Implementation of Cooperative Diversity using Message Passing in Wireless Sensor Networks

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Abstract

Cooperative diversity can be achieved by a relay node (R) assisting in a transmission between a source (S) and a destination (D). However, in general, the proposed cooperation schemes ignore the quality of the S-R channel in the decoding process. This paper introduces a novel and robust scheme for cooperative diversity that accounts for the quality of the S-R channel. In our scheme, the relaying node provides parity checks for the estimated source symbols, which are used to assist in the decoding of the source message using message passing. Performance measures, such as bit error rate, are then not limited by the S-R channel quality and improve with increasing signal-to-noise ratio on the S-D and R-D channels. No feedback signals are required, with only simple computations at the relay. Simulations show that, as with earlier proposals, our scheme achieves full diversity order in the case of a good S-R channel.

Index Terms

Cooperative diversity, parity check, message passing, sensor networks, relay channels.

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***Abstract*— Cooperative diversity can be achieved by a relay node (R) assisting in a transmission between a source (S) and a destination (D). However, in general, the proposed cooperation schemes ignore the quality of the S-R channel in the decoding process, even though it is this channel that limits the performance of cooperation schemes. This paper introduces a novel and robust scheme for cooperative diversity that accounts for the quality of the S-R channel. In our scheme, the relaying node first decodes the transmitted source symbols. It then re-encodes them to provide parity checks for the source symbols. These parity checks are used to assist in the decoding of the source message using message passing. Performance measures, such as bit error rate, are then not limited by the S-R channel quality and improve with increasing signal-to-noise ratio on the S-D and R-D channels. Feedback signals to the source node are not required, with only simple decoding and encoding required at the relay. Simulations show that, as with earlier proposals, our scheme achieves full diversity order in the case of a good S-R channel.**

I. Introduction

Advances in hardware technology have made available small, low-cost, battery-operated sensing devices that are capable of performing computations and transmitting and receiving data. Coupled with available enhancements in wireless technologies, it is now possible to implement networks of such sensor nodes, with many potential applications in environmental monitoring, military surveillance, etc. An essential limiting factor in such networks is power consumption as battery replacement can be expensive or even impossible. One proposal to conserve energy is to provide spatial diversity. In traditional cellular networks this is made possible by multiple antennas at the transmitter and/or receiver [1]. However, since sensor nodes are usually small and inexpensive, implementing multiple antennas on a single node is impractical. Cooperation diversity, introduced in [2], [3], allows sensor nodes to enlist the help of their neighbors (denoted as relays for the role they play) and use them as auxiliary antennas to provide path diversity. Because of the broadcast nature of wireless communications, the relay receives the source message "for free."

Theoretical analysis of cooperative diversity have emphasized the resulting diversity order [4], [5]. This paper focuses on schemes to *implement* cooperation in the classical relay channel where a relay node (R) helps in the transmission of data from a source node (S) to a destination node (D). Recently, Laneman *et al.* [6] suggested two simple repetition-based cooperation schemes: amplify-and-forward (AF) and decode-and-forward (DF). In using AF, the relay simply amplifies the noisy received signal. If using DF, the relay decodes the source symbols and re-encodes them for transmission to the destination. Both schemes are based on *repetition codes*, as no encoding is performed at the relay.

Other schemes for cooperation diversity include coded cooperative diversity (CCD) [7] and distributed turbo codes (DTC) [8], where encoding is performed at the relay. With CCD, each source is matched to a single potential relay. The partner node, listening in on the transmission from the source, acts as a relay only if it can decode the source message without error, confirmed using a cyclic redundancy check (CRC). This method, however, requires that the nodes be able to receive and transmit radio signals simultaneously and that the source and relay nodes be synchronized in order to reduce the required bandwidth. With DTC, the source node transmits turbo encoded codewords. Upon receiving the data, the relay node decodes the data using a Viterbi decoder, passes the decoded data through an interleaver, and re-encodes the data using a turbo encoder to provide parity bits. The need for a Viterbi decoder, however, significantly increases the required complexity of each node. Other variations include schemes that require feedback to the source node. Some examples are a hybrid DF scheme [6] and hybrid automatic-repeat-request [9]. Feedback, however, adds unnecessary overhead and complexity to the system and represents a significant waste of energy.

