Pilot-Symbol-Assisted Detection Scheme for Distributed Orthogonal Space-Time Block Coding

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Abstract—In this letter, we investigate the effect of imperfect channel estimation on the performance of distributed space-time block codes (DSTBCs) with amplify-and-forward relaying. Exploiting the orthogonality of the underlying code, we derive a maximum likelihood metric conditioned on the channel estimate acquired through the insertion of pilot symbols. For a large number of pilot symbols, we demonstrate that the proposed decoding rule coincides with the so-called mismatched receiver. On the other hand, as the number of pilot symbols decreases, the proposed decoder converges to non-coherent detector. Through Monte-Carlo simulations, we further demonstrate that the performance of the proposed scheme lies within \(0.8\) dB of the genie receiver performance bound.

I. INTRODUCTION

Cooperative diversity has been proposed as a powerful means to enhance the performance of high-rate communications over wireless fading channels [1-5]. Cooperation realizes spatial diversity advantages in a distributed network where a source node uses other nodes’ antennas to relay its message thereby creating a virtual antenna array. Although cooperative diversity has garnered much attention recently, most of the published models assume coherent detection with perfect channel state information (CSI) available at the destination and/or relay terminals. In decode-and-forward (DaF) relaying, both relay and destination require a reliable channel estimate for the decoding step. In amplify-and-forward (AaF) relaying, knowledge of CSI may be required at the relay depending on the scaling factor adopted [4, 6].

To the best of our knowledge, the channel estimation problem in the context of cooperative diversity has been first considered in [7] and [8] assuming DaF and DaF relaying modes, respectively. In [8], Chen and Laneman have proposed a sub-optimal non-coherent modulator with a piecewise-linear combiner that closely approximates the maximum likelihood (ML) detector for cooperative diversity schemes with binary frequency shift keying (BFSK) modulation. In [9], Tarasak et al. have developed a differential modulation scheme for a two-user cooperative diversity system. Several non-coherent distributed space-time block codes (DSTBCs) have also been proposed in [10], [11], [12]. The works in [8-12] focus on DaF relaying. In [13], Annavajala et al. consider AaF relaying and derive non-coherent decoding rules for BFSK and on-off keying (OOK). Our earlier work in [7] along with its journal version [14] investigates the problem of channel estimation for DSTBCs with AaF relaying and M-PSK (phase shift keying) modulation. Specifically, we have proposed non-coherent and mismatched-coherent receivers optimized for AaF fading relay channels and demonstrated, using derivations of pairwise error probability (PEP), that they are able to achieve full diversity over quasi-static fading channels. In this letter, we consider a pilot-symbol-assisted (PSA) receiver which uses an approximate ML metric conditioned on the channel estimate. The proposed receiver performs within \(0.8\) dB of the genie receiver bound (i.e., perfect channel estimation) and includes the non-coherent and mismatched-coherent receivers of [14] as special cases.

The rest of the paper is organized as follows: In Section II, we describe the transmission model for DSTBC with AaF relaying. In Section III, we derive a decoding rule conditioned on the channel estimate acquired through the insertion of pilot symbols. In Section IV, we present Monte Carlo simulation performance results of the proposed scheme. Finally, we conclude in Section V.

Notation: \((\cdot)^*, (\cdot)^T, \) and \((\cdot)^H\) denote conjugate, transpose, and conjugate transpose (Hermitian) operations, respectively. \(E[\cdot]\) denotes expectation, \(\text{diag}(\cdot)\) stands for a diagonal matrix, \(|\cdot|\) denotes the absolute value, \(|\cdot|\) denotes the Euclidean norm of a vector, \(\det(\cdot)\) denotes the determinant of a matrix, \(I_N\) denotes the identity matrix of size \(N\). Bold upper-case letters denote matrices and bold lower-case letters denote vectors.

II. TRANSMISSION MODEL

The relay-assisted transmission scenario under consideration builds upon Protocol I of Nabar et al. [5]. The network comprises a source, a relay and a destination terminal. The source terminal communicates with both the relay and destination terminals during the first signaling interval. In the second signaling interval, both the relay and source terminals communicate with the destination terminal. Let two consecutive symbols transmitted by the source terminal be denoted as \(x_1\) and \(x_2\). We assume the symbols are chosen from a \(M\)-PSK modulation set with unit energy. The received signals at the relay and destination terminals in the first time slot are given as, respectively,

\[
\begin{align*}
    r_R &= \sqrt{E_{SR}} h_{SR} x_1 + n_R, \\
    r_{D,1} &= \sqrt{E_{SD}} h_{SD} x_1 + n_{D,1}.
\end{align*}
\]

