A HYBRID D^3-SIGMA DELTA STAP ALGORITHM IN NON-HOMOGENEOUS CLUTTER

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Abstract

This paper presents a knowledge based hybrid algorithm using Sigma-Delta STAP which is practical and powerful in non-homogeneous environments. In the hybrid algorithm, statistical and non-statistical direct data domain (D^3) algorithms are combined to obtain the advantages of both approaches. In this paper a new revised D^3 algorithm which uses a maximum SINR strategy is presented. The residual interference after the D^3 process is further suppressed by the efficient ΣΔ STAP algorithm. The performance of the hybrid algorithm using D^3 – ΣΔ STAP is tested in SIRV clutter environment and compared to that of the method which employees JDL as a statistical algorithm.

1 Introduction

It is now well accepted that several fundamental issues make it impossible to implement the fully optimum space-time adaptive processing (STAP) algorithm in practical radar systems. The most obvious is the very high required computation load. However the fundamental limitation is one of limited available training. The Reed-Mallet-Brennan rule states that a reasonably accurate estimate of the interference covariance matrix requires twice the degrees of freedom (DoF). In fully adaptive STAP, the available DoF is usually too large and an adequate amount of training data is not available. Importantly, this training data is required to be statistically homogeneous, a requirement that is almost impossible to satisfy in practice. Several recent works have dealt with the issue of non-homogeneous data [2, 6-9,11]. In this regard an interesting proposal has been the development of a non-statistical direct data domain algorithm (D^3) that does not require training [12]. In [2], a hybrid algorithm was proposed, combining a D^3 algorithm with traditional, statistical, joint-domain localized (JDL) processing to achieve the benefits of both. The impact of such processing with measured data is illustrated in [11].

This paper addresses two issues raised by the two-stage hybrid algorithm as currently available in [2]. The first D^3 stage is based on a maximizing the difference between the gain on target and an interference term. Maximizing this difference leads to unstable solutions requiring a good choice of an emphasis parameter weighing one term versus the other. Here the D^3 algorithm is reformulated to maximize the signal to interference plus noise ratio (SINR) instead. While a simple extension of the available algorithm, the reformulation stabilizes the D^3 solution.

The other issue with the original hybrid algorithm is that of computation load. In the JDL stage with η_a angle bins and η_d Doppler bins in the localized processing region (LPR), the D^3 algorithm must be executed $\eta_a\eta_d$ times for each angle-Doppler bin within the LPR. The required computation load is therefore very high. In [11] this problem is addressed in a knowledge based approach by using a JDL-based non-homogeneity detector (NHD) and using the hybrid algorithm only within those range bins declared non-homogeneous.

The main contribution of this paper is a hybrid ΣΔ algorithm using the extremely computationally efficient statistical ΣΔ algorithm of [3]. The overall approach is therefore stable and efficient.

This paper is organized as follows. In Section 2 we describe the use of a NHD, the revised D^3 algorithm and the extension to the hybrid ΣΔ algorithm. Section 3 presents numerical results illustrating the efficacy of the proposed hybrid ΣΔ approach. The numerical simulations are based on the spherically invariant random variable/process (SIRV/SIRP) model of [7]. Finally, Section 4 presents some conclusions and suggestions for future directions.

2 Knowledge Based Hybrid algorithm

Typically most common environments in which STAP operates are non-homogeneous because of many complicated factors. The traditional statistical algorithms which use covariance matrix to determine the adaptive weights are not applicable in these environments.

Non-homogeneity occurs in two forms. One, variation of interference statistics results in inappropriate secondary data support and an inability to obtain an accurate estimate of the interference in the cell under test. To overcome the resulting performance degradation, a NHD is used to identify non-homogeneous range cells. Once identified these non-homogeneous range cells can be removed from the secondary data used to estimate interference statistics. The other form is discrete interferers, such as coherent repeat jammers or other local interference sources, which exist in the primary range cell. Since the secondary data has no information about these interferences, traditional statistical algorithms are not able to suppress discrete non-homogeneities. The D^3 algorithm specifically targets this form of interference.

