Optimal Relay-Subset Selection and Time-Allocation in Decode-and-Forward Cooperative Networks

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***Abstract*—We present the optimal relay-subset selection and transmission-time for a decode-and-forward, half-duplex cooperative network of arbitrary size. The resource allocation is obtained by maximizing over the rates obtained for each possible subset of active relays, and the unique time allocation for each set can be obtained by solving a linear system of equations. We also give a recursive algorithm which reduces the number of operations as well as the computational load of finding the required matrix inverses. Our results, in terms of outage rate, confirm the benefit of adding potential relays to a small network and the diminishing marginal returns for a larger network. Furthermore, optimizing over the channel resources ensures that more relays are active over a larger SNR range.**

I. Introduction

Cooperation has become a popular technique to implement diversity in the absence of multiple antennas at receiving and transmitting nodes [1–3]. In this context, resource allocation in cooperative networks has been investigated under many scenarios and metrics. In this paper, we address the problem of resource allocation, in terms of channel resources (time or bandwidth), in multi-relay networks with arbitrary connections. We describe the contributions of the paper in detail after a brief review of the pertinent literature.

For the *single-relay* case, several works have dealt with various aspects of resource allocation, in terms of power and/or bandwidth and time. Yao et al. determine the optimal power and time allocation for relayed transmissions specifically in the low-power regime [4]. Larsson and Cao present various strategies for allocating power and channel resources under energy constraints [5]. For the channel resource allocation problem, however, the authors consider selection combining only and do not address the scenario of joint decoding of the source and relay signals. The works in [6–8] address the problem of power and channel resource allocation under sum average power constraints. Optimal time and bandwidth allocation using instantaneous and average channel conditions is obtained using power control in [9]. Channel resource allocation using fractional frame slots under fixed power is developed in [10].

In networks with multiple relays, the available literature can be classified into two groups: networks where relays do not communicate with one another (parallel-relay networks), and networks without restrictions on relay communication (arbitrarily-connected networks). Resource allocation for the former has been addressed in [11–14]. Ibrahimi and Liang develop the optimal power allocation for a multi-relay cooperative OFDMA amplify-and-forward (AF) system [11]. By maximizing the channel mutual information, Anghel et al. find the optimal power allocation for multiple parallel relays in AF networks [12], [13]. A more general solution is given in [14] where the authors give the optimal power and channel resource allocation for a parallel-relay network with individual power constraints on the nodes.

To the best of our knowledge, channel resource allocation for arbitrarily-connected networks and dedicated multiple access has not been addressed. Works in the area of multi-relay systems with arbitrary links generally neglect the bandwidth penalty arising from multiple hops by assuming full-duplex nodes, a bandwidth-unconstrained system, or the availability of channel phase information at the transmitter [15–27].

These assumptions, however, are not realistic for practical wireless networks, where nodes are likely to be half-duplex, phase information is very difficult to obtain at the transmitter, and bandwidth is a scarce resource. To fill this void, in this paper we investigate the problem of resource allocation in a bandwidth-constrained, cooperative, decode-and-forward (DF), wireless network, and consider the most general setting where multiple relays can transmit and cooperate with each other. We address the joint problem of optimal selection of a relaying subset and allocation of time resources to the selected relays. The problem is framed in the context of mesh networks of relatively simple and inexpensive nodes. We concentrate on resource allocation in terms of transmission time only, removing power allocation from the optimization; furthermore, to reduce implementation complexity, we consider time-orthogonal transmissions.

This paper is structured as follows. Section II describes the system model. In Section III and IV we develop the proposed resource allocation scheme and present a significantly simplified recursive implementation. Simulation results are presented in Section V and concluding remarks are presented in Section VI.

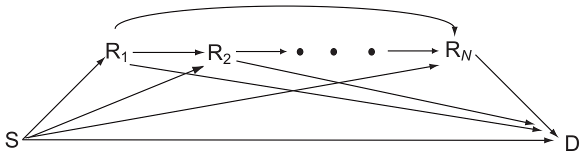

Fig. 1. Location of the relays with respect to the source and destination.

II. System Model

We consider a mesh network of static nodes comprising a source and destination node and N potential relays. The inter-node channel powers are denoted as $|a_{ij}|^2$, where i and j represent the source node s, relay nodes r_k, $k = 1 \dots N$, or the destination node d. They are assumed independent of each other and are modelled as flat, slowly-fading and exponential with parameter λ. λ is inversely proportional to the average channel power and is a function of inter-node distance, d_{ij}, through the path loss exponent p_a, e.g., $1/\lambda_{sd} = (1/d_{sd}^{p_a})$, and $1/\lambda_{r_k d} = (1/d_{r_k d}^{p_a})$. We do not include shadowing into the fading model, although this can easily be incorporated on an instantaneous basis. Because the nodes are static, the channels are assumed to change very slowly with time; we thus assume knowledge of all channel gains (although *not* channel phases) at some central decision-making node. With the aim of designing simple and cheap nodes, we assume half-duplex channels and time-orthogonal transmissions, which greatly simplifies receiver design. The relays are numbered as in Figure 1 such that relay r_j transmits after r_i if $j > i$. We also assume the DF cooperation strategy with independent codebooks, which allow for the optimization of system resources.

