A Two Stage Hybrid Space-Time Adaptive Processing Algorithm

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Abstract— This research presents two new Space-Time Adaptive Processing (STAP) algorithms; a two-dimensional non-statistical method and a hybridization of this approach with statistically based methods. The non-statistical algorithm developed here allows filtering of uncorrelated interference, such as discrete interferers, within the range cell of interest. However, performance of these algorithms in homogeneous correlated interference scenarios is inherently inferior to traditional statistical STAP algorithms.

The proposed hybrid algorithm alleviates this drawback by implementing a second stage of statistical adaptive processing. This paper illustrates the advantages of using a two stage adaptive process to combine the direct data domain and statistical algorithms. The work presented in this paper brings together two different aspects of STAP research: statistical and direct data domain processing. In doing so, this research fulfills an important need in the context of practical STAP processing.

INTRODUCTION

Space-Time Adaptive Processing (STAP) techniques promise to be the best means to suppress severe, dynamic, interference. Classical statistical algorithms achieve interference suppression through the use of the interference covariance matrix. This matrix is typically estimated using secondary data obtained from range cells close to the range cell under test. Statistical algorithms fail when the secondary data does not reflect the statistics of the interference in the range cell of interest, i.e. non-homogeneous data. Non-homogeneous data occurs in many practical situations such as airborne surveillance over land-sea interfaces, dense target environments, etc.

To minimize the loss in performance due to non-homogeneous sample support, a Non-Homogeneity Detector (NHD) can be used to identify secondary data cells that do not reflect the statistical properties of the primary data [2, 3]. These data samples are then eliminated from the estimate of the covariance matrix. However, NHDs do not specify how to detect targets within the range cells identified to be non-homogenous. The surrounding range cells do not possess information about the non-homogeneity and hence a statistical algorithm cannot suppress a discrete interferer in the primary range cell under test.

In this paper, we present a new adaptive algorithm to counter the case of a discrete interferer in the primary range cell. This research is a contribution to STAP and the broader field of Knowledge Based Adaptive Processing (KB-STAP). KB-STAP chooses the best of many possible STAP algorithms for detection with knowledge based control of algorithm parameters and selection of secondary data using NHDs [4]. Currently, KB-STAP architectures incorporate statistical STAP algorithms only and a NHD is used to eliminate non-homogeneous range bins from the secondary data support. The problem of target detection in a non-homogeneous range cell is only now being addressed. This research addresses this hitherto unsolved problem, and therefore significantly enhances the practical relevance of KB-STAP, especially in dense target environments.

A NEW NON-STATISTICAL ALGORITHM

The inability of statistical STAP algorithms to counter non-homogeneities in the range cell of interest motivates research of non-statistical or direct data

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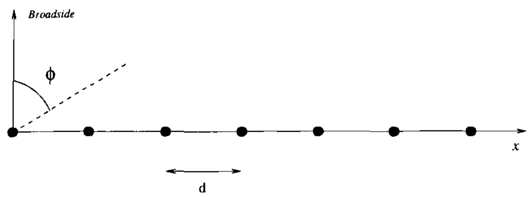

Fig. 1. A linear array of point sensors

domain algorithms. These algorithms use data from the range cell of interest only, eliminating the sample support problems associated with statistical approaches. This field of research emerged in the last few years with a focus on one-dimensional spatial adaptivity [5, 6]. This paper introduces a new two-dimensional direct data domain algorithm that reformulates earlier non-statistical attempts at adaptive processing.