The discussion above is restricted to the main themes in cooperative diversity. The reader is referred to the citations provided for references to variations on these themes. Except for CCD [7], all the schemes mentioned above assume that the relay receives the source information without error. In using CCD, this is tested using the CRC. The destination decoder therefore places equal emphasis on the data from both the source and relay nodes, and ignores the possibility of errors at the relay. The performance of these schemes is therefore restricted by the quality of the S-R channel. In this paper, we introduce a simple, yet robust, cooperation scheme that does not require synchronization between source and relay or feedback to the transmitter. Instead of the relay repeating source symbols, parity bits are formed at the relay and transmitted to the destination node. At the destination, the signals from the relay, associated with the parity bits, are used as *side information* to help with the decoding of the source symbols. The destination node decodes the source bits using message passing that accounts for the uncertainty in the side information [10].

The proposed scheme has several advantages over the currently available proposals. By using message passing in the decoding process the error performance is not limited by the S-R link quality. The overall bit error rate (BER) always decreases with an improving S-D link. Also, the source and relay nodes can send symbols independently, so synchronization between the nodes is not required. Each node, in particular the relay node, is only transmitting *or* receiving at any instant. Furthermore, since a relay node only creates parity bits, the encoding complexity is limited. Any increase in complexity is restricted to the destination decoder. This aspect of our scheme could be of significant advantage in the practical development of cooperative sensor networks based on nodes of limited complexity.

It must be noted that the motivating factors in this work, such as energy consumption and reduced complexity are also important in traditional cellular networks [7]. In this regard, while the focus here is on sensor networks, our work also has applications in other multiuser wireless communication networks.

This paper is organized as follows. Section II introduces the system model for the sensor network problem. In Section III we provide some background information on message passing, and its use in our scheme. Section IV presents some simulation results illustrating the efficacy of our scheme, allowing us to draw some conclusions in Section V.

II. System Model

The system model used (the classical relay channel), is illustrated in Fig. 1. The relay node does not have any information of its own to send and its sole purpose is to assist S in transmitting data to D. The channels between the nodes are assumed to be quasi-static Rayleigh fading channels and a block fading model is used, where the fading coefficients are constant for the entire block of length N symbols. At any instant, each node is restricted to be either transmitting or receiving. In addition, the relay is assumed to know accurately the S-R channel state information (CSI), and the destination is assumed to know the S-D and R-D CSI and average S-R signal-to-noise ratio (SNR).

At the source, the in-phase and quadrature data streams are mapped onto quadrature phase-shift keying (QPSK) symbols $s_Q[n]$ before transmission. The transmission in divided into two phases. In the first phase, symbols are transmitted by S and received by R and D. The discrete-time signals, at time index n, received by R and D respectively are

$$r_{\mathrm{SR}}[n] = h_{\mathrm{SR}} s_Q[n] + n_R[n], \quad (1)$$
$$r_{\mathrm{SD}}[n] = h_{\mathrm{SD}} s_Q[n] + n_D[n], \quad (2)$$

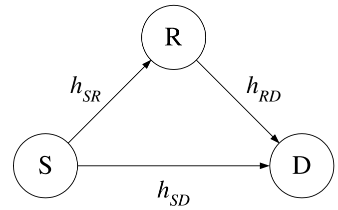

Fig. 1. System model of the sensor network.

where h_{SR} and h_{SD} are fading channel coefficients from S to R and D respectively, and $n_R[n]$ and $n_D[n]$ are independent white Gaussian noise with variance $N_{0,R}$ and $N_{0,D}$ respectively. The SNRs of the S-D and S-R links are

$$\gamma_{\mathrm{SD}} = \frac{E[|h_{\mathrm{SD}}|^2]E_S}{N_{0,D}} \qquad \gamma_{\mathrm{SR}} = \frac{E[|h_{\mathrm{SR}}|^2]E_S}{N_{0,R}}, \quad (3)$$

where $E[\cdot]$ represents statistical expectation and $E_S = |s_Q[n]|^2$ is the symbol energy. The destination D uses knowledge of γ_{SR} in the decoding process.

