Relay Selection for Low-Complexity Coded Cooperation using the Bhattacharyya Parameter

Josephine P. K. Chu*, Raviraj S. Adve* and Andrew W. Eckford†
*Dept. of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada
†Dept. of Computer Science and Engineering, York University, Toronto, Ontario, Canada
E-mail: {chuj,rsadve}@comm.utoronto.ca, aeckford@yorku.ca

***Abstract*—Demodulate-and-forward (DmF) is an attractive approach when using cooperative diversity schemes in networks where only nodes with strict complexity constraints are allowed, such as sensor networks. In using DmF, the relay only demodulates, but does not decode, the received signal from the source node. Coding can be used at the relay to improve the performance over the relay-destination link. In unrelated work, relay selection has been shown to achieve full diversity order with low overhead by choosing the best relay node out of a pool of available relays to assist the source. A simple heuristic scheme for relay selection while using DmF is available, but this involves the exchange of channel parameters between the nodes, hence increasing the overhead. In this paper, we propose the use of the Bhattacharyya parameter (BP) to facilitate relay selection. The use of BP has the distinct advantage of incorporating the specific coding scheme used while retaining low computation load. As illustrated in our simulation results, the use of BP provides frame error rates quite similar to that obtained from exhaustive search. We should note that this BP-based relay selection scheme can also be applied to cooperation schemes where decoding is performed at the relay.**

I. Introduction

The seminal work on relay channels is [1], where the upper bound on the capacity of relay channels was presented, and two different relay schemes, decode-and-forward (DF) and compress-and-forward (CF) were introduced and analyzed. Recently there has been a renewed interest in relay channels for diversity gains in mesh and sensor networks. Numerous works are now available, such as [2], [3], [4], [5]. A practical implementation of CF can be found in [6], where Wyner-Ziv coding is used.

When relays only possess simple hardware, instead of decoding the source codeword, the relay can *demodulate* the received signal [7], [8]. As illustrated in [7], diversity order of 2 can be obtained even with such a constraint. In [9], [10], coding schemes with low encoding complexity such as low-density generator matrix (LDGM) [11] and repeat-accumulate (RA) [12] codes were applied to this type of "demodulate-and-forward" (DmF) scheme to provide both coding and diversity gains. In contrast, the AF scheme is simple to implement, but does not provide coding gains. Note that DmF is a simple version of CF, where only two levels of quantization is allowed, and part of the codeword is "erased" to allow a lower code rate over the relay-destination channel [13].

In order to increase diversity order of the relay network, multiple relays can be used to assist the source in transmission. However, if orthogonal channels are being used by the source and relays to transmit, the time or bandwidth must be shared, hence limiting the transmission rate. In addition, this increases the total energy used for transmitting every source bit. As an alternative which eliminates this overhead, relay selection was introduced in [14], [15] to achieve full diversity order without sacrificing capacity or power efficiency. With relay selection, only the best relay out of a pool of available relays is chosen to assist the source node. Relay selection for DF is relatively straightforward: from the pool of relays that have decoded the source codeword correctly, the relay with the best relay-destination channel is chosen to assist the source [15]. If AF is used, the relay with the best source-relay-destination compound channel is chosen to assist the source [16].

As discussed in [17], when demodulate-and-forward is used in conjunction with coding, relay selection must account for the code structure as well. The optimal way to choose the best relay is an exhaustive search using, for example, density evolution [18], and storing all the values in a lookup table. The 3-dimensional lookup table with values for every possible channel realization can be large, hence making this not suitable for nodes with limited storage capacity. In [17], a simple scheme for relay selection was introduced, where the relay with the largest mutual information of the equivalent relay channel is chosen to assist the source. However, this calculation involves knowing the fading coefficients over all the channels. In addition to the source-relay and relay-destination channel coefficients, the source-destination channel coefficient must be acquired for the calculation, which incurs extra overhead because the communication of these values between nodes is required. In this paper, we will present a selection scheme using the Bhattacharrya parameter (BP) [19]. The use of the BP provides significant benefits: (i) the calculation only requires channel coefficients involving the relay, hence reducing the transmission overhead required to select the best relay; (ii) the BP entails a low computation load; (iii) this scheme is more general as it can be applied to any type of linear codes whose weight enumerator can be characterized, (iv) the proposed scheme is extremely flexible in that it can be used for other cooperative schemes and even scenarios where relays perform partial decoding, where only limited number of decoding iterations are performed.