Consider the linear array of equispaced, isotropic point sensors shown in Fig. 1. Each of the N elements receives returns corresponding to the M pulses transmitted per Coherent Processing Interval (CPI). This space-time data is used to decide between the presence and absence of a target at the azimuth look direction $\phi = \phi_t$ and normalized Doppler frequency $\bar{\omega} = \bar{\omega}_t$. The received data can be written as a $N \times M$ matrix $\mathbf{X}$, where $\mathbf{X_{nm}}$ represents the returns at the n -th element due to the m -th pulse. The data matrix $\mathbf{X}$ is a sum of signal, external interference, and thermal noise components. Using the desired look direction and velocity $(\phi_t, \bar{\omega}_t\ )$, the signal matrix $\mathbf{S}$ can be written, in the same matrix form of $\mathbf{X}$, as

$$\mathbf{S} = \xi_t \mathbf{a} \otimes \mathbf{b}^T, \quad (1)$$

$$\mathbf{a} = \left[1 \; z_s \; z_s^2 \ldots z_s^{(N-1)}\right]^T, \quad (2)$$

$$\mathbf{b} = \left[1 \; z_t \; z_t^2 \ldots z_t^{(M-1)}\right]^T, \quad (3)$$

$$z_s = e^{j\frac{2\pi}{\lambda} d \sin\phi_t}, \quad (4)$$

$$z_t = e^{j2\pi\bar{\omega}_t}, \quad (5)$$

where ξ_t is the signal amplitude and d is the distance between two adjacent elements. The vectors $\mathbf{a}$ and $\mathbf{b}$ form the spatial and temporal steering vectors respectively. It is important to note that target returns from an azimuth angle and/or velocity other than the look azimuth/velocity are, effectively, discrete interferers. A target detection should be declared only if it matches the look direction and velocity.

Equation (3) indicates the signal progresses by a constant phase z_s from one element to the next for each pulse. Therefore, the signal component cancels out of the term $\mathbf{X}_{nm} - z_s^{-1}\mathbf{X}_{(n+1)m}$ leaving only interference components. The entries in the $M \times (N-1)$ matrix $\mathbf{C}$, defined to be

$$\begin{bmatrix} \mathbf{X}_{00} - z_s^{-1}\mathbf{X}_{10} & \mathbf{X}_{10} - z_s^{-1}\mathbf{X}_{20} & \cdots & \mathbf{X}_{(N-2)0} - z_s^{-1}\mathbf{X}_{(N-1)1} \\ \mathbf{X}_{01} - z_s^{-1}\mathbf{X}_{11} & \mathbf{X}_{11} - z_s^{-1}\mathbf{X}_{21} & \cdots & \mathbf{X}_{(N-2)2} - z_s^{-1}\mathbf{X}_{(N-1)2} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{X}_{0(M-1)} - z_s^{-1}\mathbf{X}_{1(M-1)} & \mathbf{X}_{0(M-1)} - z_s^{-1}\mathbf{X}_{2(M-1)} & \cdots & \mathbf{X}_{(N-2)(M-1)} - z_s^{-1}\mathbf{X}_{(N-1)(M-1)} \end{bmatrix}, \quad (6)$$

are composed of interference terms only. Consider the scalar expressions

$$G_{\mathbf{w}_s} = \mathbf{w}_s^H \mathbf{a}_{N-1} \mathbf{a}_{N-1}^H \mathbf{w}_s, \quad (7)$$

$$I_{\mathbf{w}_s} = \mathbf{w}_s^H \mathbf{C}^T \mathbf{C}^* \mathbf{w}_s, \quad (8)$$

where $\mathbf{a}_{N-1}$ is the vector comprising of the first $N-1$ entries of the steering vector $\mathbf{a}$. The term $G_{\mathbf{w}_s}$ in Eqn. (7) represents the power gain in the look direction due to weights $\mathbf{w}_s$. $I_{\mathbf{w}_s}$ in Eqn. (8) represents the residual interference power. The new direct data domain algorithm obtains the set of weights that maximizes the difference between the two terms, i.e.