2.1 D^3 algorithm

Consider an *N*-element uniformly spaced array with *M* pulses in a coherent processing interval (CPI). For a look direction of ϕ_s, the signal advances from one element to the next by the same phase factor z_s=[exp($j2\pi$sin(ϕ_t))]. Defining as x_{nm} the signal received at the n-th element and m-th pulse *within the range cell under test*, the term ($x_{nm} - z_s^{-1}x_{(n+1)m}$) is free of the target and contains only interference terms. The D^3 algorithm minimizes the power in such interference terms while maintaining gain in the direction of the target.

To best present the D^3 algorithm, the data from the *N* elements due to the *M* pulses in a CPI is written as a $N\times M$ matrix **X** whose m^{th} column corresponds to the *N* returns from the m^{th} pulse. The data matrix is a sum of target and interference terms

$$\mathbf{X} = \alpha_s \mathbf{S}(\phi_t, f_t) + \mathbf{C} + \mathbf{N}. \qquad (1)$$

where α_s is the target amplitude and $\mathbf{S}(\phi_t, f_t)$ is the space-time steering matrix corresponding to a look direction of ϕ_t and look Doppler of f_t. Define the matrix

$$A = \begin{bmatrix} \mathbf{X}_{00} - z_s^{-1}\mathbf{X}_{10} & \mathbf{X}_{10} - z_s^{-1}\mathbf{X}_{20} & \cdots & \mathbf{X}_{(N-2)0} - z_s^{-1}\mathbf{X}_{(N-1)0} \\ \mathbf{X}_{01} - z_s^{-1}\mathbf{X}_{11} & \mathbf{X}_{11} - z_s^{-1}\mathbf{X}_{21} & \cdots & \mathbf{X}_{(N-2)1} - z_s^{-1}\mathbf{X}_{(N-1)1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{X}_{0(M-1)} - z_s^{-1}\mathbf{X}_{1(M-1)} & \mathbf{X}_{1(M-1)} - z_s^{-1}\mathbf{X}_{2(M-1)} & \cdots & \mathbf{X}_{(N-2)(M-1)} - z_s^{-1}\mathbf{X}_{(N-1)(M-1)} \end{bmatrix} \qquad (2)$$

and define the length *N*-1 spatial steering vector **a**(0:*N*-2) and length *N*-1 spatial weight vector $\mathbf{w}_s$. Both vectors are length *N*-1 due to the one DoF lost to the subtraction operation in the elements of the matrix **A.** The original D^3 algorithm maximizes the difference between gain on target and the interference:

$$\begin{aligned} R_{\mathbf{w}_s} &= T_{ws} - I_{ws} \\ &= \mathbf{w}_s^H \mathbf{a}_{(0:N-2)} \mathbf{a}_{(0:N-2)}^H \mathbf{w}_s - \kappa^2\, \mathbf{w}_s^H \mathbf{A}^T \mathbf{A}^* \mathbf{w}_s \end{aligned} \qquad (3)$$

where κ represents a parameter to emphasize one or the other term. The performance of the algorithm is very sensitive to the choice of this parameter, though no rigorous approach was made available.

In this paper we reformulate the D^3 algorithm using the simple and well known concept of SINR maximization [4]. I_{ws} in Equation (3) can be considered as interference power after spatial filtering. The target signal power can be defined as

$$\begin{aligned} T_{ws} &= \left\| \mathbf{w}_s^H (\alpha_s \mathbf{a}(0:N-2) \right\|^2, \\ &= |\alpha_s|^2\, \mathbf{w}_s^H \mathbf{a}(0:N-2)\mathbf{a}^H(0:N-2)\mathbf{w}_s. \end{aligned} \qquad (4)$$

When we use these two powers, the effective SINR can be defined as Equation (5)

$$\mathrm{SINR} = \frac{T_{ws}}{I_{ws}} = \frac{|\alpha_s|^2\, \mathbf{w}_s^H \mathbf{a}\mathbf{a}^H \mathbf{w}_s}{\mathbf{w}_s^H \mathbf{A}^T \mathbf{A}^* \mathbf{w}_s} \qquad (5)$$

The spatial weight vector $\mathbf{w}_s$ that maximizes this SINR is the optimal weight vector in this algorithm.