This paper is organized as follows. Section II presents the system model of the 3-node relay channel, as well as the encoding and decoding methods. A brief overview of

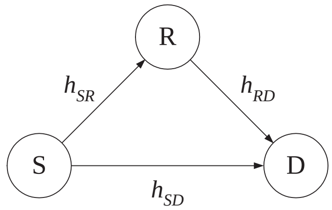

Fig. 1. System model of the 3-node relay channel.

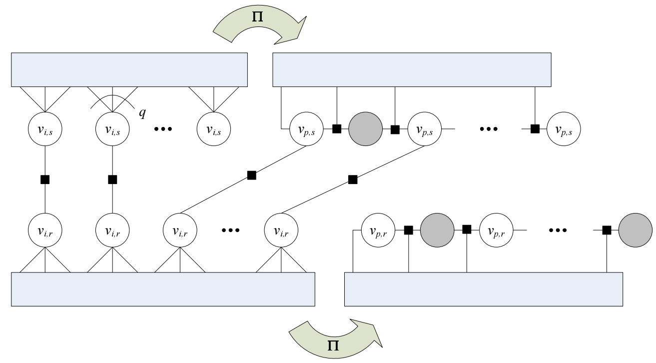

Fig. 2. Factor graph of demodulate-and-forward with the use of RA code.

cooperative RA codes, the union bound and the Bhattacharyya parameter is presented in Section III, and its application in the DmF relaying is explained in Section IV. Simulation results are presented and discussed in Section V, and finally, some concluding remarks will be drawn and some suggestions for future work are provided in Section VI.

II. System Model

In this paper the 3-node relay network of Fig. 1 is used for the analysis, as only one relay is chosen to provide diversity. In Fig. 1 S, R and D are the source, relay and destination nodes respectively. Cooperative DmF schemes are devised with sensor networks in mind, where the battery power and hardware complexity of a source or relay node are limited, but the destination is assumed to possess relatively more complex hardware, and energy consumption is not an issue. It is assumed that the relay can only transmit or receive at a time and that symbol synchronization is not available. A quasi-static Rayleigh fading channel model is used to describe the links between the nodes. It is assumed that all receivers have channel state information, and the S-R channel coefficients are known at the destination as well.

Communication from source to destination is divided into two phases. The source node first forms a codeword $\mathbf{d}_s \in \{0,1\}^{l_s}$ of rate r_s, and assuming binary phase-shift keying (BPSK) is used, the codeword is mapped to $\mathbf{c}_s \in \{+1,-1\}^{l_s}$. This mapping is denoted by the function $\xi(\cdot)$. In the first phase, the source node broadcasts the codeword $\mathbf{c}_s$ and the discrete-time received signal at R and D is given by

$$\mathbf{y}_{\mathrm{SR}} = h_{\mathrm{SR}}\mathbf{c}_s + \mathbf{n}_R \tag{1}$$

$$\mathbf{y}_{\mathrm{SD}} = h_{\mathrm{SD}}\mathbf{c}_s + \mathbf{n}_{\mathrm{SD}}, \tag{2}$$

