Blind Channel Estimation for Orthogonal STBC in MISO Systems

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***Abstract*—This paper presents a closed-form blind channel estimation scheme for Alamouti's and other orthogonal space-time block codes. Unlike other blind algorithms, the scheme is able to estimate the channels, to within a phase constant, in multiple-input single-output systems, i.e., systems that employ only one receive antenna. The channel matrix is estimated from the eigenvalue decomposition of the fourth order cumulant matrix of the output signal. The performance of the scheme depends upon the accuracy of the estimated cumulants, and thus a scheme to improve the cumulant matrix estimate is suggested. Using these improved cumulants, the algorithm performs very well in slowly fading channels.**

I. Introduction

The advantages of using multiple transmit and/or receive antennas along with space-time coding have been extensively studied [1–3] and are now well accepted. In particular, the orthogonal space-time block codes (O-STBC) of [2] and [3] have been shown to be very attractive in terms of providing full diversity with linear decoding complexity. However, the performance of these codes, as with most other space-time codes, depends on accurate knowledge of the channels between the transmit and receive antennas. The importance of channel information to space-time coding has motivated research into channel estimation for multiple-input-multiple output (MIMO) systems [4–8]. As with the single-input-single-output (SISO) case, training, blind and semi-blind techniques have been proposed.

In dealing specifically with MIMO systems using STBC, one class of approaches exploits the structure of space-time codes itself to enable channel estimation [5–8]. In [5], Budianu and Tong present a training based scheme for the orthogonal codes of Alamouti [2] and Tarokh [3]. Training bits, however, reduce throughput and such schemes are inappropriate for systems where bandwidth is scarce. By restricting themselves to real signals and transmit diversity order, Ammar and Ding estimate channels for STBC from the null space of the received signal [6]. Swindlehurst and Leus present a blind channel estimation for a generalized set of space-time codes [7].

A significant problem with these blind approaches for O-STBC is that they require the number of receive antennas to be greater than or equal to the number of transmit antennas [6, 7]. However, in a large part, space-time coding is designed for transmit diversity in the downlink. Expecting as many receive elements on a mobile device as at the base station may not be realistic. Stoica and Ganesan [8] present an iterative algorithm for blind channel estimation which poses no restrictions on the input signal or on the number of antennas. The algorithm is, however, very sensitive to initialization and the authors acknowledge the results to be unsatisfactory; they improve the algorithm through semi-blind and training-based estimation.

This paper presents a blind channel estimation technique for systems using O-STBC with an arbitrary number of receive antennas. The technique is based on the work presented by Ding and Liang in [9], who introduce a special form of the cumulant matrix. We show that, if the channel matrix satisfies certain specified conditions, multiple channels can be estimated from the eigenvectors of the cumulant matrix up to a single phase ambiguity. One important application is a system using Alamouti's scheme with two transmit antennas and one receive antenna.

This paper is structured as follows. Section II presents the data model used in this paper based on a single receive antenna and the definition of cumulants used later. Section III presents the theory of the proposed approach for MISO systems. Section IV discusses implementation issues while Section V presents simulations to illustrate the performance of the proposed scheme. Section VI concludes this work.

II. Preliminaries

A. Data and Channel Model

Consider a system with $L > 1$ transmit antennas and one receive antenna. A block of N complex data symbols, $\mathbf{s}$, is encoded over K time slots using generalized orthogonal space-time codes of [3]. The channel is modelled as flat and Rayleigh. The assumptions used in this work are:

1) source symbols are zero mean, independent, identically distributed (i.i.d.) random variables with non-zero fourth-order kurtosis, γ_4,
2) receiver noise is additive, white and Gaussian, and
3) the channel matrix is constant over the K time slots.

Under these assumptions, the length-K receive data vector $\mathbf{r}$ can be written in the most general form as

$$\mathbf{r} = \mathbf{G}_r\mathbf{h} + j\mathbf{G}_i\mathbf{h} + \mathbf{w} = \mathbf{H}_X\mathbf{s}_r + j\mathbf{H}_Y\mathbf{s}_i + \mathbf{w}, \qquad (1)$$

where $\mathbf{s}_r$ and $\mathbf{s}_i$ denote the real and imaginary parts of the transmitted signal $\mathbf{s}$ respectively, and $\mathbf{h} = [h_1, h_2, \ldots, h_L]^T$ denotes the L channels between the transmit antennas and receive antenna. The vector $\mathbf{w}$ represents the additive receiver noise. The $K \times L$ matrices $\mathbf{G}_r$ and $\mathbf{G}_i$, dependent on the particular code used, represent the encoding of the real and imaginary parts of the transmit signal vector. They are formed from real orthogonal matrices $\mathbf{X}_n$ and $\mathbf{Y}_n$, $n = 1, \ldots, N$,

$$\mathbf{G}_r = \sum_{n=1}^{N} \mathbf{X}_n s_{rn}, \qquad \mathbf{G}_i = \sum_{n=1}^{N} \mathbf{Y}_n s_{in}. \tag{2}$$