In the D^3 algorithm, assuming $\mathbf{A}^T\mathbf{A}^*$ is non-singular then weight vector can be obtained as $(\mathbf{A}^T\mathbf{A}^*)^{-1}\mathbf{a}$. If this matrix is singular then the optimal weight vector is the solution to a generalized eigenvalue problem. The overall algorithm is, therefore, very simple and the emphasis parameter κ is not required.

The temporal weights $\mathbf{w}_t$ can be determined in a similar manner using the matrix **B** and **b**(0:*M*-2), the temporal steering vector and the matrix **B** be defined as

$$B = \begin{bmatrix} \mathbf{X}_{00} - z_t^{-1}\mathbf{X}_{01} & \mathbf{X}_{01} - z_t^{-1}\mathbf{X}_{02} & \cdots & \mathbf{X}_{0(M-2)} - z_t^{-1}\mathbf{X}_{0(M-1)} \\ \mathbf{X}_{10} - z_t^{-1}\mathbf{X}_{11} & \mathbf{X}_{11} - z_t^{-1}\mathbf{X}_{12} & \cdots & \mathbf{X}_{1(M-2)} - z_t^{-1}\mathbf{X}_{1(M-1)} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{X}_{(N-1)0} - z_t^{-1}\mathbf{X}_{(N-1)1} & \mathbf{X}_{(N-1)1} - z_t^{-1}\mathbf{X}_{(N-1)2} & \cdots & \mathbf{X}_{(N-1)(M-2)} - z_t^{-1}\mathbf{X}_{(N-1)(M-1)} \end{bmatrix} \qquad (6)$$

Once we obtain the length *N*-1 spatial weight vector $\mathbf{w}_s$ and the length *M*-1 temporal weight vector $\mathbf{w}_t$ the length-*MN* space time adaptive weight vector is given by

$$\mathbf{w}(\phi_t, f_t) = \begin{bmatrix} \mathbf{w}_t \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \mathbf{w}_s \\ 0 \end{bmatrix} \qquad (7)$$

2.2 Hybrid Algorithm using ΣΔ STAP

In this section two stage hybrid algorithm will be described which uses ΣΔ STAP as statistical algorithm in second stage. Two stage hybrid algorithms have advantages of both non-statistical algorithm and statistical algorithm. The first stage of the hybrid algorithm uses D^3 algorithm to suppress discrete interferers in the cell under test. In this stage, the D^3 weights also transforms the space-time domain signal to the angle/Doppler domain which will be used in second stage [2,11]. By using the ΣΔ STAP as a second stage statistical algorithm, most processes are same as in the original hybrid-JDL algorithm. However the transformed LPR must be fitted to ΣΔ STAP. To illustrate this process, first of all, the sum and difference beamforming vectors to be used in the ΣΔ algorithm must be defined. In the original ΣΔ algorithm, the sum and difference beamforming vectors can be defined as

$$\begin{aligned} \mathbf{s}(\mathrm{u}_C) &= \{\mathbf{a}(u_L) + \mathbf{a}(u_R)\} \\ \mathbf{d}(\mathrm{u}_C) &= \{\mathbf{a}(u_L) - \mathbf{a}(u_R)\} \end{aligned} \qquad (8)$$

where $u_L = u_c - 1/(2N)$, $u_R = u_c + 1/(2N)$ and $\mathbf{a}(u)$ is the spatial steering vector for direction u.

As described in the above equations, the sum and difference beams can be formed using left and right beams. When we know the process to form sum and difference beams, the transformation matrix in two stage hybrid algorithm can be defined easily.