$$\begin{aligned} &\max_{\|\mathbf{w}_s\|_2=1}\left[G_{\mathbf{w}_s} - I_{\mathbf{w}_s}\right] = \\ &\max_{\|\mathbf{w}_s\|_2=1} \mathbf{w}_s^H \left[\mathbf{a}_{N-1}\mathbf{a}_{N-1}^H - \mathbf{C}^T\mathbf{C}^*\right] \mathbf{w}_s. \end{aligned} \quad (9)$$

The constraint $\|\mathbf{w}_s\|_2 = 1$ guarantees a finite solution. Using the Lagrange multiplier method, the weight vector that maximizes the term in Eqn. (9) is the eigenvector corresponding to the largest eigenvalue of the matrix $\left[\mathbf{a}_{N-1}\mathbf{a}_{N-1}^H - \mathbf{C}^T\mathbf{C}^*\right]$. This weight vector forms the spatial adaptive weights. It is important to note this weight vector is of length $(N-1)$ representing a loss of one degree of freedom in the spatial domain. This compares favorably with other non-statistical algorithms where close to half the degrees of freedom are lost [5].

In the temporal domain, the signal progresses by the same phase z_t from one pulse to the next at each element and therefore the signal component cancels out in terms such as $\mathbf{X}_{nm} - z_t^{-1}\mathbf{X}_{n(m+1)}$. A similar formulation to Eqns. (6)-(9) can therefore be used to obtain a $(M-1)$ length temporal weight vector $\mathbf{w}_t$. The length $N \times M$ space-time adaptive weight vector is then given by

$$\mathbf{w}(\phi_t, \bar{\omega}_t) = \begin{bmatrix} \mathbf{w}_t(\bar{\omega}_t) \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \mathbf{w}_s(\phi_t) \\ 0 \end{bmatrix}. \tag{10}$$

The zeros appended to the spatial and temporal weight vectors represent the lost degree of freedom in space and time. Using this adaptive weight vector, the statistic to be used for comparison to a threshold for detection at angle ϕ_t and normalized Doppler $\bar{\omega}_t$ is given by

$$\chi(\phi_t, \bar{\omega}_t) = \mathbf{w}^H \mathrm{vec}(\mathbf{X}), \tag{11}$$

where $\mathrm{vec}(\cdot)$ stacks the columns of $\mathbf{X}$ into a length $NM \times 1$ vector.

The above formulation effectively side-steps the high sidelobe problems associated with previous direct data domain algorithms [7]. The resulting signal estimates are free of the effects of non-homogeneities. However, direct data domain algorithms in general fail to suppress correlated interference to the degree possible with statistical STAP algorithms.

A HYBRID ALGORITHM

The main thrust of the paper is the presentation of an approach to STAP combining the benefits of both direct data domain and statistical methods. Our hybrid approach uses the non-statistical algorithm as a pre-filter to suppress discrete interferers present in the range cell of interest as shown in Fig. 2. The algorithm serves as the adaptive transformation from the space-time domain to the angle-Doppler domain.

Any STAP algorithm estimates the signal component at the look direction in angle and Doppler. STAP algorithms can therefore be viewed as an adaptive transform to this particular angle and Doppler. Creating a set of look angles and Doppler frequencies allows the STAP algorithm to perform a function similar to the Fast Fourier Transform. It must be emphasized that this transformation is non-invertible resulting in some information loss. However, this information loss may be beneficial. We take advantage of this loss to suppress discrete interferers within the range cell of interest through the use of our new direct data domain algorithm.

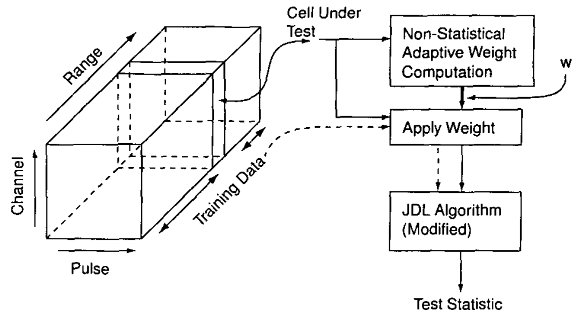

Fig. 2. Two-stage hybrid algorithm block diagram.