With these assumptions, the cooperation framework for the network is as follows. The half-duplex constraint precludes simultaneous transmissions on the same channel from a particular node, and the lack of forward-channel phase information precludes simultaneous transmissions from different nodes. Transmission between source and destination is thus divided into $N+1$ time-slots, of duration t_0, …, t_N, with $t_0 + \dots + t_N = 1$. In the first time-slot, the source transmits its data to all nodes. The first relay, r_1, decodes this data which the destination and remaining $N-1$ relays store for future processing. In the second slot, of duration t_1, the first relay re-transmits the data using an independent codebook, the second relay decodes the data from the first relay and the source, and the destination and remaining $N-2$ relays store the data for further processing. In general, each relay r_k decodes data from the source and from the previous relays $r_1 \dots r_{k-1}$ up to and including time-slot t_{k-1}. This process continues until all relays have transmitted and the destination attempts to decode the information. The overall rate is limited by the need for all decoding to succeed.

Assuming that each node uses power P and W Hz per transmission the signal to noise ratio (SNR) at node j resulting from transmission from node i can be written as $\mathrm{SNR}_{ij} = \frac{P}{N_0 W}|a_{ij}|^2$, where N_0 is the noise spectral density. In the rest of the paper, we use the short-hand notation L_{ij} to denote $\log_2(1 + \mathrm{SNR}_{ij})$, the capacity of the corresponding channel.

III. Optimal Time Allocation and Relay Selection

In this section, we solve the joint problem of time allocation and relay selection for the network discussed above, giving the values of t_i, $i = 0 \dots N$, that maximize the achievable rate between source and destination. Assuming that all relays are active, the mutual information at each relay and destination is

$$
\begin{aligned}
&I_1(t_0) = t_0 L_{sr_1},\\
&I_k(t_0,\dots,t_{k-1}) = t_0 L_{sr_k} + t_1 L_{r_1 r_k} + \dots + t_{k-1} L_{r_{k-1} r_k},\\
&\qquad\qquad\qquad\qquad\qquad\qquad k = 1,\dots,K\\
&I_D(t_0,\dots,t_k,\dots,t_{N-1},t_N) = t_0 L_{sd} + t_1 L_{r_1 d} + \dots\\
&\qquad\qquad + t_{k-1} L_{r_{k-1} d} + \dots + t_N L_{r_N d},
\end{aligned}
$$

where I_k and I_D denote the mutual information at relay r_k and the destination, respectively.

With all N relays cooperating, the maximum achievable rate under orthogonal transmissions is the minimum of the mutual information obtained at each individual relay node:

$$
\begin{aligned}
R_N = &\max_{t_0,\dots,t_N} \min\{\\
&I_1(t_0), I_2(t_0,t_1), \dots, I_{k-1}(t_0,\dots,t_{k-2}),\\
&I_k(t_0,\dots,t_{k-1}), I_{k+1}(t_0,\dots,t_k), \dots,\\
&I_N(t_0,\dots,t_k,\dots,t_{N-1}), I_D(t_0,\dots,t_k,\dots,t_{N-1},t_N)\},\\
&\text{s.t. } t_i \geq 0, \quad \forall i, t_0 + t_1 + \dots t_N \leq 1. \qquad (1)
\end{aligned}
$$

Consider the case with relay r_k removed from the network. The maximum achievable rate R_{N-1}^k becomes

$$
\begin{aligned}
R_{N-1}^k = &\max_{t_0,\dots,t_{k-1},t_{k+1},\dots t_N} \min\{I_1(t_0)\dots, I_{k-1}(t_0,\dots,t_{k-2}),\\
&I_{k+1}(t_0,\dots,t_{k-1}), \dots, I_D(t_0,\dots,t_{k-1},t_{k+1},\dots,t_N)\},\\
&\text{s.t. } t_i \geq 0, \forall i, t_0 + \dots t_{k-1} + t_{k+1} + \dots t_N \leq 1. \qquad (2)
\end{aligned}
$$

Removing relay r_k is thus equivalent to removing t_k and I_k from the optimization. Note that we use the subscript in R_{N-1}^k to denote the maximum number of *potentially* active relays, and the superscript to denote the relay removed. The maximum achievable rate can be written as the maximum of the rate obtained by using all N relays, and the rate obtained by successively removing each relay:

$$
R_T = \max\{R_N, R_{N-1}^1, R_{N-1}^2, \dots, R_{N-1}^N\}. \qquad (3)
$$

If $R_T = R_{N-1}^k$, the maximum rate can be obtained by iterating through (1) and (2), successively removing a relay each step. Note that obtaining R_{N-1}^k includes the cases where two or more relays are removed. In theory, therefore, all 2^N possible cases must be checked.

Let $(t_0^*, t_1^*, \dots, t_N^*)$ denote the time allocation that solves the optimization problem. The following proposition is an outline of the solution to the optimization problem in (1), (2) and (3).

Proposition 1: With a maximum number of potential relays N, the maximum achievable rate $R_T = R_N$ only if $t_k^* \neq 0$, $\forall k$. Otherwise, if $t_k^* = 0$, $R_T = R_{N-1}^k$.

