Fractional Cooperation using Coded Demodulate-and-Forward

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***Abstract*— Since the introduction of cooperative diversity, many different implementations have been proposed to increase the reliability and/or power efficiency of distributed networks via relaying. One simple and flexible scheme introduced has been coded demodulate-and-forward, where the relay only demodulates, instead of decodes, the received data, to create and forward a new codeword to the destination. This reduces the complexity of hardware as well as the energy consumption by the relay. In this paper, we consider another flexible feature of the coded demodulate-and-forward scheme, where the relay uses only a *fraction* of its codeword to assist the source, while using the rest of the codeword to transmit its own information. Previous schemes have generally focused on all-or-nothing cooperation where a relay either contributes all its resources or none at all to the source. Depending on the channel conditions, improved diversity order of the source codeword can be achieved with some small loss in the relay's own transmission performance. Here we identify the necessary criterion for the source to achieve a diversity order of 2.**

I. Introduction

Cooperative diversity has been shown to provide significant performance gains in distributed wireless networks where communication is impeded by channel fading. One specific motivation in this research area, and the motivation behind this paper, is reliable communications in sensor networks: distributed networks comprising simple, battery operated nodes. In such networks a relay (or group of relays) forwards information for a source node. The relay channel was first studied in [1], where the capacity of the degraded relay channel was determined. The next breakthrough came when the concept of cooperative diversity was introduced in [2], [3]. In those papers, the authors analyzed the situation with users helping each other in a cellular setting. Since then, numerous works have been presented to introduce and/or analyze cooperative schemes that provide diversity gain in fading channels.

One of the more prominent works in cooperative diversity is [4], where two schemes, amplify-and-forward (AF) and decode-and-forward (DF), were presented and analyzed. In using DF, the relay attempts to fully decode the source codeword, and re-encodes and transmits the codeword to the destination upon successful decoding. To avoid error propagation, the relay may check if it has decoded correctly before retransmission. Practical implementation of the DF schemes include, but are not limited to [5], [6], [7]. In [5], a convolutional code is used to improve the performance over all the links, and the relay uses a cyclic redundancy check (CRC) to ensure the received data is correct before forwarding to the destination. Turbo codes are used to improve the error rate over all the links in [6], whereas rateless Raptor codes are used in [7] to ensure enough code bits are received by the relay before it cooperates with the source node to transmit to the destination. In the latter two cases, the relay must perform fairly elaborate decoding processes, clearly at the expense of battery power and more complex hardware. This runs counter to the constraint of small, simple, nodes with limited battery power. On the other hand, it is clearly desirable to harness the gains due to error control coding.

In order to reduce the complexity of the hardware and the energy consumption at the relay, the relay can demodulate instead of decode the received signals. Demodulate-and-forward, independently proposed in different contexts in [8], [9], is a significantly simpler scheme; the relay does not perform any decoding, but rather just demodulates the received signals from the source, and forwards the demodulated bits. In [8], at the destination, the received signals from potentially multiple relays are processed before being summed and provided to a bit detector. In [9], a similar scheme is presented. However, instead of forwarding the demodulated bits, the relay performs simple parity encoding, and the destination uses the sum-product algorithm [10] to decode the source codeword. Analysis and simulation results for the coded demodulate-and-forward schemes are presented in [11] and [12], where low-density generator matrix (LDGM) [13] codes and repeat-accumulate (RA) [14] codes are used respectively. In all cases with one relay, a diversity order of 2 can be observed, despite the fact that only simple operations is required at the relay. In these works LDGM and RA codes are chosen for their excellent performance in additive white Gaussian noise (AWGN) channels, as well as flexibility and simplicity in their encoding procedure (as opposed to say, low density parity check codes). The goal was not to achieve capacity, but reliability while yet minimizing complexity.