In this paper the corresponding beams are formed using D[3] weight vectors for u_L , u_R and the corresponding Doppler frequency f_{-1}, f_0, f_1 in LPR (if we use 3 Doppler bins). The set of six weight vectors form the columns of the transformation matrix for the hybrid ΣΔ STAP. For instance when we get D[3] weight vectors for left and right direction corresponding to f_{-1}

$$\begin{aligned} \mathbf{w}_L(f_{-1}) &= \mathbf{w}(f_{-1}, u_L) \\ \mathbf{w}_R(f_{-1}) &= \mathbf{w}(f_{-1}, u_R) \end{aligned} \quad (9)$$

we can define sum and difference beam weights as

$$\begin{aligned} \mathbf{w}_S(f_{-1}) &= \mathbf{w}_L(f_{-1}) + \mathbf{w}_R(f_{-1}) \\ \mathbf{w}_D(f_{-1}) &= \mathbf{w}_L(f_{-1}) - \mathbf{w}_R(f_{-1}) \end{aligned} \quad (10)$$

Then the transform matrix **T** for ΣΔ STAP defined as:

$$\mathbf{T} = [\mathbf{w}_S(f_{-1}) \quad \mathbf{w}_D(f_{-1}) \quad \mathbf{w}_S(f_0) \quad \mathbf{w}_D(f_0) \quad \mathbf{w}_S(f_1) \quad \mathbf{w}_D(f_1)\ ] \quad (11)$$

The space time domain data is transformed to the LPR in the angle Doppler domain which is appropriate to ΣΔ STAP using this transformation matrix **T.** After domain transformation, this statistical algorithm is applied to the data, then we can complete the hybrid ΣΔ algorithm.

3 Simulation Results

The first simulation illustrates the performance of the D[3] algorithm which uses a SINR maximization strategy. Table 1 lists the parameters used in this simulation. The angle plot of Figure 1 represents that this D[3] algorithm also places a null in the direction of discrete interferer and jammer. The temporal beam pattern shows a null in the direction of the mainbeam clutter.

Parameter	Value	Parameter	Value
Element(N)	18	Pulse(M)	18
Element spacing	$\lambda/2$	PRF	300Hz
Array transmit Azimuth	Uniform	Main beam Look Direction	0deg
Jammer angle	-20 deg	JNR	40dB
Target Doppler frequency	100Hz	SNR(target)	0dB
Interferer Doppler frequency	100Hz	Interferer power	10dB
Interferer angle	-51deg	β (clutter slope)	1

Table 1: Parameters for simulation

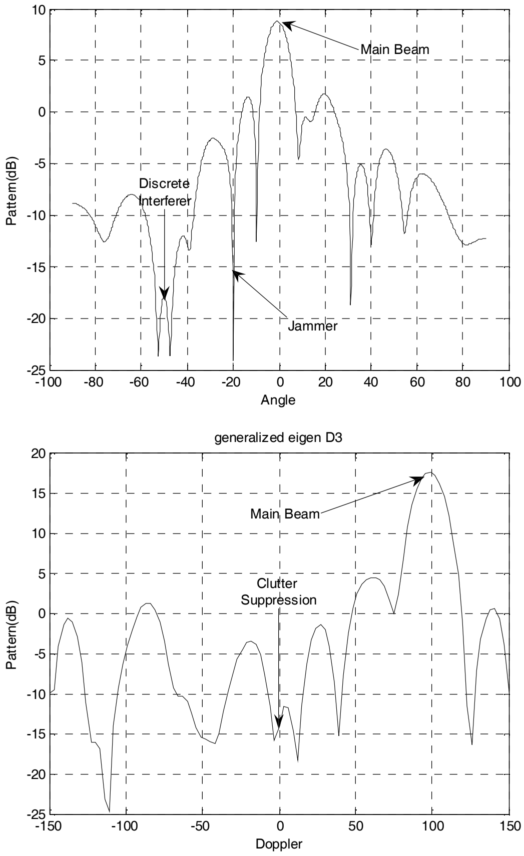

Figure 1: D[3] algorithm using SINR maximization
(A) Angle pattern (B) Doppler pattern.