Proof: With exactly N active relays, and with $k < n < N$, the resulting rate can be written explicitly as:

$$R_N = \max_{t_0,\ldots,t_N} \min\{(t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1r_2}), \ldots, (t_0 L_{sr_k} + \ldots + t_{k-1} L_{r_{k-1}r_k}), (t_0 L_{sr_n} + \ldots + t_{k-1} L_{r_{k-1}r_n} + t_k L_{r_kr_n} + t_{k+1} L_{r_{k+1}r_n} \ldots t_{n-1} L_{r_{n-1}r_n}), \ldots, (t_0 L_{sd} + \ldots + t_{k-1} L_{r_{k-1}r_d} + t_k L_{r_kr_d} + t_{k+1} L_{r_{k+1}r_d} + \ldots t_N L_{r_Nr_d})\}. \quad (4)$$

Setting $t_k = 0$ gives

$$R_N = \max_{t_0,\ldots,t_{k-1},t_{k+1},\ldots t_N} \min\{(t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1r_2}), \ldots, (t_0 L_{sr_k} + \ldots + t_{k-1} L_{r_{k-1}r_k}), (t_0 L_{sr_n} + \ldots + t_{k-1} L_{r_{k-1}r_n} + t_{k+1} L_{r_{k+1}r_n} \ldots t_{n-1} L_{r_{n-1}r_n}), \ldots, (t_0 L_{sd} + \ldots + t_{k-1} L_{r_{k-1}r_d} + t_{k+1} L_{r_{k+1}r_d} + \ldots t_N L_{r_Nr_d})\}$$
$$\leq \max_{t_0,\ldots,t_{k-1},t_{k+1},\ldots t_N} \min\{(t_0 L_{sr_1}), (t_0 L_{sr_2} + t_1 L_{r_1r_2}), \ldots, (t_0 L_{sr_n} + \ldots + t_{k-1} L_{r_{k-1}r_n} + t_{k+1} L_{r_{k+1}r_n} \ldots t_{n-1} L_{r_{n-1}r_n}), \ldots, (t_0 L_{sd} + \ldots + t_{k-1} L_{r_{k-1}r_d} + t_{k+1} L_{r_{k+1}r_d} + \ldots t_N L_{r_Nr_d})\} = R^k_{N-1}. \quad \blacksquare \quad (5)$$

To solve the optimization problem of (1) we thus require only the critical points for which $t_k^* \neq 0, \forall k$. In the following proposition, we show that for each R_N, i.e., given a set of potential relays, only one solution satisfies $t_k^* \neq 0, \forall k$.

Proposition 2: The unique solution to the minimization in (1) for which $t_k^* \neq 0, \forall k$ is given by $I_1(t_1) = I_2(t_1, t_2) = \ldots = I_N(t_1, \ldots, t_N) = I_D(t_1, \ldots, t_N)$.

Proof: Consider all possible critical points obtained from the optimization in (1). The points are obtained either by maximizing each individual term in (1) or by intersecting all possible combinations of the terms in (1). We show that the only solution leading to non-zero solutions results from intersecting every term in (1).

The critical points for the optimization problem can be obtained by solving the following:

1) Maximize the individual terms in (1) except $I_D(t_0, \ldots, t_N)$: $\forall k \leq N, \max_{t_0,\ldots,t_{k-1}} I_k(t_0, \ldots t_{k-1})$ s.t. $t_0 + \ldots + t_{k-1} \leq 1$. Because the optimization is not over $t_m, \forall k \leq m \leq N$, the solution to this problem clearly has all $t_m = 0, \forall k \leq m \leq N$, and thus cannot be a solution to the overall optimization problem.

2) Maximize $I_D(t_0, \ldots, t_N)$: $\max_{t_0,\ldots,t_N} I_D(t_0, \ldots t_N) = \max_{t_0,\ldots,t_N}\{t_0 L_{sd} + \ldots + t_N L_{r_Nr_d}\}$, s.t. $t_0 + \ldots + t_N \leq 1$. In this case, all variables are included in the optimization. It is easy to show, however, that this function is maximized by selecting the largest L value, i.e., evaluating the Kuhn-Tucker conditions leads to a solution of the form $t_m = 1, t_k = 0, \forall k \neq m$, where $m = \arg\max_k\{L_{sd}, L_{r_1d}, \ldots, L_{r_kd}, \ldots, L_{r_Nd}\}$. Therefore, this solution is also not a solution to the overall optimization problem.

3) Maximize the function that results from the intersection of all possible combinations of the functions I_k. Let $\mathcal{M}$ denote all possible subsets of $\{1 \ldots N\}$. $\mathcal{M}$ then contains 2^N such subsets, i.e., $|\mathcal{M}| = 2^N$. Consider one such subset $\delta_k = (m_1, m_2, \ldots, m_k)$, with $m_1 < m_2 < m_k$. One critical point then is $\max_{t_0,\ldots,t_{m_{k-1}}} I_{m_k}(t_0, \ldots t_{m_{k-1}})$ s.t $I_{m_1}(t_0, \ldots t_{m_1-1}) = I_{m_2}(t_0, \ldots t_{m_2-1}) = \ldots = I_{m_k}(t_0, \ldots t_{m_k-1})$. This optimization then gets repeated for all sets $\delta_k \in \mathcal{M}$. In all but one combination, this optimization is not over all the variables $\{t_0, \ldots t_N\}$. As in point (1), this maximization also leads to $t_k = 0$ for some value of k.

