Optimization for Fractional Cooperation in Multiple-Source Multiple-Relay Systems

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***Abstract*—In fractional cooperation, many relays simultaneously assist the source, and each relay is responsible to relay only a fraction of the source transmission. In this paper, the problem of fractional cooperation is considered in the presence of multiple sources and multiple relays. In particular, optimization problems are formulated that can be used to allocate the relay resources between multiple sources to either minimize the energy consumed to achieve a given probability of error threshold, or minimize the maximum probability of error experienced by each source node.**

I. Introduction

In traditional networks, multiple antennas can be used to provide spatial diversity in order to combat the detrimental effects of fading channels. However, if the system is comprised of nodes that are small, using multiple antennas may not be practical when the separation distance between antennas needed to provide independent paths is not present. Instead, nodes can recruit the help of other nodes to relay their data to the destination. The achievable rate of relay channels for decode-and-forward and compress-and-forward, as well as the upper bound on the capacity of the relay channel, can be found in [1]. The use of cooperation in cellular networks to combat fading and improve the power usage and coverage can be found in [2], [3]. In addition, the diversity-multiplex tradeoff of decode-and-forward and amplify-and-forward can be found in [4]. There are also numerous papers studying various aspects of relay channels, and the literature listed above presents only a sample of the vast amount of research performed in this area.

In most of the research performed in the area of relay networks and cooperative communications, it is assumed that the relaying node either does not relay at all or relays the complete source codeword. In [5], *fractional cooperation* was introduced, where instead of relaying the complete source codeword, relays can choose to forward only a fraction of the codeword. This allows the task of relaying for the source to be distributed among various relays, and minimizes the chance of some of the nodes draining their power much sooner than others, thereby maximizing the lifetime of the network. In addition, in the case where the relaying node has only limited resources, it need not devote its entire resources to relaying, and may instead choose the amount that it contributes. Independently, the achievable rate regions for networks where nodes transmit both data of its own as well as relayed data were found in [6].

Fractional cooperation has not previously been applied to the multiple-source, multiple-relay case. This mode of operation raises an interesting problem: in a network comprised of multiple source nodes and multiple relay nodes, it is unclear how much of its available resources each relay should devote to different source nodes. The most accurate method is to perform an exhaustive search over all possible assignments. However, this is extremely inefficient and hence impractical to implement. In [7], an upper bound on the frame error rate (FER) for fractional cooperation, derived from union-Bhattacharyya bound for parallel channels [8], was presented. With the use of this upper bound, the condition under which the average FER is below a given threshold is found. This provides us with a method of easily checking whether the given parameter and channel condition allows the average FER to be below a desired threshold, thereby guaranteeing successful communication. In this paper, our main contribution is to present two optimization problems to optimize the performance of multiple-source, multiple-relay networks where fractional cooperation is used: in the case where the available resources allows all the source codewords to be decoded correctly, the energy consumption is minimized; on the other hand, if the resources does not allow correct decoding of all the source codeword, the worst FER over all source nodes is minimized. Our approach in these problems is to optimize the union-Bhattacharyya bound, which has the advantage of being much easier to calculate than the true FER.

This paper is organized as follows. In Sec. II, we introduce the system model used throughout this paper and describe the parameters associated with fractional cooperation. Some background information on the union-Bhattacharyya bound is given in Sec. III. The optimization problem for the multiple-source multiple-relay channel is illustrated in Sec. IV. A variation on the optimization problem is introduced in Sec. V, where instead of a fixed rate, relays can form codewords based on the relayed bits, and optimize the code rate. Simulation results are shown in Sec. VI.

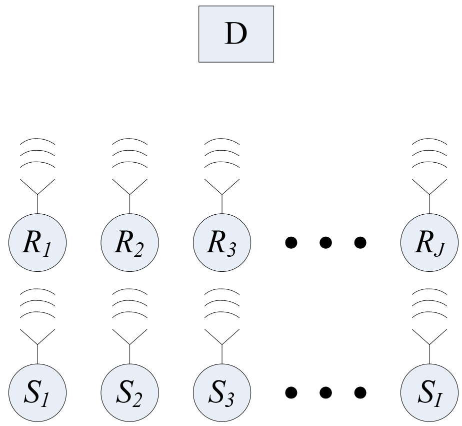

Fig. 1. System model of the multiple-source multiple-relay network.