At the relay, the transmitted symbols are estimated and separated into the in-phase and quadrature streams. Parity bits are then formed as follows

$$\hat{p}[n] = \begin{cases} \hat{s}[n] \otimes \hat{s}[n+1] & 1 \le n \le N-1, \\ \hat{s}[n] & n = N, \end{cases} \quad (4)$$

where $\hat{s}[n]$ are the estimated source bits and $\otimes$ is the XOR function. In the second phase of transmission, the two streams of decoded bits are encoded using (4) and mapped to a QPSK symbol $\hat{p}_Q[n]$ before transmission to D. The signal received at D is given by

$$r_{\mathrm{RD}}[n] = h_{\mathrm{RD}} \hat{p}_Q[n] + n_D[n], \quad (5)$$

where h_{RD} is fading channel coefficients from R to D. The QPSK symbols have unit energy, i.e., $s_Q[n], p_Q[n] \in \{\sqrt{0.5}\,(\pm 1 \pm j)\}$.

In [7], the authors suggest a similar scheme where R transmits parity bits to help decode the message. However, the scheme is binary in that the relay cooperates only if the S-R channel is good enough for it to decode the message. Similarly, the scheme described in [11] uses only data that are reliable to form parity bits at the relay. As explained in the next section, our scheme accounts for the quality of the S-R channel instead of discarding valuable source information, thereby achieving the maximum diversity order given the channel conditions.

III. Decoding With Message Passing

A. Preliminaries

Tanner graphs [12] provide a visual representation of the parity check matrix of any parity-check code [13]. A Tanner graph consists of two types of nodes: variable and function nodes. Tanner graphs are bipartite graphs, meaning that each type of nodes can only have neighbors of the other type of nodes. In our case, the variable nodes represent the symbols sent by S and R, and the function nodes are parity check functions. The Tanner graph associated with our coding scheme is

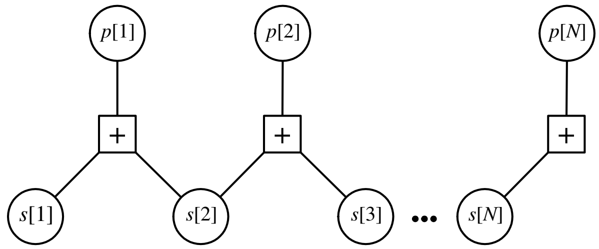

Fig. 2. Tanner graph used for message passing.

shown in Fig. 2. In this figure, $s[n]$, $n = 1, \ldots, N$, are bits formed at S and $p[n]$, $n = 1, \ldots, N$ are the associated parity bits. Symbols sent by S and R are received at D, where they are decoded to estimate the source symbols. Message passing (MP) can be used for decoding parity check codes in a Tanner graph.

The MP process is initialized by first calculating the likelihood function of each coded symbol. The probability distribution of the nth transmitted symbol $s_Q[n]$, sent by S and received by D, being x is given by

$$p(r_{\mathrm{SD}}[n]|s_Q[n] = x) = \frac{1}{\sqrt{2\pi N_{0,D}}} \exp\left\{ \frac{|r_{\mathrm{SD}}[n] - h_{\mathrm{SD}}x|^2}{2N_{0,D}} \right\}. \tag{6}$$

where $x \in \{\sqrt{0.5}(\pm 1 \pm j)\}$ is one of the four possible QPSK symbols.

For the SR-RD serial link, finding the required likelihood function involves first finding the equivalent noise distribution. We first note that the probability of bit error for a QPSK symbol in the S-R link is given by the probability of bit error in a Rayleigh fading channel [1]

$$P_{Rf,\mathrm{SR}} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_{\mathrm{SR}}/2}{1+\gamma_{\mathrm{SR}}/2}}\right). \tag{7}$$