4) Maximize the intersection of all terms in (1):

$$I_1(t_0) = \ldots = I_N(t_0, \ldots, t_{N-1}) = I_D(t_0, \ldots, t_N). \quad (6)$$

This is the only case that leads to $t_k \neq 0, \forall k = 0 \ldots N$. $\blacksquare$

Essentially, this proposition shows that if all N relays are to contribute, all terms in the minimization in (1) must be equal. Therefore, if the optimal solution has $k < N$ relays, an expression like (1) can be written for those k relays. The linear system of equations in (6) has the solution

$$\mathbf{L_{N+1}} \mathbf{t}_{N+1} = \mathbf{1}_{N+1},$$
$$\Rightarrow \mathbf{t}_{N+1} = \frac{\mathbf{L}^{-1}_{N+1}\mathbf{1}_{N+1}}{||\mathbf{L}^{-1}_{N+1}\mathbf{1}_{N+1}||_1} = \frac{\mathbf{L}^{-1}_{N+1}\mathbf{1}_{N+1}}{\mathbf{1}^T_{N+1}\mathbf{L}^{-1}_{N+1}\mathbf{1}_{N+1}}, \quad (7)$$

where $||\mathbf{v}||_1$ denotes the sum of the elements of $\mathbf{v}$, i.e., the 1-norm. $\mathbf{1}_{N+1}$ is the length-$(N+1)$ vector of ones, $\mathbf{L}_{N+1}$ is the $(N+1) \times (N+1)$ rate matrix

$$\mathbf{L}_{N+1} = \begin{bmatrix} L_{sr_1} & 0 & 0 & \ldots & 0 \\ L_{sr_2} & L_{r_1r_2} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ L_{sd} & L_{r_1d} & L_{r_2d} & \ldots & L_{r_Nd} \end{bmatrix}, \quad (8)$$

and $\mathbf{t}_{N+1} = [t_0, t_1, \ldots, t_N]^T$ is the vector of time allocations. The solution in (7) does not guarantee that the constraint $t_k > 0 \quad \forall k = 0 \ldots N$ is satisfied. To ensure that all considered solutions satisfy the constraint, again consider the set $\mathcal{M}$. Each entry in the set corresponds to a rate matrix, $\mathbf{L}_m$, similar to that in (8), formed using the relays in that entry of the set. Furthermore, let $|m|$ denote the size of the rate matrix $\mathbf{L}_m$. A relay set and its corresponding solution, denoted as $\mathbf{t}_m$, is included as a potential solution if $\mathbf{t}_m$ satisfies the constraint, i.e., $\mathbf{t}_m > \mathbf{0}_{|m|}$, where $\mathbf{0}_{|m|}$ is the all-zero vector of size $|m|$, $\mathbf{0}_{|m|} = [0, 0, 0, \ldots 0]^T$ and the inequality operates on an element-by-element basis. Let the set $\mathcal{K}$ form the subset of $\mathcal{M}$ that comprises all potential solutions. Let $\mathbf{L}_k$, $\mathbf{t}_k$ and $|k|$ denote the rate matrix, its corresponding solution and size, respectively, for each entry of the set $\mathcal{K}$. Note that the number of active relays being considered in each entry is $|k|$ - 1. Finally, the optimum solution for the time allocation vector, $\mathbf{t}^*$, can be obtained by solving (7) for all possible combinations of active relays in the set $\mathcal{K}$ i.e.,

$$\mathbf{t}^* = \max_{\mathcal{K}} \frac{\mathbf{L}_k^{-1}\mathbf{1}_{|k|}}{\mathbf{1}^T_{|k|}\mathbf{L}_k^{-1}\mathbf{1}_{|k|}}, \forall k = 1, \ldots, |\mathcal{K}|. \quad (9)$$

We note here that in arbitrary networks, some links between the nodes in the network may be unavailable. The approach to the optimization problem in this case is the same as for the fully-connected network, with the exception that the rate matrix $\mathbf{L}_{N+1}$ may not be invertible, in which case the corresponding solution is inadmissable. The remaining steps remain unchanged.

IV. Implementation with Reduced Complexity

The solution to the optimization problem in (1), (2) and (3) involves checking 2^N potential solutions, each involving the inverse of a rate matrix. In this section, we decrease the complexity of the optimization using a recursive approach. The recursive solution, which exploits the special structure of the rate matrix, greatly simplifies the matrix inversion, as well as reduces the number of possible solutions to check. Essentially, while the solution in Section III was a top-down approach, the approach we suggest here is bottom-up.

Consider a set of p relays, $\mathcal{P} = \{r_1, r_2, \ldots, r_p\}, p \geq 0$, and its corresponding rate matrix $\mathbf{L}_{p+1}^{\mathcal{P}}$, solution vector $\mathbf{t}_{p+1}^{\mathcal{P}}$ and maximum rate (if available) $R^{\mathcal{P}}$. We note that if $p = 0$ and the set is empty, the rate matrix and solution vector are constants, L_{sd} and 1, respectively. Denote as $\mathcal{P}'$ the set $\mathcal{P}$ appended with another relay, i.e., $\mathcal{P}' = \{r_1, r_2, \ldots, r_p, r_{p+1}\}$. Denote as $\mathbf{L}_{p+2}^{\mathcal{P}'}$, $\mathbf{t}_{p+2}^{\mathcal{P}'}$, and $R^{\mathcal{P}'}$ the matrix, solution vector and rate corresponding to set $\mathcal{P}'$.