In this paper, we develop *fractional cooperation*, another advantage provided by flexibility of demodulate-and-forward coupled with decoding on graphs. In most of the available literature, it is assumed that the relay either uses all of its resources to relay for the source or does not relay for the source at all, a "all-or-nothing" approach. However, this generally assumes that the relay does not have its own data. In a sensor network, on the other hand, each node in the network

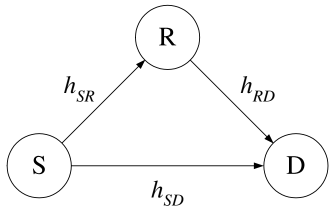

Fig. 1. System model of the 3-node relay channel.

is both a source and a potential relay. Given its own limitations (such as available battery power and latency of it own data), a "relay node" may be willing to give up only a *fraction* of its resources to the "source". We still identify these nodes as source and relay to emphasize who is relaying for whom. Using the demodulate-and-forward scheme, we will see that a relay can incorporate both its own data and that of the source into the codeword transmitted to the destination node. If the relay is closer to the destination than the source node, then the performance it sacrifices to assist the source is small compared to the performance enhancement provided to the source node. Fractional cooperation was mentioned as a possibility in [12], not developed in any detail.

This paper is organized as follows. In Sec. II, we provide the system model associated with our analysis, and the parameters used to describe the demodulate-and-forward scheme with repeat-accumulate codes. In Sec. III, we analyze this scheme to present the criterion that must be satisfied to achieve a diversity order or 2 for the source codeword, followed by simulation results and discussions in Sec. IV. Finally, we will provide some concluding remarks in Sec. V.

II. System Model and Parameters

The system model used in this paper is the 3-node relay channel illustrated in Fig. 1, although this coding scheme can be extended to networks with multiple relays. The system includes the source, relay, and destination nodes and are denoted as S, R and D in the figure. We assume that the channels between the nodes are independent quasi-static Raleigh fading channels, where the channel coefficients h_{SD}, h_{SR} and h_{RD} are constant for the whole codeword, but change from codeword to codeword. We also assume the receiver in all communications has channel state information (CSI) and the destination node has knowledge about the CSI of the S-R channel. We impose the restriction where the nodes cannot transmit and receive simultaneously, and synchronization between the nodes is not available. The relay is assumed to be too simple to perform error control decoding of the source's transmission.

When the source node has data that needed to be communicated with the destination, it first forms a codeword $\mathbf{c}_s \in \{0,1\}^{l_s}$, where l_s represents the total number of symbols transmitted by the source, and then performs the mapping from $\{0,1\}$ to $\{+1,-1\}$ to form the symbol vector to be transmitted, $\mathbf{s}$. In our scheme, transmission requires two phases. In the first phase, the source node broadcasts the source symbols $\mathbf{s}$,

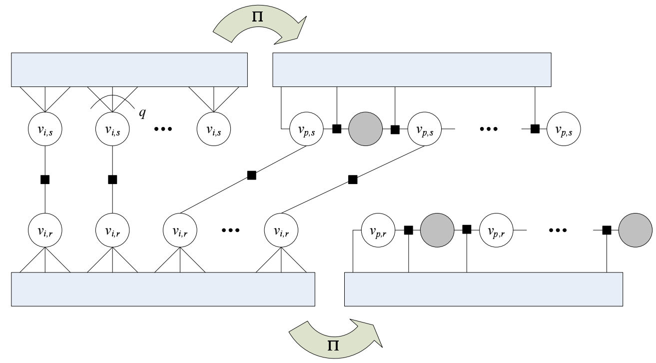

Fig. 2. Factor graph of demodulate-and-forward with the use of RA code.

and the baseband discrete-time representation of the received signals at R and D at time i can be written as

$$\begin{aligned} y_{\mathrm{SR}}[i] &= h_{\mathrm{SR}} s[i] + n_R[i], && (1)\\ y_{\mathrm{SD}}[i] &= h_{\mathrm{SD}} s[i] + n_D[i], && (2)\end{aligned}$$

where $s[i]$, $i = 1, \dots l_s$, are the binary source bits in a block of length l_s, h_{SR} and h_{SD} are fading channel coefficients on the S-R and S-D channels respectively, and $n_R[i]$ and $n_D[i]$ are independent complex white Gaussian noise with variance $N_{0,R}$ and $N_{0,D}$ respectively. The average received symbol signal to noise ratio (SNR) of the S-R channel is given by

$$\bar{\gamma}_{\mathrm{SR}} = \frac{\mathbb{E}[|h_{\mathrm{SR}}|^2]}{N_{0,R}} \qquad (3)$$

where $\mathbb{E}[\cdot]$ represents statistical expectation. The average received symbol SNR for the S-D channel, $\bar{\gamma}_{\mathrm{SD}}$, can be calculated in a similar manner.