where h_{SR} and h_{SD} are the S-R and S-D fading channel coefficients respectively, and $\mathbf{n}_R$ and $\mathbf{n}_{\mathrm{SD}}$ are independent additive white Gaussian noise with variance $N_{0,R}$ and $N_{0,D}$ respectively. After receiving $\mathbf{y}_{\mathrm{SR}}$, the relay makes hard decisions on a part of the received signal, forms a new codeword, $\mathbf{d}_r$ based on the estimated (not decoded) bits, and generates $\mathbf{c}_r = \xi(\mathbf{d}_r)$. In the second phase, the relay transmits the codeword, and the discrete time received signal at D is give by

$$\mathbf{y}_{\mathrm{RD}} = h_{\mathrm{RD}}\mathbf{c}_r + \mathbf{n}_{\mathrm{RD}}, \tag{3}$$

where h_{RD} is the R-D channel coefficient and $\mathbf{n}_{\mathrm{RD}}$ is the additive white Gaussian noise with variance $N_{0,D}$. The average received signal-to-noise ratio (SNR) of the S-R channel is given by

$$\bar{\lambda}_{\mathrm{SR}} = \mathbb{E}[\lambda_{\mathrm{SR}}] = \mathbb{E}[|h_{\mathrm{SR}}|^2]/N_{0,R}, \tag{4}$$

where $\mathbb{E}[\cdot]$ denotes statistical expectation and λ_{SR} is the instantaneous SNR of the S-R channel. The average received SNR of the S-D and R-D channel, $\bar{\lambda}_{\mathrm{SD}}$ and $\bar{\lambda}_{\mathrm{RD}}$, can be found in a similar fashion.

III. RA Codes and the Bhattacharyya Parameter

In this section we briefly review RA codes and the Bhattacharyya parameter. RA codes are used here to implement coded cooperation, though the notion of coded cooperation could be extended to any code of choice.

A. Cooperative RA Code

The cooperative DmF scheme with RA codes was first introduced in [10]. Systematic punctured RA codes are used for the flexibility in changing code rates. The codeword is generated by concatenating the information bits and the punctured parity bits. The parity bits are formed by first repeating the information bits q times, then passing the bits through a random interleaver, and finally passing the bits through a truncated rate-1 recursive convolutional encoder with transfer function $1/(1+D)$. The encoder output is used as the parity bits after the appropriate puncturing is done to obtain the required code rate. Throughout this paper, we set $q = 3$, hence allowing the code rate to range from 1/4 to 1.

In DmF, only a fraction of the received signal from the source is relayed. We introduce two parameters to help describe the transmission scheme: ϵ_i is used to describe the fraction of source *information* bits that are relayed, and ϵ_p is used to describe the fraction of source *parity* bits relayed. A factor graph of demodulate-and-forward with the use of RA codes is shown in Fig. 2. In the figure, circles and squares represent variable and check nodes respectively, where shaded variable nodes represent punctured parity bits. The labels $v_{i,s}$, $v_{p,s}$, $v_{i,r}$ and $v_{p,r}$ represent source information and parity bits, and relay information and parity bits respectively, and Π is the random interleaver. Decoding at the destination is performed using the sum-product algorithm (SPA) [20]. This algorithm can be applied easily to the cooperative coding scheme where the unreliability over the S-R link is accounted for. More information on the SPA scheme for coded demodulate-and-forward can be found in [10].

B. Union Bound and Bhattacharyya Parameter

The union bound can be used to provide upper bounds on the maximum likelihood bit error rate (BER) and frame error rate (FER) for convolutional codes [19]. Two components are required for calculating the union bound: the weight enumerator (WE) and the Bhattacharyya parameter (BP). The WE is a vector of numbers A_h that describes the number of codewords with weight h. This can be derived from the input-output weight enumerator (IOWE). The IOWE of a code is an array of numbers $A_{w,h}$ that describes the number of codewords with input weight w and output weight h. The derivation of IOWE for "turbo-like" codes was presented in [21], where RA codes were also introduced. The WE is then obtained from the IOWE by summing over all w. The second component, BP, is associated with the channel condition. The BP for binary input channels with inputs $\{0, 1\}$ and output y is given by