Proposition 3: Given $\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}$, $\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}$ can be obtained with computational complexity order of $O(p^2)$

Proof: For $p \geq 0$, the rate matrix $\mathbf{L}_{p+2}^{\mathcal{P}'}$ can be written as

$$\mathbf{L}_{p+2}^{\mathcal{P}'} = \left[\begin{array}{c|c} \mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p) & \mathbf{0}_{p\times 2} \\ \hline \mathbf{F}_{2\times p} & \mathbf{T}_2 \end{array}\right], \quad (10)$$

where $\mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p)$ denotes the first p rows and columns of the rate matrix $\mathbf{L}_{p+1}^{\mathcal{P}}$, $\mathbf{0}_{p\times 2}$ is a $(p \times 2)$ matrix of zeros, $\mathbf{T}_2$ is a (2×2) lower- triangular matrix, and $\mathbf{F}_{2\times p}$ is a $(2 \times p)$ fully-loaded matrix. Note that $\mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p)$ is triangular. Using the inverse of a partitioned matrix [28], $\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}$ can be written as

$$\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1} = \left[\begin{array}{c|c} \left(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p)\right)^{-1} & \mathbf{0}_{p\times 2} \\ \hline -\mathbf{T}_2^{-1}\mathbf{F}_{2\times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p)\right)^{-1} & \mathbf{T}_2^{-1} \end{array}\right].$$

Note that $\left(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p)\right)^{-1}$ is the inverse of a partition of the triangular matrix $\mathbf{L}_{p+1}^{\mathcal{P}}$. Using the inverse of a partitioned matrix one more time, however, it is easy to see that

$$\left(\mathbf{L}_{p+1}^{\mathcal{P}}(1:p,1:p)\right)^{-1} = (\mathbf{L}_{p+1}^{\mathcal{P}})^{-1}(1:p,1:p), \quad (11)$$

and thus

$$\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1} = \left[\begin{array}{c|c} (\mathbf{L}_{p+1}^{\mathcal{P}})^{-1}(1:p,1:p) & \mathbf{0}_{p\times 2} \\ \hline -\mathbf{T}_2^{-1}\mathbf{F}_{2\times p}(\mathbf{L}_{p+1}^{\mathcal{P}})^{-1}(1:p,1:p) & \mathbf{T}_2^{-1} \end{array}\right], \quad (12)$$

and hence obtaining $\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}$ is an $O(p^2)$ operation. ■

Using this above proposition, the solution vector $\mathbf{t}_{p+2}^{\mathcal{P}'}$ of $\mathbf{L}_{p+2}^{\mathcal{P}'}$ can be obtained from the solution vector $\mathbf{t}_{p+1}^{\mathcal{P}}$ of $\mathbf{L}_{p+1}^{\mathcal{P}}$:

$$\mathbf{t}_{p+2}^{\mathcal{P}'} = \frac{\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}\mathbf{1}_{p+2}}{\mathbf{1}_{p+2}^T\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}\mathbf{1}_{p+2}} = \left[\begin{array}{c} \mathbf{t}_{p+1}^{\mathcal{P}}(1:p) \\ \hline \mathbf{t}_{p+2}^{\mathcal{P}'}(p+1) \\ \mathbf{t}_{p+2}^{\mathcal{P}'}(p+2) \end{array}\right], \quad (13)$$

where $\mathbf{t}_{p+1}^{\mathcal{P}}(1:p)$ represents the first p entries of the already-calculated solution vector $\mathbf{t}_{p+1}^{\mathcal{P}}$, and $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+1)$ and $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+2)$ are the last two entries of the solution vector $\mathbf{t}_{p+2}^{\mathcal{P}'}$ that remain to be calculated. $R^{\mathcal{P}'} = \frac{1}{\mathbf{1}_{p+2}^T\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}\mathbf{1}_{p+2}}$ is the maximum achievable rate obtained using the set $\mathcal{P}'$ of relays. The last two entries of the solution vector $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+1)$ and $\mathbf{t}_{p+2}^{\mathcal{P}'}(p+2)$ can be written as

$$\begin{bmatrix} \mathbf{t}_{p+2}^{\mathcal{P}'}(p+1) \\ \mathbf{t}_{p+2}^{\mathcal{P}'}(p+2) \end{bmatrix} = R^{\mathcal{P}'} \times$$
$$\left[-\mathbf{T}_2^{-1}\mathbf{F}_{2\times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(1:p,1:p) \,\middle|\, \mathbf{T}_2^{-1}\right]\mathbf{1}_{(p+2)\times 1} \quad (14)$$

with a corresponding achievable rate $R^{\mathcal{P}'}$ given by

$$R^{\mathcal{P}'} = \frac{1}{\mathbf{1}_{p+2}^T\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}\mathbf{1}_{p+2}} = \left(\sum_{ij}\left(\mathbf{L}_{p+2}^{\mathcal{P}'}\right)^{-1}(i,j)\right)^{-1},$$
$$= \Big[\sum_{i,j}\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(i,j) - \quad (15)$$
$$\sum_{i,j}\mathbf{T}_2^{-1}\mathbf{F}_{2\times p}\left(\mathbf{L}_{p+1}^{\mathcal{P}}\right)^{-1}(1:p,1:p)(i,j) + \sum_{ij}\mathbf{T}_2^{-1}(i,j)\Big]^{-1},$$

where we use $\sum_{i,j}\mathbf{A}(i,j)$ to denote the summation over all the elements of matrix $\mathbf{A}$.