After receiving the transmitted source symbols, the relay demodulates them and maps them to $\{0,1\}$. In the next step, the relay takes a *fraction* of the demodulated bits, and together *with its own information bits*, forms a new codeword $\mathbf{c}_r \in \{0,1\}^{l_r}$. Similar to the source node, it performs the mapping from $\{0,1\}$ to $\{+1,-1\}$ to obtain the symbol vector $\mathbf{r}$, and in the second phase, these symbols are transmitted to the destination. The baseband discrete-time representation of the received signal at D at time i is

$$y_{\mathrm{RD}}[i] = h_{\mathrm{RD}} r[i] + n_D[i], \qquad (4)$$

where h_{RD} is fading channel coefficient on the R-D channel. Similar to (3), the average received symbol SNR of the R-D channel is given by

$$\bar{\gamma}_{\mathrm{RD}} = \frac{\mathbb{E}[|h_{\mathrm{RD}}|^2]}{N_{0,D}}. \qquad (5)$$

In this paper, RA encoders are used in both the source and relay nodes, however the concept of fractional cooperation is applicable to a much wider class of coding schemes. A repeat-accumulate codeword is generated by first repeating the information bits q times, then passing these bits through a random interleaver to form the vector $\mathbf{u}$, and finally forming the output vector $\mathbf{w}$ using the following parity equation:

$$w[i] = \begin{cases} u[i] & \text{for } i = 1, \\ u[i] \oplus w[i-1] & \text{for } i > 1, \end{cases} \quad (6)$$

where $\oplus$ is the XOR operation. One of the reasons that RA codes is chosen is because of its flexibility. The code rate can be changed on-the-fly with the use of puncturing. Note that, however, when puncturing is used, a systematic code must be used to provide good performance [15]. In our scheme, the information bits are concatenated with the punctured output from (6) to form the codeword with the desired code rate. A factor graph of demodulate-and-forward with the use of RA code is shown in Fig. 2. In the figure, circles and squares represent variable and check nodes respectively, where shaded variable nodes represent punctured parity bits. The labels $v_{i,s}$, $v_{p,s}$, $v_{i,r}$ and $v_{p,r}$ represent source information and parity bits, and relay information and parity bits respectively, and $\prod$ is the random interleaver. Another advantage provided by using RA codes is that we can make use of the sum-product algorithm [10] for decoding. This allows us to take into account the unreliability of the S-R channel while decoding. More information on the decoding processes of this type of demodulate-and-forward scheme can be found in [11].

Here we introduce some parameters that will be used in the rest of the paper to describe the demodulate-and-forward coding scheme. We use ϵ_i and ϵ_p to denote the fraction of source information and party bits relayed. Hence, if k_s denotes the number of information bits in the source codeword $\mathbf{c}_s$, and l_s is the length of the codeword, then the number of total bits relayed are $\epsilon_i k_s + \epsilon_p(l_s - k_s)$. We let k_r denote the information bits in the relay codeword $\mathbf{c}_r$, and l_r be the length of $\mathbf{c}_r$. In addition, m_r denotes the information bits *originating* from the relay. Hence $k_r = m_r + [\epsilon_i k_s + \epsilon_p(l_s - k_s)]$ must be satisfied. Finally, we let r_s and r_r denote the code rate of the source and relay codewords respectively. Throughout this paper we have set $q = 3$, hence with the use of puncturing the code rates can range from 1/4 to 1.

III. Analysis of Fractional Cooperation

In this section, we will show the conditions that must be met in order to allow the source to achieve diversity order of 2 in the frame error rate (FER) with the help of one relay. In addition, we will show that in assisting the source, the relay cannot achieve a diversity order of 2 in the FER.