The matrices $\{\mathbf{X}_n\}$ and $\{\mathbf{Y}_n\}$ satisfy the following properties:

$$\mathbf{X}_n{}^T\mathbf{X}_n = \mathbf{I}_L, \qquad \mathbf{Y}_n{}^T\mathbf{Y}_n = \mathbf{I}_L, \qquad \forall i, \tag{3}$$

$$\mathbf{X}_n{}^T\mathbf{X}_m = -\mathbf{X}_m{}^T\mathbf{X}_n, \quad \mathbf{Y}_n{}^T\mathbf{Y}_m = -\mathbf{Y}_m{}^T\mathbf{Y}_n, \quad i \neq j. \tag{4}$$

The complex channel matrices $\mathbf{H}_X$ and $\mathbf{H}_Y$ are formed from the matrices $\mathbf{X}_n$ and $\mathbf{Y}_n$ and the channel vector $\mathbf{h}$:

$$\mathbf{H}_X = \left[\begin{array}{cccc} \mathbf{X}_1\mathbf{h} & \mathbf{X}_2\mathbf{h} & \ldots & \mathbf{X}_N\mathbf{h} \end{array} \right], \tag{5}$$

$$\mathbf{H}_Y = \left[\begin{array}{cccc} \mathbf{Y}_1\mathbf{h} & \mathbf{Y}_2\mathbf{h} & \ldots & \mathbf{Y}_N\mathbf{h} \end{array} \right]. \tag{6}$$

B. Fourth Order Cumulants

The fourth order joint cumulant of four random variables x_1, x_2, x_3 and x_4, is

$$c_4(x_1, x_2, x_3, x_4) = m_4(x_1, x_2, x_3, x_4) - m_2(x_1, x_2)m_2(x_3, x_4) \\ -m_2(x_1, x_3)m_2(x_2, x_4) - m_2(x_1, x_4)m_2(x_2, x_3), \tag{7}$$

$$m_2(x_1, x_2) = E\{x_1 x_2^*\}, \tag{8}$$

$$m_4(x_1, x_2, x_3, x_4) = E\{x_1 x_2^* x_3 x_4^*\}, \tag{9}$$

where * represents complex conjugation and $E\{\}$ the expectation operator. The kurtosis of a random variable x is

$$\gamma_4 = c_4(x, x^*, x, x^*) = E\{|x|^4\} - 2E\{|x|^2\}^2 \\ -E\{xx\}E\{x^*x^*\}. \tag{10}$$

Note that the cumulants are linear in each variable and that the 4^{th}-order cumulant of jointly Gaussian random variables, such as white noise, is zero. Also, for an white process, $\{x_n\}$,

$$c_4(x_n, x_{n-n_1}, x_{n-n_2}, x_{n-n_3}) = \gamma_4 \delta(n_1)\delta(n_2)\delta(n_3). \tag{11}$$

III. Channel Estimation Using Fourth Order Cumulants

A. The Estimation Algorithm

The channel estimation algorithm presented here requires that the STBC permit a transformation of the received signal such that it can be written in terms of one $K \times N$ matrix $\mathbf{H_c}$,

$$\mathbf{r}' = \mathbf{H_c}\mathbf{s} + \mathbf{w}, \tag{12}$$

and, crucially,

$$\mathbf{H}_c^H \mathbf{H}_c = \mathbf{D}_N, \tag{13}$$

where $\mathbf{D}_N$ is a diagonal matrix. Each column of $\mathbf{H}_c$ is a permutation of the L channels and their conjugates, and possibly some zeros. These constraints are essential to the development of the algorithm. Common codes that do and do not satisfy these criteria are discussed in Section III-B.3. One important scheme that does satisfy these criteria is the Alamouti scheme. For this scheme, the received signal, over two time slots, given two transmit symbols s_1 and s_2, is given by

$$\mathbf{r} = \left[\begin{array}{c} r_1 \\ r_2 \end{array} \right] = \left[\begin{array}{c} h_1 s_1 + h_2 s_2 \\ -h_1 s_2^* + h_2 s_1^* \end{array} \right] + \mathbf{w}, \tag{14}$$

$$\Rightarrow \mathbf{r}' = \left[\begin{array}{c} r_1 \\ r_2^* \end{array} \right] = \mathbf{H}_c \left[\begin{array}{c} s_1 \\ s_2 \end{array} \right] + \mathbf{w}, \tag{15}$$

where

$$\mathbf{H}_c = \left[\begin{array}{cc} h_1 & h_2 \\ h_2^* & -h_1^* \end{array} \right], \tag{16}$$

$$\Rightarrow \mathbf{H}_c^H \mathbf{H}_c = \mathbf{H}_c \mathbf{H}_c^H = \left(|h_1|^2 + |h_2|^2 \right) \mathbf{I}_2. \tag{17}$$