$$\gamma \triangleq \sum_{y \in \mathcal{Y}} \sqrt{p(y|0)p(y|1)} \tag{5}$$

where $\mathcal{Y}$ is the alphabet of output y, and $p(y|0)$ and $p(y|1)$ are, respectively, the probability of y given 0 and 1 was sent. For the binary symmetric channel (BSC) with bit-flip probability p, $\gamma = 2\sqrt{p(1-p)}$, and for the additive white Guassian noise channel with received SNR λ, $\gamma = e^{-\lambda}$.

After deriving the WE and BP, the FER and BER bounds can be found by

$$P_f \leq \sum_{h=1}^{n} A_h \gamma^h = \sum_{h=1}^{n} \left(\sum_{w=1}^{k} A_{w,h} \right) \gamma^h \tag{6}$$

$$P_b \leq \sum_{h=1}^{n} B_h \gamma^h = \sum_{h=1}^{n} \left(\sum_{w=1}^{k} B_{w,h} \right) \gamma^h \tag{7}$$

where $B_{w,h} = A_{w,h} w/k$ and $B_h = \sum_{w=1}^{k} A_{w,h} w/k$. Note that this can be extended to scenarios where the codeword is sent through parallel channels, each with different BP [22]. Assuming the codeword has n bits, let the number of parallel channels be J, and let the set $\mathcal{I} = \{1, \ldots, n\}$ be divided among J subsets $\mathcal{I}(j), j = 1, \ldots, J$, where if $i \in \mathcal{I}(j)$, then bit i is sent through the jth channel. In this case, the upper bound for the FER is given by

$$P_f \leq \sum_{h_1=0}^{|\mathcal{I}(1)|} \cdots \sum_{h_J=0}^{|\mathcal{I}(J)|} A_{h_1,\ldots,h_J} \gamma_1^{h_1} \ldots \gamma_J^{h_J} \tag{8}$$

where $A_{h_1,\ldots,h_J}$ denote the number of codewords with Hamming weight h_j over the index set $\mathcal{I}(j)$ for $j \in \{1, \ldots, J\}$, and γ_j is the BP for the jth channel. The upper bound for the BER with parallel channels can be found in a similar manner. Note that this section only provides a brief overview of the union bound and BP. A more detailed description, including the derivation of the union bound, can be found in [19].

IV. Bhattacharyya Parameter for Relay Selection

Without loss of generality, the cooperative RA code in [10] will be used to explain the use of BP in relay selection. For simplicity, we are assuming a rate-1/4 systematic RA code is used by the relay. We also assume the "serial decoding" is performed, where the relay codeword is decoded first, before the soft output information, together with the received signal from the source, is used to decode the source codeword. In the application of (8) to the relay channel, we have $J = 2$, $|I(1)| = l_s$ and $|I(2)| = \epsilon_i k_s + \epsilon_p (l_s - k_s)$. From the source codeword point-of-view, it is first transmitted through the S-D channel, with the associated BP $\gamma_1 = e^{-\lambda_{\mathrm{SD}}}$. Then, for the bits that are relayed, they are transmitted through the S-R-D channel, with the associated BP γ_2.

As shown in (8), for a fixed code and a fixed relaying scheme, the IOWE, ϵ_i and ϵ_p remain constant, and the bound on the FER and BER decreases as the BP decreases. When choosing the optimal relay, the BP γ_1 is fixed as the S-D channel is the same for all relays; the only value that changes for different relay is γ_2. Hence, as γ_2 decreases, the UB on FER and BER goes down. We *conjecture* that choosing the relay with a link that gives the minimal γ_2 will provide the best FER and BER performance.