In the second time slot, the relay terminal normalizes the signal received in the first time slot by a factor of \(E[|r_R|^2] = \)
First replacing (1) and we obtain \[14\]

$$
Gaussian random variables with variance \( r x \) energy available at the destination terminal taking into account source-to-destination (orthogonality. For the case of single relay, we use the STBC scenarios, we assume the use of STBC to exploit its inherent constraint). The destination terminal receives a superposition of the relayed signal and the second transmitted signal, i.e.,

$$
E \rightarrow \sum \sqrt{E_{RD} h_{RD}} x_{1,m} + \sqrt{E_{SD} h_{SD}} x_{2,m} + \gamma_{1} \sqrt{E_{RD}} h_{RD} x_{1,m} + \gamma_{2} \sqrt{E_{SD}} h_{SD} x_{2,m} + \epsilon x + \epsilon_2 x,
$$

where \( \gamma_1 \) and \( \gamma_2 \) are defined respectively,

$$
\gamma_1 = \frac{E_{SD}/N_0}{1 + E_{RD}/N_0 + \gamma_{1}^{2} E_{RD}/N_0},
$$

$$
\gamma_2 = \frac{E_{SD}/N_0}{1 + E_{RD}/N_0 + \gamma_{2}^{2} E_{SD}/N_0}.
$$

In (1)-(6), \( E_{SR} \), \( E_{SD} \), and \( E_{RD} \) represent the average energy available at the destination terminal taking into account path loss and shadowing effects in source-to-relay (\( S \to R \)), source-to-destination (\( S \to D \)) and relay-to-destination (\( R \to D \)) links, respectively. We assume perfect power control where \( S \to D \) and \( R \to D \) links are balanced (i.e., \( E_{RD}/N_0 = E_{SD}/N_0 \)) and sufficiently large SNR for \( S \to R \) link (i.e., \( E_{SR}/N_0 >> E_{SD}/N_0 \)). Also, \( h_{SR} \), \( h_{SD} \) and \( h_{RD} \) denote the complex fading coefficients over \( S \to R \), \( S \to D \) and \( R \to D \) links and are modeled as complex Gaussian random variables with variance of 0.5 per dimension. \( n_R \), \( n_D \), and \( n_1 \) are the additive noise terms and modeled as zero-mean complex Gaussian random variables with variance \( N_0/2 \) per dimension.

Next, we consider the application of space-time coding across the transmitted signals \( x_1 \) and \( x_2 \). Although different space-time coding techniques can be applied to cooperative scenarios, we assume the use of STBC to exploit its inherent orthogonality. For the case of single relay, we use the STBC designed for two transmit antennas, i.e., Alamouti’s scheme [15]. Depending on the desired trade-off between diversity gain and transmission rate, other STBC techniques may be employed. Considering the broadcasting and relaying phases, we need four time slots for the transmission of two Alamouti-coded symbols. Assume that the destination terminal makes an observation for a duration length of \( N \) (\( N \) is divisible by 4). Using (2) and (4), the received signal vector over four time slots can be then written as in (7), where \( h_1 = h_{SR} h_{RD} \) and \( h_2 = h_{SD} \). In matrix form, the received signals can be rewritten as

$$
\begin{bmatrix}
r_1 \\
r_2 \\
... \\
r_{N/4}
\end{bmatrix} = \begin{bmatrix}
X_1 \\
X_2 \\
... \\
X_{N/4}
\end{bmatrix} \begin{bmatrix}
h_1 \\
h_2 \\
... \\
n_{N/4}
\end{bmatrix},
$$

where \( n \) denotes an \( N \times 1 \) noise vector and \( X_m \) is the data matrix (c.f. (9)).

III. PILOT-SYMBOL-ASSISTED ML DETECTION

Let \( X_p \) denote a matrix, such as in (8) above, comprising the pilot symbols transmitted by the source terminal. Transmission of pilot symbols is followed by data transmission. Under the quasi-static fading channel assumption, the received signals during the training and data transmission phases are given by

$$
r_p = X_p h + n_p,
$$

$$
r = X h + n,
$$

where \( n_p \) and \( n \) are the noise matrices affecting the transmission of pilot and data symbols. The receiver estimates the channel matrix \( h \) from \( r_p \) and uses the resulting estimate \( \hat{h} \) to detect the transmitted signal \( X \) according to the ML criterion [16]

$$
\hat{X} = \arg \max_X \left[ p \left( r | X, \hat{h} \right) \right].
$$

Assuming a ML channel estimator, it is shown in [14] that \( h \) is given by \( \hat{h} = h + \epsilon \) where the channel estimation error vector is complex Gaussian with covariance matrix

$$
\epsilon = (X_p^H X_p)^{-1} X_p^H n_p \quad \text{is complex Gaussian with covariance matrix}
$$

\[
E [\epsilon \epsilon^H] = \begin{bmatrix}
\frac{1}{P(E_{SD}/N_0)} & 0 \\
0 & \frac{1}{P(E_{SD}/N_0)}
\end{bmatrix},
\]

