda_{r,d}L}\lambda_{r,d}^3+7e^{2\lambda_{r,d}L}\lambda_{r,d}^3\big] \end{aligned} \tag{8}$$

$$\begin{aligned} f_r &= \frac{-7}{2L\lambda_{r,d}^4(7a^3L^3+2L^6+14a^6)}\big[\lambda_{r,d}^4Le^{\lambda_{r,d}a}(-4a^3+L^3)+4\lambda_{r,d}^3(e^{\lambda_{r,d}L}a^3-e^{\lambda_{r,d}L}L^3-a^3) \\ &\qquad +12\lambda_{r,d}^2L^2e^{\lambda_{r,d}L}-24\lambda_{r,d}Le^{\lambda_{r,d}L}+24e^{\lambda_{r,d}L}-24\big] \end{aligned} \tag{9}$$

$$\begin{aligned} \Pr\left[I_{sel}<R|\mathcal{D}(s)\right] &\approx \lambda_{s,d}\int_0^L\left[\prod_{r=1}^{|\mathcal{D}(s)|}e^{-\lambda_{r,d}a}f_r\left(a^3-y^3\right)\right]\left(1-\lambda_{s,d}y+\frac{1}{2}\lambda_{s,d}^2y^2-\frac{1}{6}\lambda_{s,d}^3y^3\right)dy, \\ &= \lambda_{s,d}\left[\prod_{r=1}^{|\mathcal{D}(s)|}f_re^{-\lambda_{r,d}a}\right]\int_0^L\left(a^3-y^3\right)^{|\mathcal{D}(s)|}\left(1-\lambda_{s,d}y+\frac{1}{2}\lambda_{s,d}^2y^2-\frac{1}{6}\lambda_{s,d}^3y^3\right)dy, \\ &= \lambda_{s,d}\left[\prod_{r=1}^{|\mathcal{D}(s)|}f_re^{-\lambda_{r,d}a}\right]\int_0^L\sum_{i=0}^{|\mathcal{D}(s)|}\binom{|\mathcal{D}(s)|}{i}(a^3)^{|\mathcal{D}(s)|-i}(-y^3)^i\left(1-\lambda_{s,d}y+\frac{1}{2}\lambda_{s,d}^2y^2-\frac{1}{6}\lambda_{s,d}^3y^3\right)dy, \\ &= \lambda_{s,d}\left[\prod_{r=1}^{|\mathcal{D}(s)|}f_re^{-\lambda_{r,d}a}\right]\sum_{i=0}^{|\mathcal{D}(s)|}\binom{|\mathcal{D}(s)|}{i}a^{3(|\mathcal{D}(s)|-i)}(-1)^i\left(\frac{L^{3i+1}}{3i+1}-\lambda_{s,d}\frac{L^{3i+2}}{3i+2}+\frac{1}{2}\frac{\lambda_{s,d}^2L^{3i+3}}{3i+3}-\frac{1}{6}\frac{\lambda_{s,d}^3L^{3i+4}}{3i+4}\right). \end{aligned} \tag{10}$$

simulated results are presented in Figures 1 and 2. Both plots show the simulated and analytical outage probability for a selection cooperation system with 6 relays and rate $R = 1$ b/s/Hz. Although this rate is kept constant, we note that for the purpose of the approximation, increasing the rate has the same effect as decreasing the SNR (decreasing a); it is thus sufficient to evaluate the approximations by considering the low-SNR regime.

Figure 1 plots the outage probabilities for the i.i.d. case where all inter-node channels have unit power. With the exception of using (2), this plot is similar to the one given in [2]. The approximations developed in this paper are accurate for all SNR values, while the high-SNR approximations are accurate in the range of outage probabilities $P_{out} = 10^{-2} - 10^{-3}$.

As shown in Figure 2, the importance of the approximation in (10) is clearly seen in a more realistic setting. In this figure, the inter-channel quality is set to $\lambda_{r,d} = [1, 2, 3, 4, 5, 6]$, and $\lambda_{s,r} = [6, 5, 4, 3, 2, 1]$. We provide the results both for a strong and weak source-destination channel, i.e., $\lambda_{s,d} = 0.5$ and $\lambda_{s,d} = 5$, respectively. In each one, the approximations developed in this paper track the simulated plots at all SNRs, and crucially, at the practical values of outage probability, $P_{out} = 10^{-2} - 10^{-3}$. The high-SNR approximations, however, only begin to converge at much lower outage probabilities.

V. Conclusions

Motivated by a lack of analysis of selection cooperation for arbitrary SNR and practical outage probability values, we have developed approximations applicable in these scenarios. The approximations, based on the Taylor series and a third-order approximation of the exponential, are shown to closely follow the simulated curves. The approximations presented here can be used as closed form expressions with applications in system design, such as maximizing rate given a target outage probability.

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