Let $p_Q[n]$ be the QPSK symbol associated with $p[n]$, the correct parity bit formed by substituting $s[n]$ into $\hat{s}[n]$ in (4). Then the received signal at D in (5) can be rewritten as

$$r_{\mathrm{RD}}[n] = h_{\mathrm{RD}}p_Q[n] + h_{\mathrm{RD}}(\hat{p}_Q[n] - p_Q[n]) + n_D[n], \tag{8}$$

where the equivalent noise is the sum of the latter two terms. Hence the equivalent noise variance is given by

$$\tilde{N}_{0,D} = d^2(p_Q[n], \hat{p}_Q[n])E[|h_{\mathrm{RD}}|^2] + N_{0,D}, \tag{9}$$

where $d^2(p_Q[n], \hat{p}_Q[n]) \in \{0, 2, 4\}$ is the squared Euclidean distance between $p_Q[n]$ and $\hat{p}_Q[n]$. To find the distribution of the noise, we first find all the combinations of $(\mathbf{k_S}, \mathbf{k_P})$ for the block with block size N, where $\mathbf{k_S} = (k_{S,0}, k_{S,2}, k_{S,4})$ is the vector comprising the number of decoded symbols $\hat{s}_Q[n]$ with $d^2(s_Q[n], \hat{s}_Q[n]) = 0, 2, 4$ respectively. Similarly, the vector $\mathbf{k_P} = (k_{P,0}, k_{P,2}, k_{P,4})$ comprises the number of encoded parity symbols $\hat{p}_Q[n]$ with $d^2(p_Q[n], \hat{p}_Q[n]) = 0, 2, 4$ respectively. We derive an expression analogous to (6) for the R-D link by averaging over all combinations of $(\mathbf{k_S}, \mathbf{k_P})$

$$p(r_{\mathrm{RD}}[n]|p_Q[n] = x) = \sum_{(\mathbf{k_P},\mathbf{k_S})} \left(\prod_{i\in\{0,2,4\}} p_{e,i}^{k_{S,i}} \right) \times \left(\sum_{i\in\{0,2,4\}} \frac{k_{P,i}}{N} \frac{1}{\sqrt{2\pi(N_{0,D} + iE[|h_{\mathrm{RD}}|^2])}} \times \exp\left\{ -\frac{|r_{\mathrm{RD}}[n] - h_{\mathrm{RD}}x|^2}{2(N_{0,D} + iE[|h_{\mathrm{RD}}|^2])} \right\} \right), \tag{10}$$

where $p_{e,i}$, the probability that $d^2(s_Q, \hat{s}_Q[n]) = i$, is given by

$$p_{e,i} = \begin{cases} (1 - P_{Rf,\mathrm{SR}})^2 & i = 0, \\ (1 - P_{Rf,\mathrm{SR}})P_{Rf,\mathrm{SR}} & i = 2, \\ P_{Rf,\mathrm{SR}}^2 & i = 4. \end{cases} \tag{11}$$

In implementing this approach, the actual distribution is estimated using a small block size as finding all combinations of $(\mathbf{k_S}, \mathbf{k_P})$ is computationally intensive for large N. There is minimal change in the distribution as N increases.

B. Message Passing

The MP process is divided into two steps. In the first step, the probabilities are propagated forward (to the right in Fig. 2). We first calculate the message passed from a variable node v to a function node f, $m_{v\to f}$. This is obtained by

$$m_{v\to f}(x) = \prod_{f'\in n(v)\backslash f} m_{f'\to v}(x), \tag{12}$$

where $n(v)$ are all the neighbors of v, and $m_{f'\to v}$ are the messages from function node f' to variable node v. A variable node can send a message to a function node neighbor once the messages from all the other function node neighbors have been received. The MP process is initialized by assigning the likelihood function as one of the incoming messages for each variable node. If a variable node has only one neighbor, it sends to the function node its likelihood function.