As in [9], define the joint cumulant matrix of the vector $\mathbf{r}'$ as

$$\mathbf{C}_4^{[k]} = c_4(\mathbf{r}', \mathbf{r}'^H, r'_k, r'^*_k) \qquad k = 1, \ldots, K, \tag{18}$$

$$\Rightarrow \mathbf{C}_4^{[k]}(i, j) = c_4(r'_i, r'^*_j, r'_k, r'^*_k). \tag{19}$$

Proposition: The length L channel, $\mathbf{h}$, can be extracted from the eigenvectors of the joint cumulant matrix $\mathbf{C}_4^{[k]}$.
Proof: See Appendix.

The cumulant matrix $\mathbf{C}_4^{[k]}$ has as its final two arguments r'_k and r'^*_k, corresponding to the k^{th} received symbol. Theoretically, any choice of k, $k = 1 \ldots K$ provides the required channel estimate. In practice, due to noise and the finite data record used to estimate the cumulants, each choice of k in the matrix $\mathbf{C}_4^{[k]}$ provides a different channel estimate. Clearly, at the expense of complexity, one could obtain better channel estimates by repeating the process for all valid k and averaging the results.

The steps of the algorithm are therefore:

1) Using a block of data, estimate the cumulant matrix $\mathbf{C}_4^{[k]}$ for $k = 1$ using (7) and (18).
2) Perform an eigendecomposition of this matrix.
3) To within a multiplicative constant and a permutation, the channel is given by a suitably chosen eigenvector.
4) Repeat for $k = 2, \ldots K$ and average.

This procedure leaves the channel estimate ambiguous up to a single multiplicative factor. This ambiguity is common to all blind channel estimation techniques and may be resolved using a *single* pilot symbol inserted at the beginning of the data block.

B. Applicable and Inapplicable Codes

The scheme presented above assumes the received data can be manipulated to fit (12). Also, the channel matrix $\mathbf{H}_c$ must satisfy (13). Clearly Alamouti's scheme satisfies these constraints. This section shows that all real orthogonal codes and the complex rate-1/2 codes of [3] also satisfies this constraint. However, the rate-3/4 codes in the same reference do not.

1) Real Orthogonal Codes: If the symbol alphabet is real, (1) is reduced to $\mathbf{r} = \mathbf{H}_X\mathbf{s_r} + \mathbf{w}$. The real and imaginary parts of the received signal arising from the complex channel can be processed independently, i.e.,

$$\mathbf{r}_r = \mathbf{H}_{X\mathbf{r}}\mathbf{s_r} + \mathbf{w_r}, \tag{20}$$

$$\mathbf{r}_i = \mathbf{H}_{X\mathbf{i}}\mathbf{s_r} + \mathbf{w_i}, \tag{21}$$

where $\mathbf{H}_{X\mathbf{r}}$ and $\mathbf{H}_{X\mathbf{i}}$, themselves real, are the real and imaginary parts of the complex channel $\mathbf{H}_X$, respectively. Focusing on the real channel matrix, $\mathbf{H_c} = \mathbf{H}_{X\mathbf{r}}$ and using (4) and (5), the $(i,j)^{\text{th}}$, $(i \neq j)$, entry of the matrix $\left(\mathbf{H_c}^H\mathbf{H_c}\right)$ is

$$\begin{aligned} m_{ij} &= \mathbf{h}^H \mathbf{X}_i{}^H \mathbf{X}_j \mathbf{h}, & (22)\\ \Rightarrow m_{ij}^* &= \mathbf{h}^H \mathbf{X}_j{}^H \mathbf{X}_i \mathbf{h} = -\mathbf{h}^H \mathbf{X}_i{}^H \mathbf{X}_j \mathbf{h} = -m_{ij} & (23)\end{aligned}$$

Thus m_{ij} is purely imaginary when $i \neq j$. However, m_{ij} is real for all i and j, as the matrix $\mathbf{H_c}$ is real. It thus follows that $m_{ij} = 0 \;\forall i \neq j$, and the matrix $\mathbf{H_c}^H\mathbf{H_c}$ is diagonal,with diagonal entries

$$\begin{aligned} m_{ii} &= \mathbf{h}_r^H \mathbf{X}_i{}^H \mathbf{X}_i \mathbf{h}_r = \mathbf{h}_r^H \mathbf{h}_r, \\ &= (|h_{r1}|^2 + |h_{r2}|^2 + \ldots + |h_{rL}|^2)\mathbf{I}_N. & (24)\end{aligned}$$

The same argument is valid for the imaginary matrix $\mathbf{H}_{X\mathbf{i}}$. Real codes thus satisfy the constraints of (12) and (13) because the real and imaginary components can be processed separately.