Using the above, the optimization problem for a network of N potential relays can be solved recursively as follows:

1) Determine the set of all potential relay combinations. Sequence the set as:

$$\mathcal{M} = \{(r_1), (r_1, r_2), (r_1, r_2, r_3), \ldots (r_1, r_2, \ldots, r_N), \ldots$$
$$(r_1, r_3), (r_1, r_3, r_4), \ldots, (r_1, r_3, \ldots, r_N), \ldots (r_1, r_N),$$
$$(r_2), (r_2, r_3), (r_2, r_3, r_4), \ldots (r_2, r_3, \ldots, r_N), (r_2, r_4),$$
$$(r_2, r_4, r_5), \ldots, (r_2, r_4, \ldots, r_N), \ldots (r_2, r_N), \ldots (r_{N-1}, r_N)\}.$$

   Note that each "row" of $\mathcal{M}$ is a subset of relay combinations in which each element is formed from the previous element by adding a relay.

2) In each "row", obtain the rate matrix, its respective optimized time allocation vector and achievable rate for each element (i.e., relay combination) recursively using (12), (13), (14) and (15).

3) Check that for each particular set $\mathcal{P}$ of p relays, the solution $\mathbf{t}_p$ and achievable rate R_p satisfies the constraints $R^{\mathcal{P}} \geq 0$, and $\mathbf{t}_{p+1}^{\mathcal{P}} > \mathbf{0}_{p+1}$. If both constraints are satisfied, place the solution in the potential set of valid solutions $\mathcal{K}$, advance elements and return to step (1). If $\mathbf{t}_{p+1}^{\mathcal{P}} > \mathbf{0}_{p+1}$ is not satisfied, check which element of the the allocation vector $\mathbf{t}_p$ does not satisfy the constraint.
   - If any of the first $(p-1)$ entries of $\mathbf{t}_p$ are less than zero, i.e., $\mathbf{t}_p(1:p-1 < \mathbf{0}_{p-1})$, this constraint will not be satisfied for any other relay combinations in this "row". Advance rows and return to item (1).
   - If the constraint is not satisfied by either of the last two items in the solution vector, discard the solution but check the other elements in the "row".

4) From the set $\mathcal{K}$, pick the highest achievable rate and its corresponding time allocation.

The recursive algorithm given above simplifies the optimization problem in two ways:

1) It reduces the computation load of determining successive matrix inverses by writing each matrix inverse as a function of another, already known, matrix inverse, and two other matrices obtained through simple matrix multiplication. It is straightforward to show that the resulting complexity order of calculating each rate and solution vector is $O(q^2)$. Without the recursion, this complexity is of order $O(q^3)$, resulting from the inverse of the rate matrix. The recursion thus introduces significant savings in terms of complexity.

2) It may eliminate infeasible solutions by discarding relay combinations which do not satisfy constraints. For example, if the relay combination (r_1, r_2, r_3) does not satisfy the constraints, the combination (r_1, r_2, r_3, r_4) may be automatically discarded.

V. Simulations

In this section, we present results of the resource allocation scheme discussed in Section III for networks with 1 to 6 relays arranged linearly. The figure of merit is the achievable rate R_a with an outage probability of 10^{-3}, i.e. $\Pr[R^* < R_a] = 10^{-3}$. A closed form expression for the outage probability of optimized cooperation is very complicated and beyond the scope of the paper. The outage probability and rate are thus obtained numerically.

The relays are equispaced on a line between the source and destination, as in Figure 1, and we use an attenuation exponent of $p_a = 2.5$. This choice is motivated by the application of static mesh-nodes installed on posts; transmissions between such nodes should undergo little shadowing and a lower attenuation exponent. From 60000 fading realizations we obtain the cumulative density function of the instantaneous rate $F_R(r)$. The outage rate is the rate for which the probability of outage is 10^{-3}, i.e., $F_R^{-1}(10^{-3})$.

Figures 2 and 3 plot the outage rate as a function of the average end-to-end SNR, $\frac{P}{N_0 W}$, for optimized and non-optimized cooperation, respectively. As expected the rate increases with increasing number of potential relays. The rate for the optimized cooperation is obtained from (9). Non-optimized cooperation uses equal time allocation, i.e., the rate for a particular relay set is simply the minimum of the mutual information at each node. Non-optimized cooperation, however, does optimally select relays by choosing the best, in terms of rate, of the 2^N relay combinations. Comparing Figure 2 and Figure 3 shows that optimizing resources increases rates significantly, as expected. The outage rate increases as a function of nodes available to relay. We also note the typical phenomenon of decreasing marginal returns: the gains of adding each additional relay decreases with increasing number of relays.