As "serial decoding" is used at the destination node, the calculation of γ_2 can be separated into two steps. In the first step, we need to obtain the distribution of the output signal after decoding the relay codeword. This can be obtained from density evolution, with the value stored in a one-dimensional lookup table. In our case, these values can also be approximated with the simpler algorithm as outlined in [23], as no puncturing is used. Only the variance of the channel distribution need to be tracked in order to obtain the output distribution. Let the output distribution for the R-D channel be modeled as

$$\begin{aligned} p(y|c_r = 1) &= f(y; 1, \sigma_{R,i}^2) \\ p(y|c_r = -1) &= f(y; -1, \sigma_{R,i}^2) \end{aligned} \tag{9}$$

where $f(t; \mu, \sigma^2)$ is the distribution of the Gaussian random variable t with mean μ and variance σ^2. The relationship of the channel distribution before and after decoding the rate-1/4 RA code is plotted in Fig. 3, where $\sigma_{R,i} = \frac{|h_{\mathrm{RD}}|^2}{N_{0,R}}$ is the variance for channel R-D, and $\sigma_{R,o}$ is the equivalent variance after decoding.

In the second step, we use the information obtained from the first step, together with the S-R channel condition, to calculate γ_2. As hard detection is used for decoding the source code bits at the relay, the S-R channel can be modeled as a BSC with bit-flip probability $p_{\mathrm{SR}} = \frac{1}{2}\mathrm{erfc}\sqrt{\lambda_{\mathrm{SR}}}$, where $\mathrm{erfc}(\cdot)$ is the complementary error function. After the equivalent channel noise variance $\sigma_{R,o}^2$ has been found through a lookup table at the relay node, given the R-D channel coefficient, the likelihood probability for the equivalent S-R-D channel after

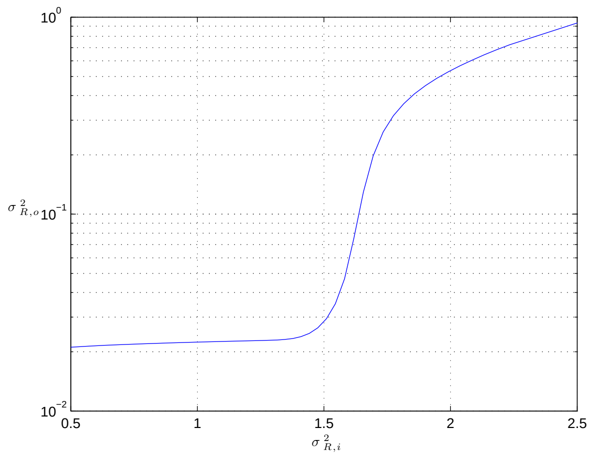

Fig. 3. Plot of equivalent channel noise variance before and after decoding.

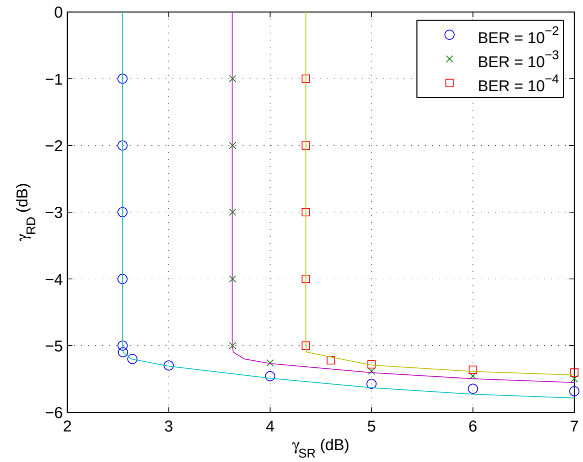

Fig. 4. Contours of BER from density evolution (*Markers*) and from (13) (*Lines*).

decoding is given by

$$p(y|c_s) = \begin{cases} (1-p_{\text{SR}})f(y;1,\sigma^2_{R,o}) + p_{\text{SR}}f(y;-1,\sigma^2_{R,o}) & \text{for } c_s = 1 \\ p_{\text{SR}}f(y;1,\sigma^2_{R,o}) + (1-p_{\text{SR}})f(y;-1,\sigma^2_{R,o}) & \text{for } c_s = -1. \end{cases} \tag{10}$$