Similarly, a function can send a message to a variable node only after it has collected messages from all the other neighboring variable nodes. The message sent by the function node f to variable node v is given by

$$m_{f\to v}(x) = \sum_{v'\in n(f)\backslash v} \prod_{(x,x')} m_{v'\to f}(x')I(x, x'), \tag{13}$$

where the indication function is given by

$$I(x, x') = \begin{cases} 1 & \text{if } (x, x') \text{ satisfies the parity check,} \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

In the second step, the messages are propagated backwards (to the left in Fig. 2). As with forward propagation, the messages $m_{v\to f}$ and $m_{f\to v}$ are calculated accordingly and passed to the appropriate nodes. Since there are no cycles in the Tanner graph of Fig. 2, only one iteration is needed to acquire the exact *a posteriori* probabilities. After the backward propagation, the probability for each symbols are obtained by multiplying all the messages incoming to the variable node

$$p(v = x|\mathbf{r_D}) = \prod_{f \in n(v)} m_{f \to v}(x), \tag{15}$$

where $\mathbf{r_D}$ includes all the received signals at D. A hard decision is made by choosing the symbol with highest probability

$$\tilde{s}_Q[n] = \arg\max_x p(s_Q[n] = x|\mathbf{r_D}). \tag{16}$$

The discussion above serves as a brief overview of MP as applied to our relay problem. The reader may refer to [10] and [13] for more details on message passing. By using MP, the information provided by the parity bits are used as extrinsic (side) information, and *their reliability is accounted for.*

IV. Simulation Results

This section presents results of simulations that illustrate the efficacy of our proposed MP scheme. In all examples, the block size N is set to 100. In practice it is logical to use a relay "closer" to the destination, i.e., $\gamma_{\mathrm{RD}} \geq \gamma_{\mathrm{SD}}$. To eliminate one variable from our simulations, we set $\gamma_{\mathrm{RD}} = \gamma_{\mathrm{SD}}$. We begin by illustrating the drawback with earlier approaches.

A. Previously Proposed Schemes

Fig. 3 presents simulation results for a maximal ratio combining (MRC) scheme. In MRC, the relay uses repetition coding, where it decodes and re-transmits the data. This method assumes that no error is made during the S-R transmission and equal emphasis is placed on the data received from both S and R. At D, the received data from both S and R are summed together before decoding. Also shown are the results when relaying is not used. When the S-R channel quality is poor, direct transmission without relaying is preferred. This is reasonable since if the S-R channel quality is low, the relay often decodes erroneously, thereby limiting the performance of the system. However, crucially, for all cases with relaying, an error floor always appears, even with γ_{SR} as high as 30 dB.

The simulation results for distributed turbo coding is illustrated in Fig. 4. This scheme shares some characteristics with our system in that encoding is done in a distributed fashion. As in Fig. 3, the BER does not improve even as the S-D SNR increases, due to the performance limitation introduced by the bit errors occurring at the relay decoder. Notice, however, that the BER curves plateau at a lower BER than the MRC case, showing that channel coding improves the performance. In practice therefore, a relaying scheme that does not account for the S-R channel quality may not be very effective.

B. Cooperation Using Message Passing: Ideal Case

This section presents results in the ideal case where the destination has accurate knowledge of γ_{SR}, the average S-R SNR.

Simulation results, for the same situation as Fig. 3, using our method are shown in Fig. 5. It is assumed that the destination node has perfect knowledge of the S-D and R-D CSI and the average channel SNRs, γ_{SR}, γ_{RD} and γ_{SD}. As shown in the plot, the error performance is slightly worse than the MRC case in the low SNR regime. However, unlike the MRC case, there is no error floor with increasing γ_{SD}. Indeed, there is no crossover between the curves for the cases with and without relaying. With a good S-R channel, i.e., $\gamma_{\mathrm{SR}} = 30$ dB, the diversity order is slightly less than 2. Limited by a poor S-R channel (e.g. γ_{SR} = 10 dB), our scheme is slightly better than the case without relays. Note the significantly improved performance in all cases as compared to MRC performance in Fig. 3.

The frame error rate (FER) corresponding to Fig. 5 is shown in Fig. 6. As with the BER simulation results, there are no crossovers or error floors, with improving performance with increasing SNR. There are differences, however, when the performance of the systems is evaluated using the BER or FER criteria. If the performance is evaluated based on BER, relaying with γ_{SR} = 10 dB is better than the case without relaying, while the opposite is true if the performance is