II. System Model

A multiple-source multiple-relay system is illustrated in Fig. 1. The system comprises of I source nodes (S_i), J relay nodes (R_j) and one destination node (D). The transmission of the source nodes are received by all the relays and the destination node. It is assumed that each relay can only transmit or receive at a time and that symbol synchronization is not available. Orthogonal channels are used to facilitate transmissions from different nodes. A quasi-static Rayleigh fading channel model is used to describe the links between the nodes. It is assumed that all receivers have channel state information, and the instantaneous SNR between source nodes and all the relays are known at the destination as well.

The source node S_i forms a codeword $\mathbf{d}_i^{(S)} \in \{0,1\}^{l_{S_i}}$ of rate r_{S_i}, and assuming binary phase-shift keying (BPSK) is used, the codeword is mapped to $\mathbf{c}_i^{(S)} \in \{+1,-1\}^{l_{S_i}}$. This mapping is denoted by the function $\xi(\cdot)$. The transmission is separated into two phases. In the first phase, the source node S_i broadcasts the codeword $\mathbf{c}_i^{(S)}$ and the discrete-time received signal at relay j and D are given by

$$\mathbf{y}_{S_i,R_j} = h_{S_i,R_j}\mathbf{c}_i^{(S)} + \mathbf{n}_{S_i,R_j} \quad (1)$$

$$\mathbf{y}_{S_i,D} = h_{S_i,D}\mathbf{c}_i^{(S)} + \mathbf{n}_{S_i,D}, \quad (2)$$

where h_{S_i,R_j} and $h_{S_i,D}$ are the fading channel coefficients between source node S_i and relay node R_j and between source node S_i and D respectively, and $\mathbf{n}_{S_i,R_j}$ and $\mathbf{n}_{S_i,D}$ are independent additive white Gaussian noise (AWGN) with variance N_{R_j} and N_D respectively.

After receiving $\mathbf{y}_{S_i,R_j}$, the relay node R_j processes the received data. Instead of relaying the complete source codeword, each relay may choose to only relay a *fraction* of it. Let $\epsilon_{j,i}$ be the fraction of $\mathbf{c}_i^{(S)}$ relayed by relay R_j. Using all the bits that it has decided to relay, R_j forms a new symbol vector $\mathbf{c}_j^{(R)}$, which is then transmitted to the destination node. The length of the new signal formed by different relays can be different for different relays. The discrete time signal transmitted by R_j and received by D is given by

$$\mathbf{y}_{R_j,D} = h_{R_j,D}\mathbf{c}_j^{(R)} + \mathbf{n}_{R_j,D}, \quad (3)$$

where $h_{R_j,D}$ is the channel coefficient between relay R_j and D, and $\mathbf{n}_{R_j,D}$ is the AWGN with variance N_D. The average received signal-to-noise ratio (SNR) of the channel between S and relay k is given by

$$\bar{\gamma}_{S_i,R_j} = \mathbb{E}[\gamma_{S_i,R_j}] = \mathbb{E}[|h_{S_i,R_j}|^2]/N_{R,j}, \quad (4)$$

where $\mathbb{E}[\cdot]$ denotes statistical expectation and γ_{S_i,R_j} is the instantaneous SNR of the channel between S_i and the R_j relay. The average received SNR of the S_i-D and R_j-D channel, $\bar{\gamma}_{S_i,D}$ and $\bar{\gamma}_{R_j,D}$, can be found in a similar fashion.

Here we assume that demodulate-and-forward (DemF), a low-complexity relaying scheme [5] is used for processing at the relays. In DemF, the relay performs symbol-by-symbol decoding of the source codeword. As BPSK is used, this is equivalent to performing a two-level quantization for each bit. The relay then chooses at random a fraction of the source codeword. Depending on the amount of contribution provided by R_j to assist its neighbors, the number of bits chosen by the relay for transmitting can vary. Note that with fractional cooperation, no coordination between the relays is required. Each relay node chooses the fraction independently. There is, however, a need to describe this information to the destination node to ensure correct decoding.

At the destination node, sum-product algorithm (SPA) [9] is used for decoding. With the use of the SPA, the reliability of the source-relay links can be taken into account in the decoding process, and the effects of error propagation is mitigated. As the source-relay link quality is needed for the decoding process, this information must be transmitted by the relays to the destination node.