2) Half-Rate Complex Orthogonal Codes: In using rate-1/2 codes [3], N complex symbols are transmitted using $2N$ time slots. Conjugating the last N received symbols, the received signal can be written in terms of a single channel matrix $\mathbf{H}_c$,

$$\begin{aligned} \mathbf{r}' &= \begin{bmatrix} \mathbf{H}_N \\ \mathbf{H}_N^* \end{bmatrix} \mathbf{s} + \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2^* \end{bmatrix}, \\ &= \mathbf{H_c s} + \mathbf{w}, & (25)\end{aligned}$$

where $\mathbf{H}_N$ is the channel matrix used in Section III-B.1 for real symbols. Therefore,

$$\begin{aligned} \mathbf{H_c}^H \mathbf{H_c} &= \begin{bmatrix} \mathbf{H_N}^H & \mathbf{H_N}^T \end{bmatrix} \begin{bmatrix} \mathbf{H_N} \\ \mathbf{H_N}^* \end{bmatrix}, \\ &= \mathbf{H}_N^H \mathbf{H}_N + \mathbf{H}_N^T \mathbf{H}_N^* = 2\Re\{\mathbf{H}_N^H \mathbf{H}_N\}. & (26)\end{aligned}$$

where $\Re$ denotes the real part of a complex number. As shown in Section III-B.1, the $(i,j)^{\text{th}}$ entry of $\mathbf{H}_N^H\mathbf{H}_N = \mathbf{h}^H\mathbf{X}_i^H\mathbf{X}_j\mathbf{h}$, which is purely imaginary if $i \neq j$. Hence, the off-diagonal entries of $\mathbf{H}_c^H\mathbf{H}_c$ are zero, making $\mathbf{H}_c^H\mathbf{H}_c$ diagonal. The rate-1/2 codes, therefore, satisfy both (12) and (13) and the procedure of Section III-A can be applied.

3) Inapplicable Codes: Consider a space-time block code wherein no transformation of the received signal $\mathbf{r}$ will reduce it to the form in (12), $\mathbf{r}' = \mathbf{H_c s} = \mathbf{H}_c[\mathbf{s}_r + j\mathbf{s}_i] + \mathbf{w}$. The sporadic rate-3/4 complex codes in [3] are examples of such a code. It is always possible to write the received signal as

$$\mathbf{r} = [\mathbf{H}_X \;\; j\mathbf{H}_Y] \begin{bmatrix} \mathbf{s}_r \\ \mathbf{s}_i \end{bmatrix} + \mathbf{w} = \mathbf{H}_Z \begin{bmatrix} \mathbf{s}_r \\ \mathbf{s}_i \end{bmatrix} + \mathbf{w}, \quad (27)$$

where the new $K \times 2N$ composite channel matrix $\mathbf{H}_Z = [\;\mathbf{H}_X \quad j\mathbf{H}_Y\;]$. It is easy to show that the algorithm in Section III-A is valid if $\mathbf{H}_Z^H\mathbf{H}_Z$ is diagonal, $\mathbf{H}_Z$ satisfies the constraint in (13). However,

$$\mathbf{H}_Z^H \mathbf{H}_Z = \begin{bmatrix} \mathbf{H}_X^H \mathbf{H}_X & j\mathbf{H}_X^H \mathbf{H}_Y \\ -j\mathbf{H}_Y^H \mathbf{H}_X & \mathbf{H}_Y^H \mathbf{H}_Y \end{bmatrix}. \quad (28)$$

Using (4), the $(i,j)^{\text{th}}$ entry $\mathbf{H}_X^H\mathbf{H}_Y$ is

$$\left[\mathbf{H}_X^H \mathbf{H}_Y\right]_{ij} = \mathbf{h}^H \mathbf{X}_i^H \mathbf{Y}_j \mathbf{h} \neq 0 \qquad \forall i,j. \quad (29)$$

Therefore, a code, such as the rate-3/4 codes of [3], which cannot be written as $\mathbf{r} = \mathbf{H_c s} + \mathbf{w}$, cannot satisfy the constraint of (13). The algorithm in Section III-A cannot be used to estimate the required channel information.