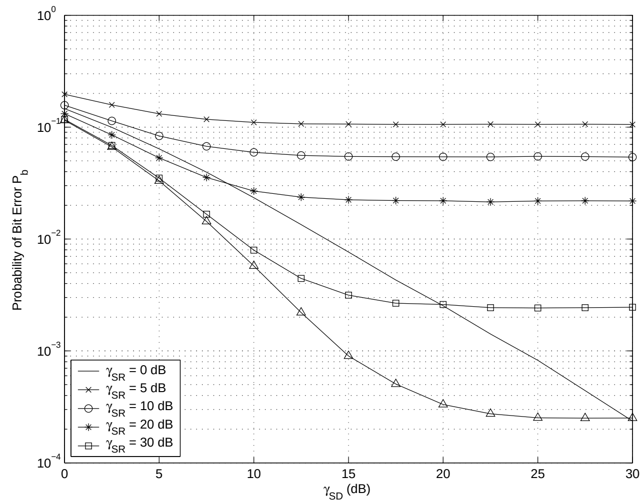

Fig. 3. Bit error rate with maximal ratio combining.

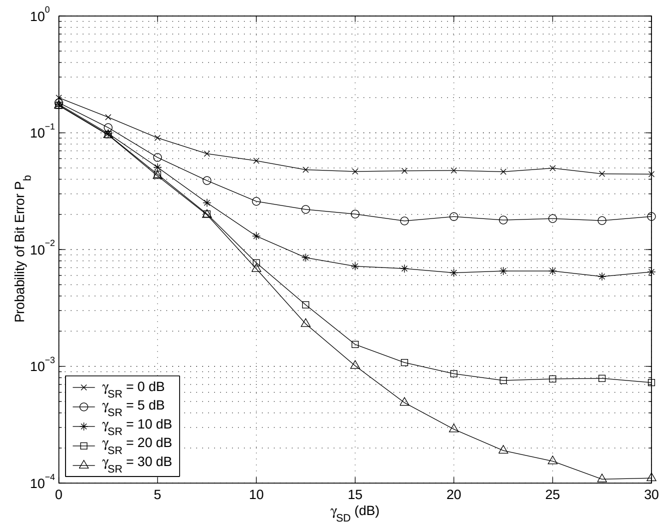

Fig. 4. Bit error rate with distributed turbo codes.

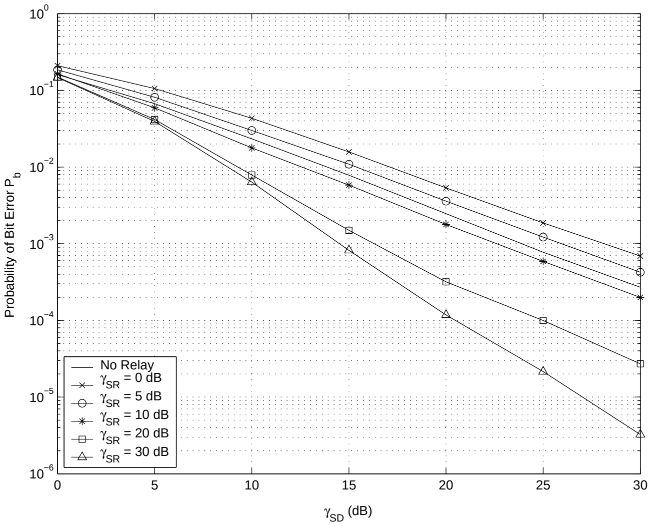

Fig. 5. Bit error rate with MP with knowledge of γ_{SR}.

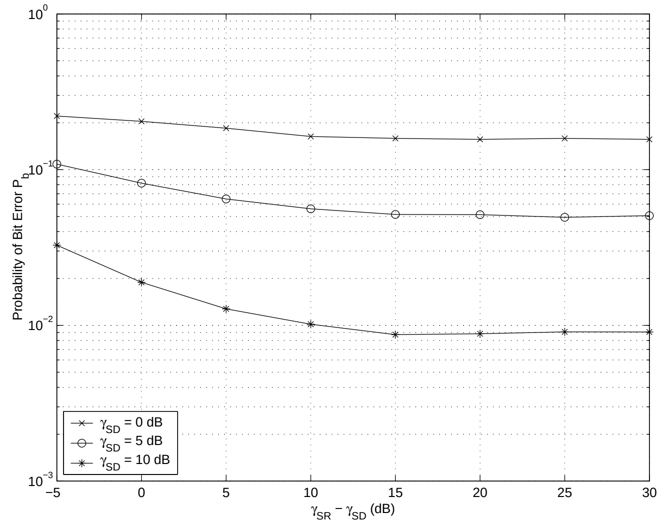

Fig. 7. Bit error rate using MP without knowledge of γ_{SR}.