III. Union-Bhattacharyya Bound

The union-Bhattacharyya (UB) bound can be used to provide upper bounds on the maximum likelihood (ML) bit error rate (BER) and FER for linear codes [10]. The UB bound for the FER of a point-to-point channel is given by

$$P_f \leq \sum_{h=1}^{n} A_h \beta^h. \quad (5)$$

As illustrated in the equation, two components are required for calculating the UB bound: the weight enumerator (WE) A_h and the Bhattacharyya parameter (BP) β. The WE is a vector of numbers A_h that describes the number of codewords with weight h. The derivation of the WE averaged over all possible codebooks for “turbo-like” codes was presented in [11], where repeat-accumulate (RA) codes were also introduced. The second component, BP, is associated with the channel condition. The BP for binary input channels with inputs $\{0,1\}$ and output y is given by

$$\beta \triangleq \sum_{y \in \mathcal{Y}} \sqrt{p(y|0)p(y|1)} \quad (6)$$

where $\mathcal{Y}$ is the alphabet of output y, and $p(y|0)$ and $p(y|1)$ are, respectively, the probability of y given 0 and 1 was sent. For example, for the binary-input AWGN with SNR γ, the BP is given by $\beta_{Gauss} = \exp\{-\gamma\}$. For the source-relay-destination link where DemF is used, the BP is given by [7]

$$\beta_{DemF} = \sqrt{\frac{\gamma_{\rm RD}}{\pi}} \exp\left\{-\gamma_{\rm RD}(y^2+1)\right\} \int_{-\infty}^{\infty} [((1-p_{\rm SR})e^{2\gamma_{\rm RD}y} + p_{\rm SR}e^{-2\gamma_{\rm RD}y}) (p_{\rm SR}e^{2\gamma_{\rm RD}y} + (1-p_{\rm SR})e^{-2\gamma_{\rm RD}y})]^{1/2} dy \quad (7)$$

where $p_{\rm SR}$ is the bit-flip probability over the source-relay channel, and $\gamma_{\rm RD}$ is the SNR over the relay-destination channel.

From (5), it can be seen that the smaller β is, the smaller the upper bound. These results can be extended to scenarios where the codeword is sent through multiple parallel channels [8]. Assume that there are K parallel channels, and let α_k be the probability of a bit being transmitted through channel k, where $\sum_{k=k}^{K} \alpha_k = 1$. By averaging over all possible channel assignments for the codeword bits, the authors derived the parallel-channel UB bound on the FER

$$P_f \le \sum_{h=1}^{n} A_h \left(\sum_{k=1}^{K} \alpha_k \beta_k\right)^h \quad (8)$$

where β_k is the BP for channel k. The incremental redundancy cooperative coding scheme, where the source codeword is divided into subsets, with each subset relayed by at most one relay, is analyzed using UB bound in [12].

The UB bound can also be applied to fractional cooperation [7]. Assume that R_j relays each bit of S_i with probability $\epsilon_{j,i}$. After some simple manipulation, it can be shown that for a multiple-source multiple-relay system

$$P_{f,i} \le \sum_{h=1}^{n} A_h \left(\beta_{{\rm SD},i} \prod_{j=1}^{J} \left(1-\epsilon_{j,i}\left(1-\beta_{j,i}\right)\right)\right)^h \quad (9)$$

where $P_{f,i}$ is the probability of frame error for source i, $\beta_{{\rm SD},i} = \exp\{-\gamma_{S_i,D}\}$ is the BP for the link between S_i and the destination node, and $\beta_{j,i}$ refers to the BP for the link between S_i and the destination node via R_j.

As illustrated in (9), the average frame error is below the threshold $P_{f,t}$, if

$$\beta_{{\rm SD},i} \prod_{j=1}^{r} \left(1-\epsilon_{j,i}(1-\beta_{j,i})\right) \le \beta_t \quad (10)$$

where $P_{f,t} = \sum_{h=1}^{n} A_h \beta_t^h$.

IV. Optimization in Fractional Cooperation

In a multiple-source multiple-relay system, it is desirable to optimize the allocation of the relay resources to obtain the best results. In this section, we present two optimization problems. In the first problem, we assume that there is a minimum FER threshold that must be satisfied to guarantee no outage. Assuming that the amount of resources available is more than what is required to satisfy the FER threshold for all the source node, it would be optimal to minimize the amount of energy expanded while still satisfying the FER threshold. In the second optimization problem, it is assumed that resources available cannot allow the FER threshold to be satisfied by all the source nodes. Instead of optimizing the energy consumption, it is now desirable to ensure the relay resources are distributed in a manner such that the maximum FER over all source codewords is minimized. The formulation of the two problems are presented below.