Theorem 1: In a one-relay network, let r_s be the rate of the source codeword, and ϵ_i and ϵ_p be the fraction of the source information and parity bits included in the relay codeword. Then

$$r_s \leq \frac{\epsilon_p}{1 - \epsilon_i + \epsilon_p} \quad (7)$$

must be satisfied to obtain a FER with diversity order of 2 for the source codeword.

Proof: Let k_s be the number of source information bits, and $l_s - k_s$ be the number of parity bits. Then since $r_s = k_s/l_s$, the number of source bits that are included in the relay codeword is given by

$$\epsilon_i k_s + \epsilon_p(l_s - k_s) = (\epsilon_i r_s + \epsilon_p - \epsilon_p r_s) l_s. \quad (8)$$

Because only some of the source's symbols are selected for relaying, the S-R-D channel as seen from the source codeword point of view is an erasure channel, with the erasure probability given by

$$e = 1 - [(\epsilon_i - \epsilon_p) r_s + \epsilon_p], \quad (9)$$

which is independent of the channel conditions. Given an erasure channel with erasure probability e, the capacity is $1-e$ [16]. This means that the code rate, r_s, must be less than or equal to $1 - e$ for successful decoding. Thus, for successful decoding on the relay link, this translates to the condition

$$\begin{aligned} r_s &\leq 1 - e \\ &= (\epsilon_i - \epsilon_p) r_s + \epsilon_p, \end{aligned} \quad (10)$$

and the condition in (7) follows from minor manipulation of this expression.

In order to achieve diversity order of 2, the decoder must have the ability to decode the codeword in the event of an outage over the S-D channel, or equivalently, when the instantaneous received symbol SNR of the S-D channel, γ_{SD}, is 0 (otherwise, a minimum SNR is required on the S-D link, which is the definition of a system with diversity order 1.) Hence, in order for successful decoding in the event of an outage over the S-D channel, the condition in the above equation must be satisfied. Thus, with the S-D channel present, the condition must be met in order to achieve a diversity order of 2. ■

A scenario where the above condition is met is when $\epsilon_i = 1$ and $\epsilon_p = 0$, and for any r_s. In this case a diversity order of 2 can be achieved with the use of 1 relay. Another example that a diversity order of 2 can be achieved is when $r_s = 1/2$ and $\epsilon_i + \epsilon_p = 1$.

Corollary 2: The frame error rate of the *relay codeword* has diversity order less than 2 for the demodulate-and-forward coding scheme.

Proof: In order for the relay to achieve a diversity order of 2, the condition in (7) must be satisfied for the relay codeword as well. From the setup of the scheme, the corresponding ϵ_p for the relay, $\tilde{\epsilon}_p$, is 0, since its parity bits are not relayed by any other nodes, and (7) becomes

$$r_r \leq \tilde{\epsilon}_i r_r, \quad (11)$$

where $\tilde{\epsilon}_i$ is the fraction of relay information bits that are also transmitted by the source, i.e., bits from the source that are relayed by R. This translates to $\tilde{\epsilon}_i \geq 1$. However, since $\tilde{\epsilon}_i < 1$ as long as the relay is using part of its codeword to relay the source bits, diversity order of 2 cannot be achieved for the relay codeword. ■

This corollary, at first glance, seems obvious since no other node is forwarding information for the relay. However, as we will see in the simulation results, the relay can achieve a diversity order of greater than 1. Since the relay includes the source's information, decoding the source's codeword helps in decoding the relay's codeword as well.

IV. Results and Discussions

A. One-Relay Network

The simulation results for two different scenarios are shown in this section. Both scenarios use a path loss exponent of $\alpha = 4$ [17]. In the first scenario we have the relay closer to the source than it is to the destination, where if we let the distance between the source and destination be d_{SD}, then the distance between the source and relay, d_{SR}, is $0.4d_{\mathrm{SD}}$, and that between the relay and destination, d_{RD} is d_{SD}. The received signal energy is proportional to $d^{-\alpha}$, where d is the distance between the transmitter and receiver, the relationship between the average received symbol SNR of the various channels is given by

$$\bar{\gamma}_{\mathrm{SD}} = (0.4)^4 \bar{\gamma}_{\mathrm{SR}} = \bar{\gamma}_{\mathrm{RD}}. \tag{12}$$

In the second scenario the relay is closer to the destination than it is to the source, where $d_{\mathrm{SR}} = d_{\mathrm{SD}}$ and $d_{\mathrm{RD}} = 0.4d_{\mathrm{SD}}$. The relationship between the average received symbol SNR of the various channels is given by

$$\bar{\gamma}_{\mathrm{SD}} = \bar{\gamma}_{\mathrm{SR}} = (0.4)^4 \bar{\gamma}_{\mathrm{RD}}. \tag{13}$$

In both cases, the rate of the source codeword r_s is set to 1/2.