The hybrid algorithm presented here adaptively processes space-time data in two stages. The first stage is the direct data domain adaptive transform mentioned above. The output of the first stage lends itself to the application of a post-Doppler, beamspace statistical algorithm forming the second stage of adaptive processing. An enhanced version of the joint domain localized (JDL) algorithm [8, 9] is used as the second stage. JDL suppresses interference in a Localized Processing Region (LPR) of the angle-Doppler domain. Figure 2 shows the block diagram of the two-stage hybrid algorithm.

Mathematically, the transformation to a predetermined LPR is accomplished through a matrix operator $\mathbf{T}$. The angle-Doppler data is given by $\tilde{\mathbf{x}} = \mathbf{T}^H \mathrm{vec}(\mathbf{X})$. The steering vector is transformed in the same manner. An example of $\mathbf{T}$ for a 3×3 LPR is

$$\begin{aligned} \mathbf{T} = [\ & \mathbf{w}(\phi_{-1}, \bar{\omega}_{-1}) \ \ \mathbf{w}(\phi_{-1}, \bar{\omega}_t) \ \ \mathbf{w}(\phi_{-1}, \bar{\omega}_{+1}) \\ & \mathbf{w}(\phi_t, \bar{\omega}_{-1}) \ \ \mathbf{w}(\phi_t, \bar{\omega}_t) \ \ \mathbf{w}(\phi_t, \bar{\omega}_{+1}) \\ & \mathbf{w}(\phi_{+1}, \bar{\omega}_{-1}) \ \ \mathbf{w}(\phi_{+1}, \bar{\omega}_t) \ \ \mathbf{w}(\phi_{+1}, \bar{\omega}_{+1})\ ]. \end{aligned} \tag{12}$$

Because the JDL algorithm only operates within a localized region of the angle-Doppler domain, few degrees of freedom are used and secondary data support requirements are correspondingly reduced [8]. These advantages are carried over to the hybrid algorithm.

NUMERICAL EXAMPLE

The hybrid algorithm is tested on data generated using the physical model presented by Jaffer [10] and Ward [11], and implemented by Roman and Davis [12]. Comparison of adapted beam patterns associated with JDL, the new direct data domain, and the hybrid algorithms illustrate the motivation for, and improved performance due to, the hybrid algorithm.

The adapted antenna pattern plots presented in this paper are the mean pattern over 200 independent realizations. Vertical bars represent the standard deviation over these 200 trials. This method was necessitated because the direct data domain algorithm is non-statistical and is based solely on a single data set/realization. Operating with known covariance to obtain an ideal pattern as in JDL or other statistical algorithms is not an option.

The simulation includes the effects of clutter, white noise, two barrage noise jammers, and a discrete interferer. The simulated antenna array is linear with $N = 18$ elements and a coherent processing interval (CPI) of $M = 18$ pulses. Two 40 dB jammers are located at $45°$ and $-20°$. The discrete interferer is simulated by an injected 40 dB target at the same normalized Doppler as the look Doppler but a different azimuth angle of $\phi = -51°$. The look direction is set to an azimuth angle of $\phi_t = 0°$ and normalized Doppler $\bar{\omega}_t = 1/3$. The JDL algorithm uses 3 angles and 3 Doppler frequencies centered around the look direction for a total of $N_{\text{DOF}} = 9$. The number of secondary data vectors used to estimate the covariance matrix is set to $2N_{\text{DOF}} = 18$.

Figures 3 and 6 illustrate the antenna patterns for the standard JDL algorithm [8] along target azimuth and Doppler. Figure 3 shows the algorithm has placed distinct nulls in the two jammer locations. The discrete interferer, i.e. off azimuth target, does not contribute to the covariance matrix estimate and therefore is not nulled by the algorithm. Figure 6 shows a null at $\bar{\omega} = 0$ to suppress mainlobe clutter. The mainlobe is formed at the Doppler look direction of $\bar{\omega}_t = 1/3$.