The theory developed above focuses on the MISO case. With multiple receive antennas, the procedure above can be repeated at each element. If the number of transmitters is greater than the number of receivers, the procedure above appears to be the only effective blind scheme available. However, if there are at least as many receive as transmit antennas, other blind channel estimation techniques have been proposed [5–7].

IV. Implementation Issues

A. Ambiguity Resolution

The proposed channel estimation scheme extracts channel estimates from the eigenvector of the cumulant matrix. Each eigenvector is a permutation of a column of the channel matrix, $\mathbf{H}_c$. The scheme, however, does not identify the permutations. The O-STBC encodes N data symbols over K epochs and transmits the code over L antennas. Only for square STBC ($N = K = L$) schemes are the columns of the channel matrix the permutations of the channel vector $\mathbf{h} = [h_1, h_2, \ldots, h_L]$. This is the case for the Alamouti code, as well as for real square codes utilizing 4 and 8 antennas. This case is investigated in more detail in this section. However, we begin with the simpler case of generalized rectangular block codes.

For such codes, the columns of the channel matrix, and thus its eigenvector, are augmented with $(N - L)$ zeros. In the case of the full-rate real code for 3 antennas, for example, each column of the channel matrix is augmented with one zero:

$$\mathbf{H}_c = \begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2 & -h_1 & 0 & -h_3 \\ h_3 & 0 & -h_1 & h_2 \\ 0 & h_3 & -h_2 & -h_1 \end{bmatrix}. \quad (30)$$

The locations of the zero in the eigenvector can help resolve the permutation, i.e., identify which entry in the eigenvector corresponds to which channel. In the case of the real rectangular code shown, for example, the column number can be identified as $K - p_z + 1$, where p_z is the location of the zero in the column. This is also true for the generalized complex O-STBC.

For a space-time code with $N = K = L$, such as Alamouti's scheme, the eigenvector provides estimates of the L channels. However, no information in the code itself allows a correct assignment between the estimated eigenvectors of the cumulant matrix and the columns of the channel matrix. A resolution of this problem therefore requires the use of one time slot for a single pilot transmission. Consider, for example, the case of the Alamouti code with two transmit and one receive antennas. Symbols are transmitted through channels h_1 and h_2. The algorithm has determined the two channels h_a and h_b, but cannot assign them to the channels h_1 and h_2. This assignment is possible, however, if a single pilot tuple ($s_1 = -1$, $s_2 = 1$) is inserted into the first time slot of data transmission . In such a case, the first received signal is $-h_1 + h_2$, and the two channels can be distinguished because $-h_a + h_b$ is different than

$-h_b + h_a$. The channels, including the phase ambiguity, are then resolved for that window of data.

The cumulant matrix is estimated using a window of n output bits. The resulting channel estimate is applied to this entire window of data, i.e., inside this window, the channel is assumed constant. If the actual channel is assumed fixed over the entire block of data (composed of a number of windows), the channel can be estimated for any time index within the block using only a single pilot-tuple at the beginning of the first window.

B. Cumulant Estimation

Using (7), the fourth-order cumulant matrix is:

$$\begin{aligned}\mathbf{C}_4^{[k]} &= c_4\left(\mathbf{r}, \mathbf{r}^H, r_k, r_k^*\right) \\ &= m_4(\mathbf{r}, \mathbf{r}^H, r_k, r_k^*) - m_2(\mathbf{r}, \mathbf{r}^H) m_2(r_k, r_k^*) \\ &\quad - m_2(\mathbf{r}, r_k) m_2(\mathbf{r}^H, r_k^*) - m_2(\mathbf{r}, r_k^*) m_2(\mathbf{r}^H, r_k), \\ &= m_4(\mathbf{r}, \mathbf{r}^H, r_k, r_k^*) - m_2(\mathbf{r}, \mathbf{r}^H) m_2(r_k, r_k^*) \\ &\quad - m_2(\mathbf{r}, r_k) m_2^H(\mathbf{r}, r_k) - m_2(\mathbf{r}, r_k^*) m_2^H(\mathbf{r}, r_k^*). \quad (31)\end{aligned}$$

Note that $m_2(\mathbf{r}, r_k^*)$ and $m_2(r_k, r_k^*)$ compose the k^{th} column and diagonal entry of $m_2(\mathbf{r}, \mathbf{r}^H)$, respectively. Thus to determine the cumulant matrix, only the following three terms need be calculated: $m_4(\mathbf{r}, \mathbf{r}^H, r_k, r_k^*)$, $m_2(\mathbf{r}, \mathbf{r}^H)$, and $m_2(\mathbf{r}, r_k)$. Other efficient estimates of the fourth order cumulants are also possible [10].