where \( P \) denotes the number of pilot symbols. Conditioned on \( h_{RD} \), it can be easily seen that \( r \) and \( \hat{h} \) are jointly Gaussian with auto-correlation and cross-correlation matrices of \( R_{rr} = N_0 I_N + X R_h X^H \), \( R_{rh} = X R_h \), \( R_{hh} = R_h + \)
\[ \hat{X} = \arg \max_X \left\{ \mathbb{E}_{|h|} \left[ \pi_X \det(\Sigma)^{-1} \exp \left(-tr \left[ (r - X\hat{h})^H \Sigma^{-1} (r - X\hat{h}) \right] \right) \right] \right\} \quad (14) \]

\[ \hat{X} = \mathbb{E}_{|h|} \left[ \arg \max_X \left\{ \pi_X \det(\Sigma)^{-1} \exp \left(-tr \left[ (r - X\hat{h})^H \Sigma^{-1} (r - X\hat{h}) \right] \right) \right\} \right] \quad (15) \]

\[ \hat{X} = \mathbb{E}_{|h|} \left[ \arg \min_X \left\{ \frac{1}{N_0} \|r - X\hat{h}\|^2 - \frac{1}{N_0^2} (r - X\hat{h})^H X\Sigma X^H (r - X\hat{h}) \right\} \right] \quad (16) \]

\[ \hat{X} = \arg \min_X \left\{ \frac{1}{N_0} r^H X\hat{h} - \frac{1}{N_0} \hat{h}^H M H^H X^H r - \frac{1}{N_0} \hat{h}^H X\hat{h} X^H r + \frac{1}{N_0} \hat{h}^H \hat{h} X^H r + \frac{1}{N_0} r^H X\hat{h} \right\} \quad (19) \]

\[ N_0 (X^H X)_{\text{P}}^{-1}\] where \( R_h = \text{diag} \left( |h|_{RD}^2, 1 \right) \). Conditioned on \( h \), \( r \) is complex Gaussian with mean \( m_{rh} = R_{rh} R_h^{-1} \hat{h} = X\hat{h} \) and covariance matrix \( \Sigma = R_r - R_{rh} R_h^{-1} R_{hr} = N_0 I_N + X R_h X^H - X M \Sigma X^H \). Here \( M \) is given by \( M = R_h \left( R_h + N_0 (X^H X)^{-1} \right)^{-1} \) [16] and further simplifies to

\[ M = \begin{bmatrix} (P/2) E_{SD}/N_0 |h|_{RD}^2 & 0 \\ (P/2) E_{SD}/N_0 |h|_{RD}^2 + 1 & E_{P/SD}/N_0 \end{bmatrix}, \]

after noting \( X^H X_{\text{P}} = \text{diag} \left((P/2) E_{SD}, P E_{SD} \right) \). Replacing the pdf of \( r \) conditioned on \( \hat{h} \) and \( X \) in (12), we obtain (14) which can be found on the top of the page. The expectation with respect to \( h_{RD} \) required in (14) does not readily yield a closed-form solution. As an alternative solution, we interchange the order of the maximum and expectation operators and modify the decision metric as in (15) given on top of the page.

Taking the logarithm of the inner term in (15) and dropping unnecessary terms yields (16).

Using the matrix identity \( \det (I + AB) = \det (I + BA) \) [17] and exploiting the orthogonality structure of the underlying STBC, i.e., \( X^H X = \text{diag} \left((N/2) E_{SD}, N E_{SD} \right) \), the determinant term in \( \lambda (X) \) can be shown to be independent of the data sequence. Further using the matrix inversion lemma, \( (A + BCD)^{-1} = A^{-1} - A^{-1}B (C^{-1} + DA^{-1}B)^{-1} DA^{-1} \) [17], \( Q = \Sigma^{-1} \) can be rearranged as

\[ Q = I_N - \frac{1}{N_0} X\bar{K} X^H, \]

(17)

where \( K \) is given by

\[ K = \begin{bmatrix} (P/2) E_{SD}/N_0 |h|_{RD}^2 & 0 \\ (P/2) E_{SD}/N_0 |h|_{RD}^2 + 1 & E_{P/SD}/N_0 \end{bmatrix}. \quad (18) \]

Replacing (17) in (16) and dropping unnecessary terms which do not affect maximization, we obtain (19). Noting \( (P + N) > 1 \), dropping unnecessary terms which do not affect minimization, interchanging the order of the maximum and expectation operators, and finally performing expectation with respect to \( h_{RD} \), we obtain (20), where \( \tilde{K}, \tilde{M}, \) and \( \tilde{\Omega} \) are scaling matrices as a function of the number of pilot symbols and signal-to-ratios and are given by

\[ \tilde{K} = \begin{bmatrix} \frac{2}{(P+N) E_{SD}/N_0} & 0 \\ 0 & \frac{1}{(P+N) E_{SD}/N_0} \end{bmatrix}. \quad (21) \]