Here we present and compare simulation results for two different cases. In both cases $\epsilon_p = 0$, $k_s = 2000$ and $l_s = 4000$. In order to provide a fair comparison l_r is fixed at $l_r = 4000$ such that the amount of energy used for transmission in both cases are the same. The parameters ϵ_i and r_r are varied to show two different ways to employ fractional cooperation. In the first case, we set the rate of the relay code $r_r = 1/2$, and ϵ_i, the fraction of source information bits, is adjusted according to allow the required relay information bits, m_r, to be transmitted. In the second case, instead of varying ϵ_i, we set $\epsilon_i = 1$, and adjust r_r accordingly to allow the required m_r to be transmitted. Hence, in the first case, when we have $m_r = 1000$ and $m_r = 500$, $\epsilon_i = 1/2$ and $\epsilon_i = 3/4$ respectively. In the second case, for $m_r = 1000$ and $m_r = 500$, $r_r = 3/4$ and $r_r = 5/8$ respectively. The simulation results are shown in Fig. 3. The solid lines represent FER for the source information bits, and the dash-dot lines represent FER for the relay information bits.

When $r_r = 1/2$, the source and relay FER are almost the same, whereas the source and relay FER are vastly different when r_r is allowed to be varied in order to relay all the source information bits. From the plot, we can see that the diversity order is approximately 1 for all the curves, except the ones represent the FER for the source codeword with $\epsilon_i = 1$. This comes at a cost of increasing the relay code rate, and the loss is evident from the shift of the relay FER curves. The loss, compared with its constant r_r counterpart, is about 2 dB when r_r is reduced from 1/2 to 3/4, and about 1 dB when it is reduced to 5/8. There is also a slight decrease in the source

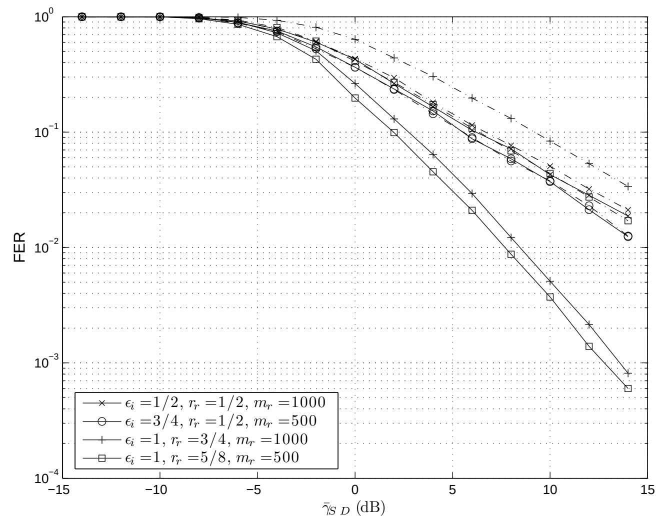

Fig. 3. Plot of frame error rate with 1 relay, where the relay is closer to the source node.

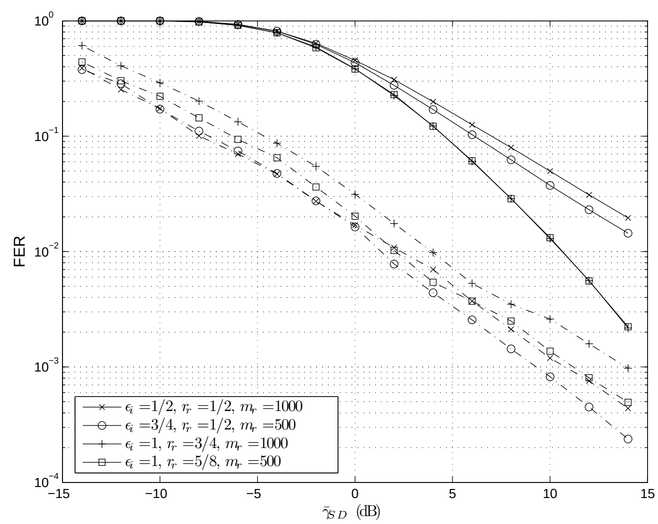

Fig. 4. Plot of frame error rate with 1 relay, where the relay is closer to the destination node.