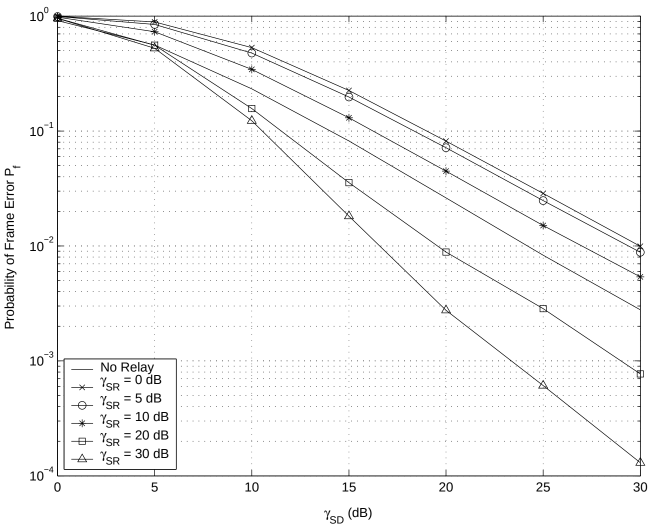

Fig. 6. Frame error rate using MP with knowledge of γ_{SR}.

evaulated using the FER. Again, $\gamma_{\mathrm{SR}} = 30$ dB results in a diversity order of slightly less than 2.

C. Cooperation Using Message Passing: Sensitivity to γ_{SR}

One significant issue with the scheme proposed in Section III is that the destination requires knowledge of the γ_{SR}, the average S-R SNR. Since relaying γ_{SR} represents an additional overhead, we investigate the sensitivity of our scheme to the assumed value of γ_{SR}. The decoder assumes that S-R channel quality is at least as good as the S-D channel quality, and uses $\gamma_{\mathrm{SR}} = \gamma_{\mathrm{SD}}$ in the MP process. Note that since the receiver D is assumed to know the S-D CSI, estimating γ_{SD} is not difficult.

Figure 7 plots the BER versus the difference between the γ_{SR} and γ_{SD}, which is equivalent to the difference between the actual and assumed γ_{SR}. As illustrated in the plot, the performance is effectively insensitive to the mismatch between the actual and assumed values of γ_{SR}. As expected when the S-R channel is worse than that assumed (the relay data is given greater credence than it should receive), the error rate worsens. However, assuming that a relay node is closer to the source than the destination node, this scenario is rather unlikely. In summary, the scheme in Section III can be used even when the true γ_{SR} is unavailable at D.

V. Conclusion

In this paper, we have presented a novel cooperation scheme that can be used to provide spatial diversity in a sensor network to conserve energy. The proposed scheme has some significant advantages over the previous proposals to *implement* cooperative diversity. The scheme requires extremely simple decoding and re-encoding at the relay node, limiting the required complexity within each sensor node. With a moderate source-relay channel, unlike with cases that equally emphasize source and relay data, performance improves over the case without relaying. As with other schemes, the simulations show that our scheme provides full diversity with a good S-R channel. The scheme is robust in that, in all cases, there is never an error floor in the BER/FER, i.e., performance is not limited by the S-R channel. The scheme does not require synchronization between source and relay and each node is either transmitting *or* receiving at any given time.

In many practical applications, the destination node is not energy limited and not as complexity limited as the source or relay node. This is the case in uplink wireless communications and surveillance networks, where the destination is the base station or a data processing center. The scheme proposed here takes advantage of this fact by shifting the processing burden to the destination node. The only drawback is the need for CSI and average S-R channel quality at the receiver. However, as illustrated in our simulations, our scheme works well even when the S-R channel quality is unavailable at the destination, provided that the S-R channel quality is at least as good as the S-D channel quality.

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