The antenna patterns resulting from the implementation of the two-dimensional direct data domain algorithm are presented in Figs. 4 and 7. It bears repeating that a direct data domain algorithm uses only data from the range cell of interest and hence does not require any secondary data. Figure 4 shows the direct data domain algorithm is effective in countering a discrete interferer in the range cell of interest. The adapted angle pattern shows a distinct null in the direction of the discrete at $-51°$. However, Figs. 4 and 7 also illustrate the limitations of the direct data domain algorithm. The nulls in the direction of the jammers are not as deep as in the case of Fig. 3. The null at $\bar{\omega} = 0$ in the clutter spectrum is also not as deep, i.e. the mainbeam clutter is not suppressed as effectively as by the JDL algorithm.

The results of Figs. 3-7 provide the motivation for the development of the hybrid algorithm. The direct data domain algorithm is used as the first stage to screen out discrete interferers. A statistical algorithm, such as JDL, is then used to suppress correlated interference. Figures 5 and 8 show the antenna beam patterns resulting from the use of the hybrid algorithm. Figure 5 shows the hybrid algorithm combines the advantages of both statistical and non-statistical adaptive processing. The adapted azimuth pattern shows deep nulls at $-51°$, $-20°$ and $45°$; the directions of the discrete interferer and the two jammers. Figure 8 shows the adapted pattern has a deep null at $\bar{\omega} = 0$ resulting in effective nulling of the mainbeam clutter.

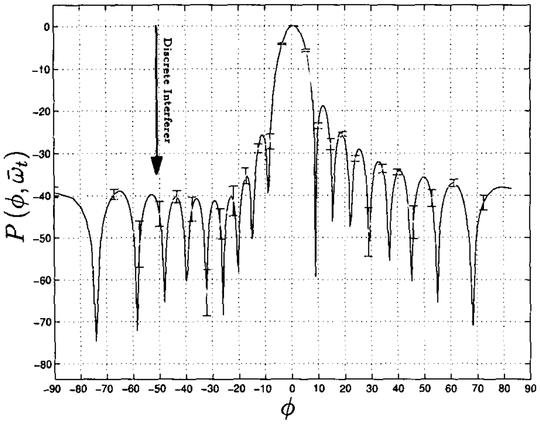

Fig. 3. The JDL algorithm antenna pattern at the target Doppler.

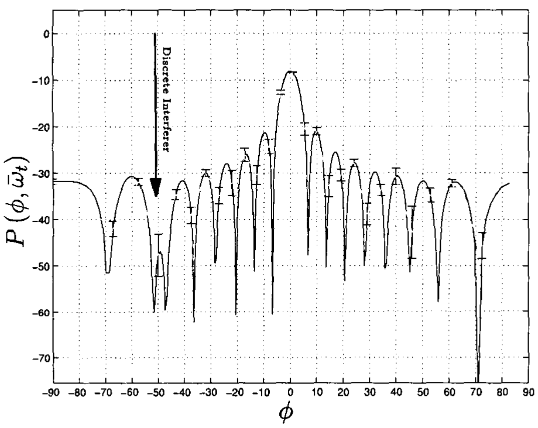

Fig. 4. The direct data domain algorithm antenna pattern at the target Doppler.

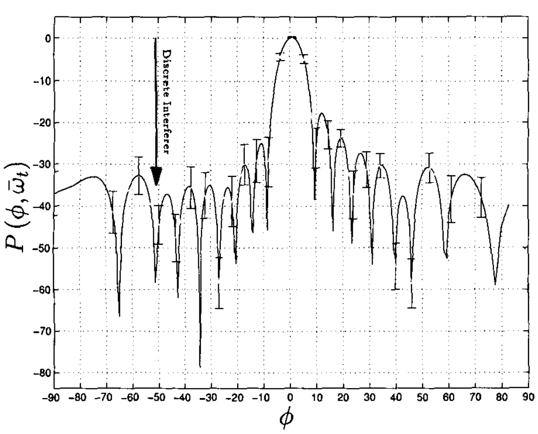

Fig. 5. The hybrid algorithm antenna pattern at the target Doppler.