For the Alamouti scheme specifically, cumulant estimates can be improved by further investigating the properties of the cumulant matrix. As shown in (37), the fourth order cumulant can be written as $\mathbf{C}_4^{[k]} = \gamma_4 \mathbf{H_c} \mathbf{S}_k \mathbf{H_c}^H$. For example, for Alamouti's scheme

$$\mathbf{S}_1 = \text{diag}(|h_1|^2, |h_2|^2), \qquad \mathbf{S}_2 = \text{diag}(|h_2|^2, |h_1|^2). \quad (32)$$

Therefore,

$$\begin{aligned}\Rightarrow \mathbf{C}_4^{[1]} + \mathbf{C}_4^{[2]} &= \quad (33) \\ &\gamma_4 \mathbf{H_c} \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \mathbf{H_c}^H, \\ &= \gamma_4(|h_1|^2 + |h_2|^2) \mathbf{H_c} \mathbf{H_c}^H = \gamma_4(|h_1|^2 + |h_2|^2)^2 \mathbf{I}_2 (34)\end{aligned}$$

The sum of the two cumulant matrices is thus a scaled identity matrix. Given two cumulant estimates $\mathbf{C}_4^{[1]}$ and $\mathbf{C}_4^{[2]}$, this information can be used to generate two new cumulant matrices, $\tilde{\mathbf{C}_4}^{[1]}$ and $\tilde{\mathbf{C}_4}^{[2]}$ in the following manner:

1) Generate the cumulant matrices $\mathbf{C}_4^{[1]}$ and $\mathbf{C}_4^{[2]}$. Let $\mathbf{A} = \mathbf{C}_4^{[1]} + \mathbf{C}_4^{[2]}$ and $\mathbf{B} = \mathbf{C}_4^{[1]} - \mathbf{C}_4^{[2]}$.
2) Create a new diagonal matrix $\tilde{\mathbf{A}}$ by replacing the diagonal entries of $\mathbf{A}$ with zeros.
3) Form $\tilde{\mathbf{C}_4}^{[1]} = (\tilde{\mathbf{A}} + \mathbf{B})/2$, $\tilde{\mathbf{C}_4}^{[2]} = (\tilde{\mathbf{A}} - \mathbf{B})/2$.

Channel estimation continues using the eigenvectors of one of the newly estimated matrices. It should be noted that this scheme to improve cumulant estimates is valid for the Alamouti code only.

We note here that the use of the improved cumulant matrices increases the complexity of the algorithm by imposing four additions each time a new set of cumulant matrices is estimated.

Fig. 1. Time Invariant Channel. Phase Error vs. SNR using 100-, 500-, 1000- and 3000-point cumulant estimates.

As shown in the following section, however, this scheme significantly improves the performance of the estimation scheme, thus decreasing the amount of points required in the cumulant estimates. It can be argued, therefore, that for the same performance, the estimation scheme using the improved cumulant matrices has lower complexity than that achieved by using regular cumulant matrices.

V. Numerical Examples

In this section, the channel estimation algorithm presented in Section III is verified with simulations. All examples use a BPSK i.i.d. input signal transmitted through a slow, flat Rayleigh fading channel. The performance of the proposed scheme is evaluated using the normalized mean squared error and the resulting change in bit error rate. The only other known blind estimation algorithm applicable to O-STBC for MISO systems is presented in [8]. Those results are not used as comparison in this paper, however, as the authors acknowledge that their blind scheme is unsatisfactory, and improve it with training bits. Instead, the results presented in this paper are compared to the standard bit error rate (BER) plots of the clairvoyant receiver using perfect knowledge of the channel.

A. Static Channel

In the following simulations, the channel to be estimated is held constant over one window of data. All data points in a window are used to estimate the cumulant matrix. As discussed in Section IV-A, the data in the first time slot is assumed known to allow for channel resolution. The channel changes for each window and the results are averaged over 10^6 Monte Carlo simulations.

1) Two Transmit Antennas: The first example simulates the Alamouti case with two transmit antennas and one receive antenna. The improved cumulant estimates described in Section IV-B are used.

Fig. 1 shows the pahse error (PE) between the phases of true and estimated channels, as a function of signal-to-noise ratio

Fig. 2. Time Invariant Channel. BER vs. SNR using 100-, 500-, 1000- and 3000-point cumulant estimates.