Figures 4 and 5 show the average number of relays that are active from the set of potential relays for optimized and non-optimized cooperation. For each network size, this number is a decreasing function. Interestingly, the number of active relays decreases much faster for non-optimized as compared to optimized cooperation, suggesting that optimizing resources distributes the relaying burden more effectively.

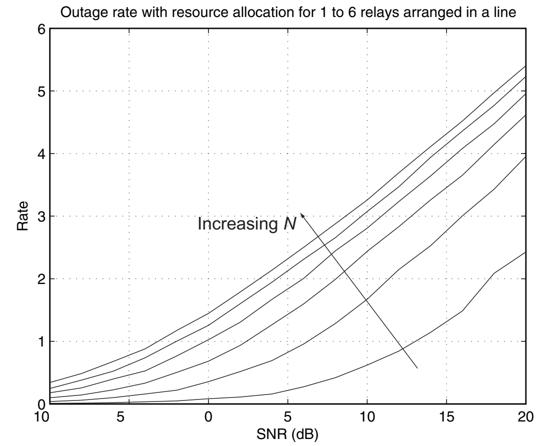

Fig. 2. Outage rate vs. SNR using $1, \dots 6$ potential relays and with resource allocation.

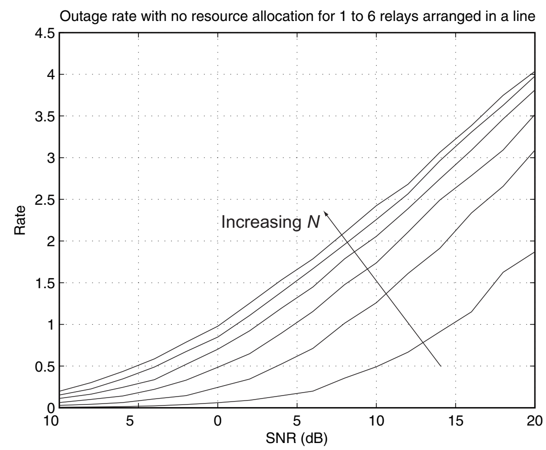

Fig. 3. Outage rate vs. SNR using $1, \dots 6$ potential relays and without resource allocation.

VI. Conclusions

In this paper, we determined the optimal channel resource allocation, in terms of time allocation, for the N-node cooperative diversity, multihop network using DF and independent codebooks. For a particular network, i.e., set of potential relays, the unique solution for a particular relay numbering scheme is obtained by taking the inverse of the triangular rate matrix, and the optimal solution is found by maximizing over the rate for each possible network, given its maximum size. We show that by exploiting the special structure of the rate matrix, the optimization can be performed in a recursive fashion which decreases the computation load of the rate matrix inverse and the number of required iterations. Node selection is inherent to the optimization strategy. Simulation

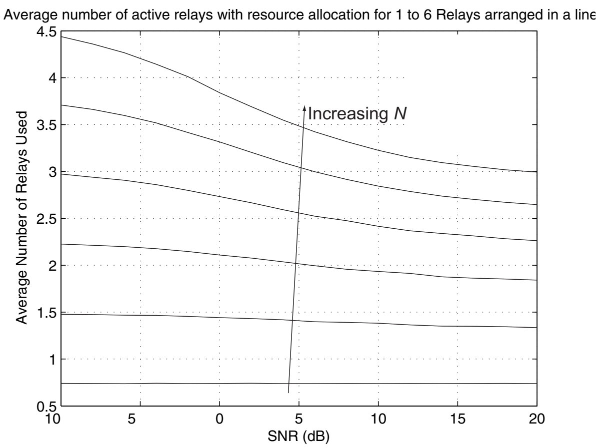

Fig. 4. Average number of active relays with $1, \ldots 6$ potential relays and with resource allocation.

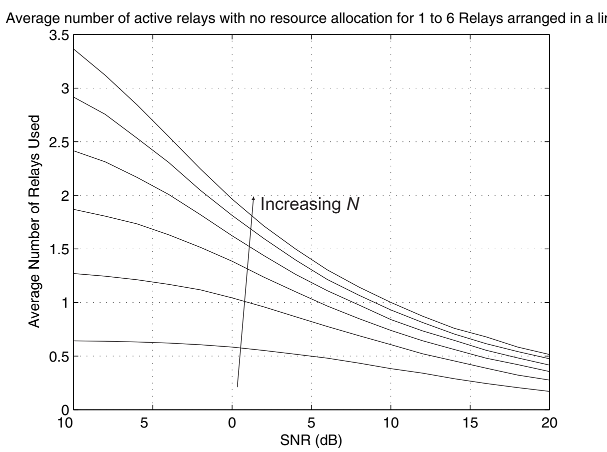

Fig. 5. Average number of active relays with $1, \ldots 6$ potential relays and without resource allocation.

results show significant gains in achievable rate due to resource allocation, but diminishing marginal returns as a function of network size.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity - part I, II," *IEEE Trans. Commun.*, vol. 51, pp. 1927 – 1948, November 2003.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2415 – 2425, October 2003.