For simplicity, it is assume that the length of the source codewords are the same and is denoted as l_S. Let $\bar{\epsilon}_j$ be the maximum contribution relay j is willing to contribute. In addition, it is assumed that no encoding is available at the relay. A variation on these problems, where it is assume that the relays can change the code rate of the relay codeword, is presented in a later section.

A. Optimizing Energy Consumption

In this optimization problem our goal is to minimize the total energy consumption while still achieving the threshold $P_{f,t}$ for all the source nodes. Given the threshold β_t, the optimization problem can be formulated as follows

$$\begin{aligned} &\min_{\epsilon_{j,i}} \sum_{i=1}^{I}\sum_{j=1}^{J} \epsilon_{j,i} \\ &\text{subject to } \beta_{SD,i}\prod_{j=1}^{J}(1-\epsilon_{j,i}(1-\beta_{j,i})) \le \beta_t \qquad i=1,\ldots,I \\ &\qquad \sum_{i=1}^{I} \epsilon_{j,i} \le \bar{\epsilon}_j \qquad j=1,\ldots,J \\ &\qquad \epsilon_{j,i} \ge 0. \end{aligned} \quad (11)$$

The solution to this optimization problem can be found using the steepest descend method.

B. Optimizing Error Rate

In this optimization problem, the goal is to minimize the maximum error rate over all source nodes, while not exceeding the amount of resources available. The optimization problem is formulated as follows

$$\min_{\epsilon_{j,i}} \quad \tilde{\beta}_t \quad (12)$$

$$\begin{aligned} &\text{subject to } \beta_{SD,i}\prod_{j=1}^{J}(1-\epsilon_{j,i}(1-\beta_{j,i})) \le \tilde{\beta}_t \qquad i=1,\ldots,I \\ &\qquad \sum_{i=1}^{I} \epsilon_{j,i} \le \bar{\epsilon}_j \qquad j=1,\ldots,J \\ &\qquad \epsilon_{j,i} \ge 0 \end{aligned} \quad (13)$$

With the use of the dummy variable $\tilde{\beta}_t$, we ensure that the relay assistance is distributed as to minimize the maximum error rate. Similar to the previous optimization problem, the solution to this optimization problem can be found using the steepest descent method.

V. Optimization with Relay Encoding

Instead of using repetitive codes, where the relay merely repeats the bits chosen to relay, DemF allows the relays to form codewords based on the chosen bits and transmit to the destination node. Some examples include low-density generator matrix (LDGM) and RA codes [5]. These codes are chosen because the rate of these codes can be adjusted easily by the relay, depending on the relay-destination channel quality. In addition, they have excellent performance in AWGN channels. In this case, the cost of transmitting one bit are not equal for all the nodes anymore, and is scaled by the rate of the channel code used. If a rate $r_{j,i}$ code is used for the bits relayed by relay j for source i, the optimization becomes

$$\begin{aligned} &\min \sum_{i=1}^{I}\sum_{j=1}^{J}\frac{\epsilon_{j,i}}{r_{j,i}} \\ &\text{subject to } \beta_{SD,i}\prod_{j=1}^{J}(1-\epsilon_{j,i}(1-\beta_{j,i})) \leq \tilde{\beta}_t \qquad i=1,\ldots,I \\ &\qquad \sum_{i=1}^{I}\frac{\epsilon_{j,i}}{r_{j,i}} \leq \bar{\epsilon}_j \qquad j=1,\ldots,J \\ &\qquad \epsilon_{j,i} \geq 0. \end{aligned} \tag{14}$$

where in this case, $\beta_{j,i}$ is a function of $r_{j,i}$.

It is not straightforward to characterize the relationship between $\beta_{j,i}$ and $r_{j,i}$ as the relationship is also dependent on the channel code used. The optimization problem can be solved by separating it into two subproblems. In the first subproblem, the optimal values of $\epsilon_{j,i}$ are obtained from solving (14) for a fixed rate $r_{j,i}$ (and hence fixed $\beta_{j,i}$) for each pair of source and relay nodes. The optimal values of $\epsilon_{j,i}$ are denoted as $\epsilon^*_{j,i}$, and the value used in the optimization problem is denoted as $r^*_{j,i}$. In the second subproblem, optimal values of $\epsilon_{j,i}$ and $r_{j,i}$ are solved for each source-relay pair, given the channel qualities over the source-relay and relay-destination link. This is done by solving the following optimization problem

$$\min \quad (1-\epsilon_{j,i}(1-\beta_{j,i})) \tag{15}$$

$$\text{subject to} \quad \frac{\epsilon_{j,i}}{r_{j,i}} \leq \frac{\epsilon^*_{j,i}}{r^*_{j,i}} \tag{16}$$

such that the fraction used by each relay for each source is fixed. After obtaining the optimal $r_{j,i}$ for each source-relay pair, the value is substituted into the first subproblem and the process is repeated until the change in $\sum_{i,j}\epsilon_{j,i}/r_{j,i}$ is less than δ, where $\delta > 0$. As both of these optimization subproblems provide solutions that are non-increasing, a local minimum can be found.