(23), and (24) which can be found on top of the next page. In (22) and (23), \( \Gamma (\cdot, \cdot) \) denotes the incomplete gamma function [18]. When \( P \to \infty \), we have \( K = 0 \) and \( M = I_2 \). Therefore, (19) in this case reduces to

\[ \hat{X} = \arg \min_X \left\{ \frac{1}{N_0} \|r - X\bar{h}\|^2 \right\}. \quad (24) \]

which coincides with the so-called mismatched-coherent receiver in [14, Eq. (33)]. Without any channel knowledge (i.e., \( P \to 0 \)), we have \( M = 0 \). Hence, after performing expectation with respect to \( h_{RD} \), (19) reduces to

\[ \hat{X} = \arg \max_X \left\{ \frac{1}{N_0^2} r^H X\bar{K} X^H r \right\}. \quad (25) \]

which is the non-coherent receiver given by [14, Eq. (25)].
\[
\tilde{M} = \begin{bmatrix}
1 - (\frac{P}{T}E_{SD}/N_0)^{-1} e^{(\frac{P}{T}E_{SD}/N_0)^{-1} \Gamma \left(0, \left(\frac{P}{T}E_{SD}/N_0\right)^{-1}\right)} & 0 \\
0 & \frac{PN E_{SD}/N_0}{(P+N)(P E_{SD}/N_0+1)}
\end{bmatrix}
\]

\[
\tilde{\Omega} = \begin{bmatrix}
\left(\frac{N}{N+T}\right) \left(1 - (\frac{P}{T}E_{SD}/N_0)^{-1} e^{(\frac{P}{T}E_{SD}/N_0)^{-1} \Gamma \left(0, \left(\frac{P}{T}E_{SD}/N_0\right)^{-1}\right)}\right) & 0 \\
0 & \frac{PN E_{SD}/N_0}{(P+N)(P E_{SD}/N_0+1)}
\end{bmatrix}
\]

Fig. 1. BER performance of the proposed and genie decoders for DSTBC.

![Fig. 1. BER performance of the proposed and genie decoders for DSTBC.](image1)

**IV. NUMERICAL RESULTS**

In this section, we present results of Monte-Carlo simulations to demonstrate the performance of the proposed receiver over quasi-static fading channels. In our simulations, we consider BPSK modulation and assume \(E_{SD} = E_{RD}\) (i.e., \(S \rightarrow D\) and \(R \rightarrow D\) links are balanced) which can be achieved through power control. We further assume \(E_{SR}/N_0 = 35\)dB and a frame length of 500 symbols.

Fig.1 plots the error rate performance of the proposed approximate ML decoding rule, i.e., (20), for various number of pilot symbols \(P\) assuming one and two receive antennas. As a benchmark, the performance of the genie coherent receiver which assumes perfect CSI is included. We have further included the performance of ML-optimum decoder given by (14) for \(P=2\). It is observed from Fig. 1 that the optimum and proposed decoders yield nearly identical performance. Furthermore, the performance of our decoder lies within 0.8dB and 1dB of the genie bound, respectively, assuming one and two receive antennas at the destination terminals for \(P=2\). Performance gap further decreases with increasing \(P\). Specifically, for \(P=8\), we observe a performance loss of merely 0.15dB and 0.4dB for one and two receive antennas.

Fig. 2 illustrates the bit error rate (BER) performance versus the number of pilot symbols. Here, we fix \(E_{SD}/N_0\) at 12 dB, assume one receive antenna at the destination terminal and compare the performance of ML-optimum, approximate ML, mismatched and non-coherent decoders given by (14), (20), (24), and (25), respectively. As with Fig. 1, it is observed from Fig. 2 that the optimum and proposed decoders yield almost identical performance. For small \(P\) values, it is observed that approximate ML and non-coherent decoders demonstrate a similar performance. As the number of pilot symbols becomes larger, the derived approximate ML decoder converges to mismatched-coherent receiver while both schemes outperform the non-coherent decoder.

![Fig. 2. Comparison of mismatched, non-coherent, and proposed decoders for DSTBC.](image2)

**V. CONCLUSION**

We have studied pilot-symbol-assisted receivers for a single relay-assisted transmission scheme using DSTBC operating in AaF mode. The proposed receiver relies on a ML decoding rule conditioned on the channel estimate acquired through the insertion of pilot symbols. The decoding rule coincides with the so-called mismatched receiver performance for large number of pilot symbols while it converges to non-coherent detection as the number of pilot symbols goes to zero. For the specific case of two training symbols, our Monte-Carlo simulation results indicate that the performance of the proposed receiver lies within 0.8dB of the genie receiver bound assuming one receive antenna.
REFERENCES