Fig. 3. Time-Invariant Channel. Complex Half-Rate STBC. Channels estimated using 3000 point cumulants

(SNR). The phase information is presented as it is more relevant to communications performance, especially BER. Given estimate $\hat{\mathbf{h}}$ of the true channel $\mathbf{h}$, the phase error is defined as

$$\mathrm{PE} = \sum_{l=1}^{L} |\angle h_l - \angle \hat{h}_l|. \tag{35}$$

The cumulants are estimated using 100, 500, 1000 and 3000 data points. As expected, the phase error is a decreasing function of SNR. Although it is true that in theory cumulant estimates are zero for Gaussian noise and should be insensitive to SNR, this occurs when the number of points used in the cumulant estimates is very large (we note here that 4^{th} order cumulants have a large variance and a very large number of points, of the order of thousands or more, is required for the estimated cumulants to converge to their true statistics). When using a smaller number of points, however, noise will affect the accuracy of the estimation, and thus the estimates are sensitive to SNR. The PE is also a function of increasing window size: the more points are used the more accurate the cumulant estimate and the more accurate the channel estimate. The channel estimate is one important step towards decoding the transmitted data. The next plot, shown in Fig. 2 demonstrates the efficacy of the channel estimation in terms of the resulting bit error rate (BER). The BER is compared to that obtained by the clairvoyant receiver, which has knowledge of the true channel. As with the PE plot, the results are shown for window sizes 100, 500, 1000 and 3000, respectively. Depending on the number of points used in cumulant estimates, the system BER when using estimated channels closely tracks that of the clairvoyant receiver up to certain BER. With 3000-point cumulant estimates, this threshold occurs at about 2×10^{-5}, and with 100-points estimates, it occurs at about 2×10^{-4}. In static or very slowly changing channels, the performance of the algorithm could always be improved by increasing the number of points used in cumulant estimates.

2) Three Transmit Antennas: The second example uses identical channel conditions with three transmit antennas and one receive antenna. The rate-1/2 complex code of [3] is used to encode the data. The BER vs. SNR curve is depicted in Fig. 3. Again, the performance of the receiver using the estimated channels is compares well to that of a clairvoyant receiver up to a BER of 10^{-4}.

B. Time-Varying Channel

The efficacy of the proposed channel estimation algorithm is now examined when used in a slowly fading environment. The data is sampled at a rate of 20 MHz and is modulated using a carrier with a frequency of 5.5 GHz. The mobile is assumed to be moving at 100 km/h.

For each Monte Carlo run, 2×10^6 bits are corrupted by the time varying channel. The channel estimate for a particular window of data is obtained from the cumulant estimate resulting from that same window. Channel estimates in one window are static, but change from window to window. To correctly identify the channels, a pilot symbol is inserted at the beginning of every window. We focus here on the BER, which is more meaningful in terms of practical implications. As long as the BER using the channel estimates is close to the BER of the clairvoyant receiver, the estimates are "good enough", and improving the estimates past this point would be wasteful. The results are averaged over 200 Monte Carlo runs.

The BER for window sizes of 100, 250, 500, 1000 and 3000 is shown in Fig. 4. The BER curves using 100, 250 500 points tracked the ideal BER to approximately 3×10^{-4}. The results, however, seem surprising: contrary to what was determined in section Section V-A, the BER decreases for increasing window sizes. This occurs because when the statistics of the input change over time, the cumulant estimate does not necessarily become more accurate for longer inputs. The variation in signal statistics therefore imposes a limit on the maximum number of points that can be used in cumulant estimates. The faster these channel variations, the fewer the points that can be used. In this example, inputs of 250 and 500 points generate the best bit error rates, up to 10^{-4}. We note that when the velocity of

Fig. 4. Time-Varying Channel. BER vs. SNR using 100-, 500-, 1000- and 3000-point cumulant estimates.

the mobile decreases, so does the channel fading, and the results approach those obtained in Section V-A.

VI. Conclusions

This paper presents a blind channel estimation algorithm for block coded data in the important MISO situation, i.e., where the number of receiving antennas is fewer than the number of transmit antennas. The algorithm is based on fourth order cumulants of the received data. While the scheme is restricted to orthogonal codes, it is applicable to such important schemes as Alamouti's scheme. Other applicable codes are the real and complex half-rate orthogonal codes of [3].

A significant cost of the algorithm is the complexity involved in estimating the required cumulants. Good estimation results, however, are obtained when using as few as 250 points in the cumulant estimates. Complexity is further by decreased by exploiting the symmetry inherent in the definition of the cumulant matrix. Note that to date, this algorithm is the only effective blind channel estimation scheme applicable to Alamouti's code that is suitable for use with only one receive antenna.