VI. Simulation Results

In this section, we will show that by optimizing the distribution of relay resources, we can improve the performance of the system while minimizing the energy consumption. In all the simulations presented in this paper, a systematic rate-1/2 RA code is used for encode the source symbols, where $l_s = 4000$. In Fig. 2, the FER in an AWGN channel for a two-source two-relay system is shown. The SNR between the nodes are given by

$$\begin{aligned} \gamma_{S_1,D} &= -4 \text{ dB} & \gamma_{S_2,D} &= -2 \text{ dB} \\ \gamma_{S_1,R_1} &= -2 \text{ dB} & \gamma_{S_2,R_1} &= -1 \text{ dB} \\ \gamma_{S_1,R_2} &= -2 \text{ dB} & \gamma_{S_2,R_2} &= -1 \text{ dB} \\ \gamma_{R_1,D} &= 0 \text{ dB} & \gamma_{R_2,D} &= 0 \text{ dB} \end{aligned}$$

In this case, the resources available are not adequate to ensure correct encoding at both source nodes. All the resources are exhausted to minimize the maximum FER over both source nodes, as indicated in (13). In the plot, the contours for the maximum FER over both source nodes for different $\epsilon_{1,1}$ and $\epsilon_{2,1}$ values are shown, where $\bar{\epsilon}_1 = \bar{\epsilon}_2 = 0.8$. As all the resources are exhausted in this scenario, $\epsilon_{1,2} = \bar{\epsilon}_1 - \epsilon_{1,1}$ and $\epsilon_{2,2} = \bar{\epsilon}_2 - \epsilon_{2,1}$. In the plot, the marker indicates the optimal values of $\epsilon_{1,1}$ and $\epsilon_{2,1}$ obtained from the optimization problem (13), where $\epsilon_{1,1} = \epsilon_{2,1} = 0.63$, and the FER is 0.2933 for both source nodes. As shown in the plot, the minimization of the maximum FER is achieved, where resources are distributed such that both source nodes have the same FER, even though S_2 has better links to the relays and the destination node than S_1. This optimization is therefore extremely useful when the link quality is very different for the source nodes and fair performance is desired.

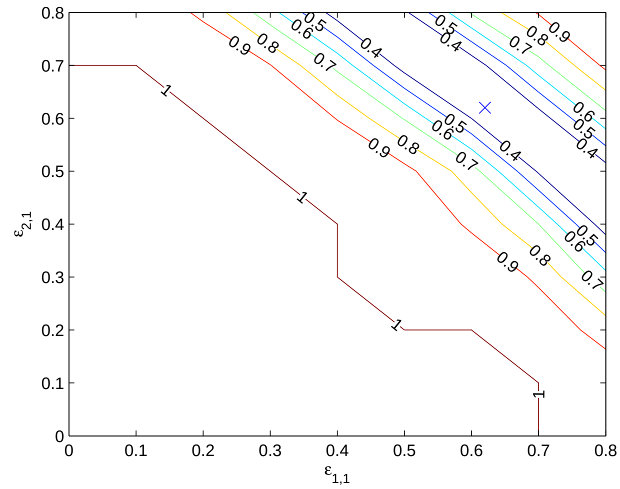

Fig. 2. Contours of maximum FER for two-source two-relay network with AWGN.