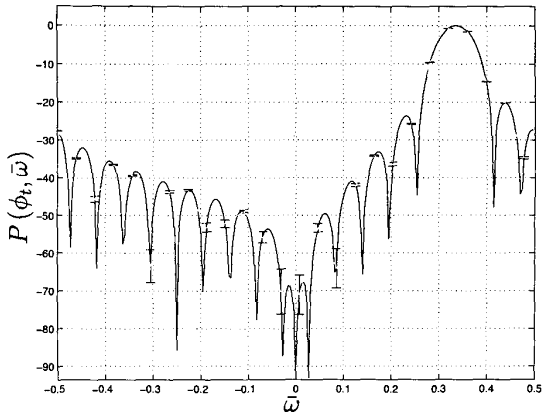

Fig. 6. The JDL algorithm antenna pattern at the target azimuth.

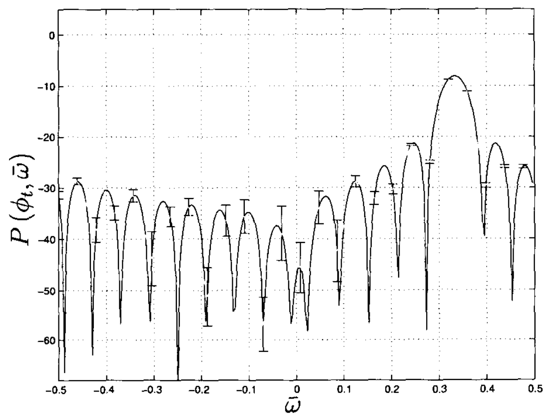

Fig. 7. The direct data domain algorithm antenna pattern at the target azimuth.

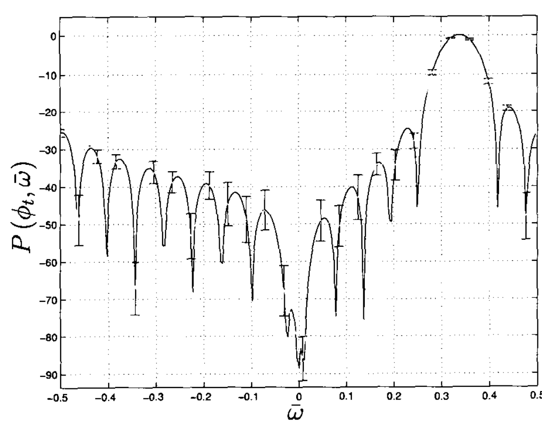

Fig. 8. The hybrid algorithm antenna pattern at the target azimuth.

CONCLUSIONS

In this paper, we have developed two new algorithms; a two-dimensional non-statistical approach to STAP and a hybridization of this approach with statistically based methods. The non-statistical algorithm developed here allows filtering of discrete interferers within the range cell of interest. However, performance of direct data domain algorithms in homogeneous correlated interference scenarios is inferior to traditional statistical STAP algorithms.

The proposed hybrid algorithm alleviates this drawback by implementing a second stage of statistical adaptive processing. Figures 3 through 8 illustrate the advantages of using a two stage adaptive process to combine the direct data domain and statistical algorithms. The direct data domain method is particularly effective at countering non-homogeneous clutter. The statistical STAP algorithm then improves suppression of the correlated interference.

The work presented in this paper brings together two different aspects of STAP research: statistical and direct data domain processing. In doing so, this research fulfills an important need in the context of practical STAP processing.

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