Appendix

The proposition in Section III-A states that the L channels can be extracted from the eigenvectors of the fourth order cumulant matrix $\mathbf{C}_4^{[k]}$. We now prove this claim.

¿From the definition in (7), the joint cumulant is linear in each of its arguments. As in [9], the cumulant matrix can therefore be decomposed as

$$\begin{aligned}\mathbf{C}_4^{[k]} &= c_4(\mathbf{r}', \mathbf{r}'^H, r'_k, r'^*_k) = c_4(\mathbf{H_c s}, \mathbf{s}^H \mathbf{H_c}^H, r'_k, r'^*_k) \\ &= \mathbf{H}_c c_4(\mathbf{s}, \mathbf{s}^H, r'_k, r'^*_k)\mathbf{H_c}^H = \mathbf{H}_c \mathbf{S}_k \mathbf{H}_c^H. \quad (36)\end{aligned}$$

The entry in row i and column j of the inner matrix, $\mathbf{S}_k$, is given by $c_4(s_i, s_j^*, r'_k, r'^*_k)$. From (11), this term is non-zero only for $i = j$, i.e., $\mathbf{S}_k$ is diagonal. Therefore, using (15), the fourth-order cumulant matrix can be written as

$$\mathbf{C}_4^{[k]} = \gamma_4 \mathbf{H_c S}_k \mathbf{H_c}^H, \quad (37)$$

$$\mathbf{S}_k = \mathrm{diag}\left(|h_{k1}|^2, |h_{k2}|^2, \ldots, |h_{kN}|^2\right), \quad (38)$$

where $\mathbf{h}_k = [h_{k1}, h_{k2} \ldots h_{kN}]$ is the k^{th} row of $\mathbf{H}_c$. $\mathbf{S}_k$ is an $N \times N$ diagonal matrix whose entries are a permutation of the L channels and $(N - L)$ zeros. In a full-rate code, $N = L$ and $\mathbf{S}_k$ is full-rank. The eigenvalues λ_j and eigenvectors $\mathbf{v}_j$ of the cumulant matrix, $\mathbf{C}_4^{[k]}$, are determined using the constraint of (13)

$$\begin{aligned}\mathbf{C}_4^{[k]}\mathbf{v}_j = \lambda_j \mathbf{v}_j &\Rightarrow \gamma_4 \mathbf{H_c S}_k \mathbf{H_c}^H \mathbf{v}_j - \lambda_j \mathbf{v}_j = 0, \quad (39) \\ &\Rightarrow \gamma_4 \mathbf{H_c}^H \mathbf{H_c S}_k \mathbf{H_c}^H \mathbf{v}_j - \mathbf{H_c}^H \lambda_j \mathbf{v}_j = 0, \\ &(\gamma_4 \mathbf{D}_N \mathbf{S}_k - \lambda_j \mathbf{I}_N)\mathbf{H_c}^H \mathbf{v}_j = 0, \\ &\Rightarrow (\gamma_4 \mathbf{F}_N - \lambda_j \mathbf{I}_N)\mathbf{H_c}^H \mathbf{v}_j = 0, \quad (40)\end{aligned}$$

where, $\mathbf{F}_N = \mathbf{D}_N \mathbf{S}_k$ is a $N \times N$ diagonal matrix with entries $f_j, j = 1, \ldots, N$. The eigenvalue $\lambda_j = \gamma_4 f_j$ reduces the rank of $\mathbf{F}_N$ by one and consequently $\mathbf{H_c}^H \mathbf{v}_j$ is proportional to $\mathbf{p}_j$, the j^{th} column of the size-N identity matrix, i.e.,

$$\mathbf{H_c}^H \mathbf{v}_j = \beta \mathbf{p}_j. \quad (41)$$

Substituting this result into (39) and using (38),

$$\begin{aligned}\gamma_4 \mathbf{H_c S}_k \beta \mathbf{p}_j &= \lambda_j \mathbf{v}_j, \\ \Rightarrow \mathbf{v}_j &= \frac{\beta \gamma_4 |h_{kj}|^2}{\lambda_j} \mathbf{H_c p}_j. \quad (42)\end{aligned}$$

Thus $\mathbf{p}_j$ extracts successive columns of the channel matrix $\mathbf{H}_c$ and the L channels can be identified from the eigenvectors of the cumulant matrix up to a scalar ambiguity: each eigenvector represents a scaled version of a column of $\mathbf{H_c}$. Since each column of $\mathbf{H}_c$ is a permutation of the length L channel, and possibly some zeros, each eigenvector is a scaled version of a permutation of the channel. ■

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