In the next set of simulation results, we show how the optimization can help us minimize the amount of energy consumed while minimizing the maximum FER in fading channels. Similar to the previous plot, this system consists of two source and two relay nodes. Also, $\bar{\epsilon}_1 = \bar{\epsilon}_2 = 0.8$. In Fig. 3, the maximum FER over the two source nodes are shown. The average SNR for the source-destination links are

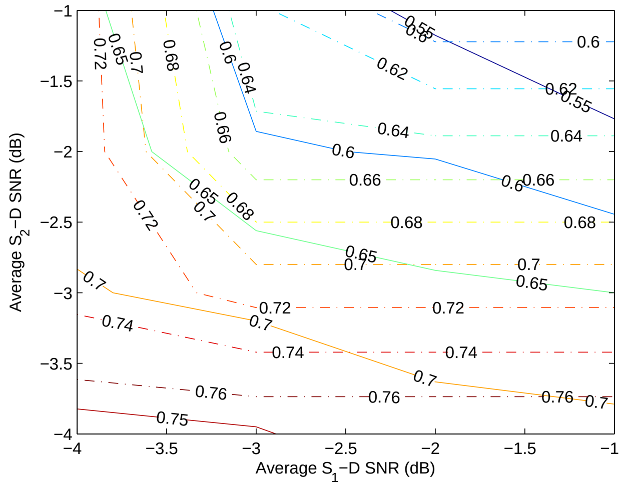

Fig. 3. Contours of maximum FER for two-source two-relay network with fading channels using optimization (solid) and equal allocation (dash-dot).

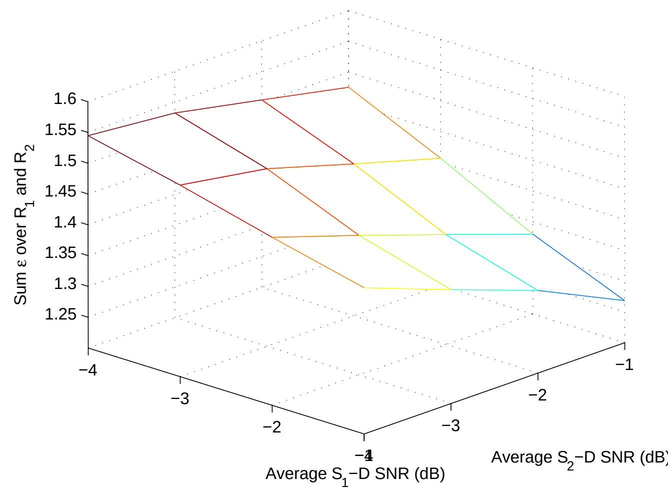

Fig. 4. Plot of $\sum_{i,j} \epsilon_{j,i}$ for two-source two-relay network in fading channels.

varied, while the average SNR over the rest of the links are

$$\bar{\gamma}_{S_1,R_1} = -2 \text{ dB} \qquad \bar{\gamma}_{S_1,R_2} = -2 \text{ dB}$$
$$\bar{\gamma}_{S_2,R_1} = -1 \text{ dB} \qquad \bar{\gamma}_{S_2,R_2} = -1 \text{ dB}$$
$$\bar{\gamma}_{R_1,D} = 0 \text{ dB} \qquad \bar{\gamma}_{R_2,D} = 0 \text{ dB}$$

In the plot, the solid lines represent contours the maximum FER for the case where optimization is performed, and the dash-dot lines represent contours for the maximum FER for the case where the resources are distributed evenly among the two source nodes.

In Fig. 4, the value of $\sum_{i,j} \epsilon_{j,i}$ corresponding to Fig. 3 is shown. From these plots, it can be observed that with optimization the amount of energy used is smaller than that if the resources were distributed evenly without optimization, and the maximum FER is smaller compared to the case where no optimization is performed as well, where $\sum_{i,j} \epsilon_{j,i} = \bar{\epsilon}_1 + \bar{\epsilon}_2 = 1.6$.

VII. Conclusions and Future Work

In this paper, we have provided the formulations of two optimization problems using the union-Bhattacharyya bound. In the first optimization problem, assuming that there is an excess of relay resources than that is required to achieve a given FER threshold. In this case, the optimization problem solves for the minimum amount of energy required to achieve the FER threshold. In the second optimization problem, it is assumed that the resources available are not adequate to achieve the FER threshold. In this case, the goal is to minimize the maximum FER experienced by each source. This is achieved by allocating the resources in a fair manner such that the source nodes that need most help gets the assistance needed. In a fading channel, these two optimization problems can be used to minimize the energy consumption while minimizing the maximum FER, allowing the efficient use of relay resources. Note that even though the optimization of system resource allocation is illustrated using DemF, this is applicable to other relaying schemes, such as decode-and-forward and amplify-and-forward. Finally, as fractional cooperation is a distributed scheme where coordination between relay nodes are not required, it is desirable to optimize the allocation of relay resources in a distributed manner.

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