The next simulation verifies the performance of the hybrid algorithm to detect targets in the non-homogeneous environment. In this simulation, two targets are injected to range bin 50 and 56, the power of targets are 0dB and 8dB respectively. Since two target signals are present in range bins which are close to each other, the interference estimation was strongly affected by other signal.

The clutter environment is assumed to be non-homogeneous and the strong discrete interferer is injected at range bin 100.

This non-homogeneous environment is based on the SIRV clutter model [7-9].

In this example we use $\eta_a = 3$ and $\eta_d = 3$ for the JDL algorithm and $\eta_d = 3$ for the ΣΔ algorithm. 36 range cells are used to estimate the covariance matrix in the JDL case while only 24 are used to estimate the covariance matrix in the case of the hybrid ΣΔ algorithm.

Figure 2 represents the performance of the JDL hybrid algorithm while Figure 3 represents the performance of the hybrid ΣΔ algorithm. The performance of the ΣΔ Hybrid algorithm is similar to that of the original hybrid-JDL, however at significantly lower computation cost (6 instead of 9 D^3 solutions are required).

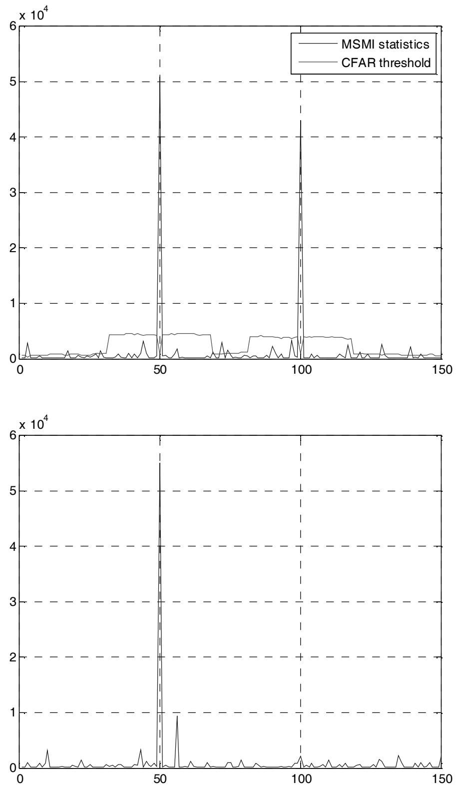

Figure 2: Hybrid JDL for target detection
(A) JDL only (B) Two stage Hybrid algorithm using JDL.

As shown in Figure 2(A) and 3(A), when we use statistical algorithm only, the weak target is obscured because of the strong target effect. Since the strong target data is the one of the secondary data, the weight computation is affected by that signal. The obtained weights have a null in the direction of weak target. If the NHD is used, the strong target range bin is identified as non-homogeneous cell and removed from secondary data then the weak target can be detected. However NHD could not be enough to deal with the discrete interferer. When we use the hybrid algorithm for this environment we can detect both two targets (strong and weak) and handle the discrete interferer well. This result is illustrated in Figure 2(B) and 3(B).

4 Conclusions

This paper has presented a revision of the available hybrid JDL algorithm designed for non-homogeneous clutter. The hybrid algorithm using the revised D^3 algorithm as the underlying non-statistical algorithm and ΣΔ STAP as second stage statistical algorithm is developed here and shown to be very effective. The revised D^3 algorithm is significantly more stable since it does not require an emphasis parameter. The hybrid ΣΔ algorithm has significantly lower computation load, addressing one of the key drawbacks of the original hybrid algorithm.

In this paper, the non-homogeneous clutter environment was modelled as SIRV and JDL or ΣΔ STAP was used as NHD. There are several NHD schemes for SIRV clutter model which has good performance result. Then these kinds of NHD can be applied to implemented hybrid algorithms. Another non-statistical algorithm which replaces D3 will be studied and used to hybrid algorithm.