FER with $\epsilon_i = 1$ when r_r is reduced, as illustrated by the FER curve shift in the plot.

The simulation results for the scenario where the relay is closer to the destination node is shown in Fig. 4. When $r_r = 1/2$, similar to the previous case, the FER performance for both the source and relay are better with a larger ϵ_i. The figure also illustrates an interesting result: the FER of the relay codewords has diversity order of approximately 1.4 for all cases even though no other node forwards messages for the relay. This is because of the relationship between the source and relay codewords. Decoding the source's codeword significantly reduces the rate of the relay codeword, helping decoding. In addition, the FER of the source codewords with $\epsilon_i = 1$ has diversity order of 2. Together with the simulation results from Fig. 3, all these observations confirm the results from Sec. IV.

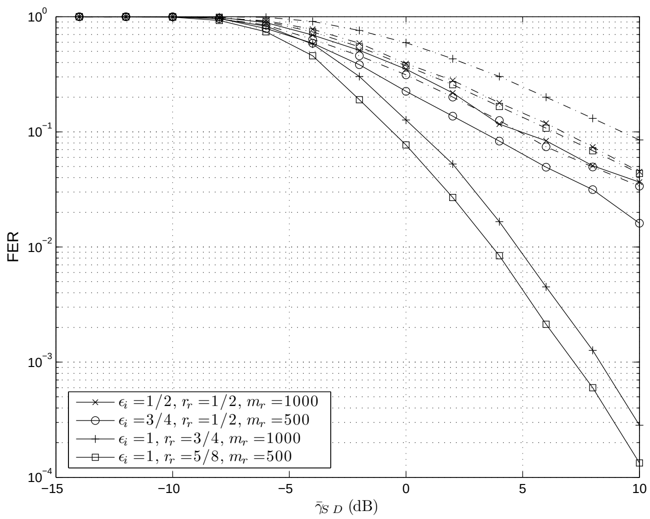

Fig. 5. Plot of frame error rate with 2 relays, where the relays are closer to the source node.

Other than the fact that the performance of the relay codeword is better when the relay is placed closer to the destination, another major difference between the simulation results for the two different scenarios is that the source FER is the same when $\epsilon_i = 1$ and r_r is varied. This is probably due to the fact that the $\bar{\gamma}_{\mathrm{RD}}$ is much greater than $\bar{\gamma}_{\mathrm{SR}}$, and hence the performance is limited by the S-R channel, where the difference in the r_r becomes irrelevant. In addition, when $\epsilon_i = 1$, the source benefits more from the relay being closer to the source then the destination, although there is minimal difference between the two scenarios when $\epsilon_i < 1$.

B. Two-Relay Network

The notion of fractional cooperation is fairly general and can be extended to networks with multiple relays. Here we include simulation results to show the performance of fractional cooperation when two relays are used. In Fig. 5, the FER for the case where 2 relays are placed closer to the source is plotted, where the relationships between the average channel SNR described in (12) still holds. As illustrated in the plot, a source FER with diversity order of 3 can be observed when $\epsilon_i = 1$. Unlike the 1-relay cases, there is a slight gain in the source FER performance when compared to its relay counterpart with $\epsilon_i < 1$. This is probably due to the assistance it receives from the extra relay. Note that the performance of the relay is almost identical to the scenario where only 1 relay is used. Adding an extra relay only changes the source FER, and the relay FER is not affected. This, again, confirms our results in III, where we stated that the relay codeword cannot achieve a FER with diversity order of 2.