[3] E. Beres and R. Adve, "Selection cooperation in multi-source cooperative networks," *IEEE Trans. Wireless Communications*, vol. 7, pp. 118 – 127, Januaryr 2008.

[4] Y. Yao, X. Cai, and G. B. Giannakis, "On energy efficiency and optimum resource allocation of relay transmissions in the low-power regime," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 2917– 2927, November 2005.

[5] E. G. Larsson and Y. Cao, "Collaborative transmit diversity with adaptive radio resource and power allocation," *IEEE Commun. Lett.*, vol. 9, pp. 511–513, June 2005.

[6] D. Gunduz and E. Erkip, "Opportunistic cooperation by dynamic resource allocation," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1446 – 1454, April 2007.

[7] J. Yang, D. Gunduz, D. Brown III, and E. Erkip, "Resource allocation for cooperative relaying," in *Proc. of Conference on Information Sciences and Systems (CISS 2008)*, March 2008.

[8] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inf. Theory*, vol. 51, pp. 2020 – 2040, June 2005.

[9] L. Xie and X. Zhang, "Opportunistic cooperation for quality of service provisionings over wireless relay networks," in *Proc. of IEEE Int. Conf. on Communications*, June 2007.

[10] Y. Ning, T. Hui, C. Shasha, and Z. Ping, "An adaptive frame resource allocation strategy for TDMA-based cooperative transmission," *IEEE Commun. Lett.*, vol. 11, pp. 417 – 419, May 2007.

[11] M. Ibrahimi and B. Liang, "Efficient power allocation in cooperative OFDM system with channel variation," in *IEEE International Conference on Communications (ICC)*, May 2008.

[12] P. A. Anghel, M. Kaveh, and Z. Q. Luo, "Optimal relayed power allocation in interference-free non-regenerative cooperative systems," in *Proc. of IEEE Workshop on Signal Processing Advances in Wireless Communications*, July 2004.

[13] ——, "An efficient algorithm for optimum power allocation in a decode-and-forward cooperative system with orthogonal transmissions," in *Proc. of the IEEE International Conference on Acoustics, Speech and Signal Processing ICASSP*, May 2006.

[14] Y. Liang, V. Veeravalli, and H. V. Poor, "Resource allocation for wireless fading relay channels: Max-min solution," *IEEE Trans. Inf. Theory*, vol. 53, pp. 3432 – 3453, October 2007.

[15] J. Boyer, D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, pp. 1820 – 1830, October 2004.

[16] A. K. Sadek, W. Su, and K. J. R. Liu, "Multinode cooperative communications in wireless networks," *IEEE Trans. Signal Process.*, vol. 55, no. 1, pp. 341–355, January 2007.

[17] Z. Yang and A. Host-Madsen, "Routing and power allocation in asynchronous Gaussian multiple-relay channels," *EURASIP Journal on Wireless Communications and Networking*, 2006.

[18] J. Zhang and T. M. Lok, "Performance comparison of conventional and cooperative multihop transmission," in *Proc. of the Wireless Communications and Networking Conference (WCNC)*, April 2008.

[19] S.-H. Chen, U. Mitra, and B. Krishnamachari, "Cooperative communication and routing over fading channels in wireless sensor network," in *Proc. of the IEEE International Conference on Wireless Networks,Communications, and Mobile Computing (WirelessCom)*, June 2005.

[20] X. Fang, T. Hui, Z. Ping, and Y. Ning, "Cooperative routing strategies in Ad Hoc networks," in *Proc. of the IEEE Vehicular Technology Conference (VTC-Spring 2005)*, June 2005.

[21] A. Khandani, J. Abounadi, E. Modiano, and L. Zhang, "Cooperative routing in wireless networks," in *Proc. of Allerton Conference on Communications, Control and Computing*, October 2006.

[22] F. Li, A. Lippman, and K. Wu, "Minimum energy cooperative path routing in wireless networks: An integer programming formulation," in *Proc. of the 63th IEEE Vehicular Technology Conference (IEEE VTC '2006)*, May 2006.

[23] Y. Yuan, Z. He, and M. Chen, "Virtual MIMO-based cross-layer design for wireless sensor networks," *IEEE Transactions on Vehicular Technology*, vol. 55, pp. 856– 864, May 2006.

[24] Y. Yuan, M. Chen, and T. Kwon, "A novel cluster-based cooperative MIMO scheme for multi-hop wireless sensor networks," *EURASIP Journal on Wireless Communications and Networking*, vol. 2006, pp. Article ID 72 493, 9 pages, 2006.

[25] V. Srinivasan, P. Nuggehalli, C.-F. Chiasserini, and R. Rao, "An analytical approach to the study of cooperation in wireless Ad Hoc networks," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 722 – 733, March 2005.

[26] L. Ong and M. Motani, "Optimal routing for decode-and-forward based cooperation in wireless networks," in *Proc. of the IEEE Conference on Sensor, Mesh and Ad Hoc Communications and Networks (SECON)*, June 2007.

[27] A. Aksu and O. Ercetin, "Reliable multi-hop routing with cooperative transmissions in energy-constrained networks," *IEEE Trans. Commun.*, vol. 7, no. 8, pp. 2861–2865, August 2008.

[28] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.