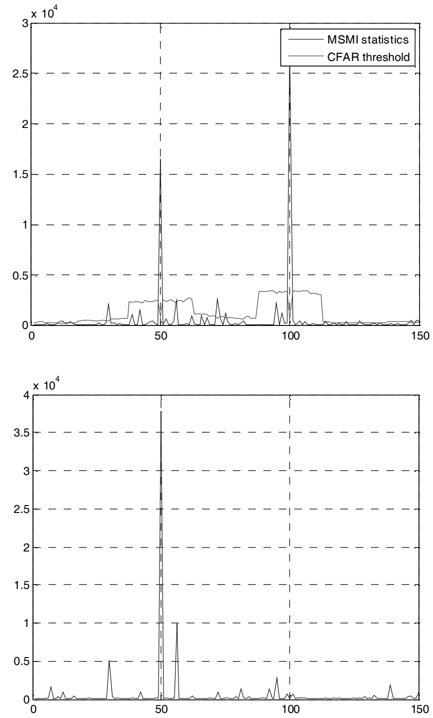

Figure 3: Hybrid ΣΔ STAP for target detection
(A) ΣΔ STAP only (B) Two stage Hybrid algorithm.

References

[1] R.S. Adve, T.B. Hale, and M.C. Wicks, "Practical joint domain localised adaptive processing in homogeneous and non-homogeneous environments Part I: Homogeneous environments",IEE Proc.Radar. Sonar Navig., v.147, no.2, pp.57-65, April. 2000.
[2] R.S. Adve, T.B. Hale, and M.C. Wicks,"Practical joint domain localised adaptive processing in homogeneous and non-homogeneous environments Part II:Non-homogeneous environments",*IEE Proc. Radar. Sonar Navig.*, v.147, no.2, pp.66-73, April. 2000.
[3] R.D. Brown, R.A. Schneible, M.C. Wicks, H.Wang and Y.Zhang, "STAP for clutter suppression with Sum and Difference Beams", *IEEE Transactions on Aerospace and Electronic Systems* Vol. 36, No.2. April 2000

[4] G.H. Golub and C.F. Van Loan, "Matrix Computation", Johns Hopkins University Press, 1996

[5] W. Koch and R. Klemm, "Ground target tracking with STAP radar", *IEE Proc.-Radar. Sonar Navig.,* v.148, no.3, June. 2001.

[6] W.L. Melvin, "A STAP Overview", *IEEE Aerospace and Electronic Systems Magazine*, v.19, No.1, Jan.2004

[7] M. Rangaswamy, D. Weiner, A. Ozturk, "Non-Gaussian random vector identification using Spherically Invariant Random Processes", *IEEE Transactions on Aerospace and Electronic Systems*, Vol.29, No.1, January 1993

[8] M. Rangaswamy, D. Weiner, A. Ozturk, "Computer Generation of Correlated Non-Gaussian Radar Clutter", *IEEE Transactions on Aerospace and Electronic Systems*, Vol.31, No.1, January 1995

[9] M. Rangaswamy, "Statistical Analysis of the non-homogeneity detector for Non-Gaussian Interference Backgrounds", *IEEE Transactions on Signal Processing*, Vol.53, No.6, June 2005

[10] J. Ward, "Space-time adaptive processing for airborne radar", Tech. Rep. F19628-95-C-0002, MIT Lincoln Laboratory, December 1994.

[11] M. C. Wicks, M. Rangaswamy, R. S. Adve, and T. B. Hale, "Space-Time Adaptive Processing ; A knowledge-based perspective for airborne radar" ,*IEEE Signal Processing Magazine*, Jan. 2006.

[12] T.K. Sarkar and N. Sangruji, "An adaptive nulling system for a narrow-band signal with a look-direction constraint utilizing the conjugate gradient method", *IEEE Transactions on Antennas and Propagation*, Vol. 37, No.7, July 1989.