V. CONCLUSION

In this paper we have developed the concept of fractional cooperation, where the relay only uses a *fraction* of its codeword to assist the source, leaving "space" to transmit its own information. As described above, this coded demodulate-and-forward scheme is extremely simple, where simple hardware is required at the relay and the energy consumption is significantly reduced; only demodulation, instead of decoding, is required at the relay. In addition, by using systematic RA codes and puncturing, this scheme is extremely flexible. Hence the relay can provide as much help as its own constraints allow, constraints such as available batter power, latency of its own data, etc. We have provided the criterion that must be met in order for the source FER to have a diversity order of 2 at high SNR, and shown that the diversity order of the relay FER must be less than 2. And finally, the simulation results show the increase in diversity order of the source FER that can be achieved with 1 or 2 relays, by ensuring the required criterion are satisfied.

REFERENCES

[1] T. Cover and A. E. Gamal, "Capacity Theorems for the Relay Channel," *IEEE Trans. Inform. Theory*, vol. 25, pp. 572–584, Sep. 1979.
[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity – Part I: System Description," *IEEE Trans. on Comm.*, vol. 51, pp. 1927–38, November 2003.
[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity – Part II: Implementation Aspects and Performance Analysis," *IEEE Trans. on Comm.*, vol. 51, pp. 1939–48, November 2003.
[4] J. Laneman, G. Wornell, and D. N. C. Tse, "An Efficient Protocol for Realizing Cooperative Diversity in Wireless Networks," in *Proc. 2001 IEEE Int. Symp. on Info. Th.*, vol. 1, p. 294, June 2001.
[5] T. E. Hunter and A. Nosratinia, "Performance Analysis of Coded Cooperation Diversity," in *Proc. IEEE Int. Conf. on Comm. 2003*, vol. 4, pp. 2688–2692, ICC'03, 11-15 May 2003.
[6] B. Zhao and M. C. Valenti, "Distributed Turbo Coded Diversity for Relay Channel," *Electronics Letters*, vol. 39, pp. 786–787, 15 May 2003.
[7] J. Castura and Y. Mao, "Rateless Coding over Fading Channels," *IEEE Commun. Letters*, vol. 10, pp. 46–48, Jan. 2006.
[8] D. Chen and J. N. Laneman, "Modulation and Demodulation for Co-operative Diversity in Wireless Systems," *IEEE Trans. Wireles Comm.*, vol. 5, pp. 1785–1794, July 2006.
[9] J. P. K. Chu and R. S. Adve, "Implementation of Co-operative Diversity Using Message-Passing in Wireless Sensor Networks," *Proc. 2005 IEEE Globecom*, 2005.
[10] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor Graphs and the Sum-Product Algorithm," *IEEE Trans. Inform. Theory*, vol. 47, pp. 498–519, Feb. 2001.
[11] A. Eckford, J. Chu, and R. Adve, "Low Complexity Cooperative Coding for Sensor Networks Using Rateless and LDGM Codes," in *IEEE International Conf. on Communications*, pp. 1537–1542, 2006.
[12] A. W. Eckford and R. Adve, "A Practical Scheme for Relaying in Sensor. Networks Using Repeat-Accumulate Codes," in *Proc. Conf. on Inform. Sci. and Systems*, Mar. 2006.
[13] T. R. Oenning and J. Moon, "A Low Density Generator Matrix Interpretation of Parallel Cconcatenated Single Bit Parity Codes," *IEEE Trans. Magnetics*, vol. 37, pp. 737–741, Mar. 2001.
[14] D. Divsalar, H. Jin, and R. J. McEliece, "Coding Theorems for 'Turbo-Like' Codes," in *Proc. 36th Annual Allerton Conference on Communications, Control, and Computing,* Monticello, IL, USA, pp. 201–210, 1998.
[15] A. Abbasfar, D. Divsalar, and K. Yao, "Maximum Likelihood Decoding Analysis of Accumulate-Repeat-Accumulate Codes," in *Proc. Global Telecomm. Conf.*, 2004.
[16] T. Cover and J. Thomas, *Elements of Information Theory*. Wiley and Sons, 1 ed., 1991.
[17] T. S. Rappaport, *Wireless Communications–Principles and Practice*. Prentice Hall, 1996.