The relay then calculates γ_2 by substituting (10) into (5)

$$\gamma_2 = \int_{-\infty}^{\infty} \sqrt{p(y|d_s=0)p(y|d_s=1)}dy \tag{11}$$

$$= \int_{-\infty}^{\infty} \{[(1-p_{\text{SR}})f(y;1,\sigma^2_{R,o}) + p_{\text{SR}}f(y;-1,\sigma^2_{R,o})]\times [(1-p_{\text{SR}})f(y;-1,\sigma^2_{R,o}) + p_{\text{SR}}f(y;1,\sigma^2_{R,o})]\}^{1/2}dy \tag{12}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2_{R,o}}} \int_{-\infty}^{\infty} \exp\left\{-\frac{y^2+1}{2\sigma^2_{R,o}}\right\} \times [4p_{\text{SR}}(1-p_{\text{SR}})\sinh^2(y/\sigma^2_{R,o})+1]^{1/2}dy. \tag{13}$$

Note that as p_{SR} becomes very small, the S-R-D channel becomes an additive white Gaussian noise channel, with $\gamma_2 = \exp\{-1/2\sigma^2_{R,o}\}$. Similarly, as $\sigma_{R,o}$ becomes very small, the S-R-D channel becomes a BSC, with $\gamma_2 = 2\sqrt{p_{\text{SR}}(1-p_{\text{SR}})}$. A closed-form expression of the integration in (13) does not exist, but it can be approximated using Taylor series expansion of $\sqrt{1+x}$. Even though slightly more complex hardware is required at the relays for the computation of BP, it is justified by the performance improvement and the energy savings that accompany it.

As stated earlier, performing density evolution on the complete factor graph in Fig. 2 provides us the best information to selecting the optimal relay to assist the source, at the cost of high complexity or large storage requirement. A comparison of this optimal method and the calculations using BP is shown in Fig. 4, where the channel conditions that give the same BER are illustrated. The parameters ϵ_i and ϵ_p are set to be 1 and 0, and the rate of the source codeword $r_s = 1/2$. In addition, the R-D channel instantaneous SNR λ_{RD} is set to -6 dB. In the figure, the markers represent values of λ_{SR} and λ_{RD} that give the BER indicated through the use of density evolution on the factor graph, and the lines show the contours representing values of λ_{SR} and λ_{RD} that give the same γ_2 values found using the lookup table and (13). As illustrated in the plot, the calculation of γ_2 through the lookup table and (13) does a good job of characterizing the S-R and R-D channel conditions that would give a certain BER, and it shows that as γ_2 decreases, the value of BER improves.

One of the advantages provided by using the Bhattacharyya parameter is that the calculation does not need to be done in a centralized manner. Only knowledge of the S-R and R-D channel condition is required, hence the BP calculation can be performed at each relay. For example, after the source has sent out a request for assistance, each relay that are able to help will calculate its associated BP, and send this information to either the source or destination node. After collecting all the γ_2 from the relays, the source or destination node will then send a bit to each of the relay that are ready to help to indicate whether it is the chosen relay. There is, however, the need for transmit channel state information, as the relays need to know the R-D channel condition. It is assumed that this information is available at the relay, and the its acquirement is outside of the scope of this paper.

V. Simulation Results

As when plotting the contours in Fig. 4, we set $\epsilon_i = 1$ and $\epsilon_p = 0$. The rate of the source code word is $r_s = 1/2$, with $l_s = 4000$, and the rate of the relay codeword is $r_r = 1/4$, with blocklength $l_r = 8000$. In this scenario, we assume that all the channels have the same average received SNR. The FER for 1, 2, or 3 relays while using the lookup table and (13) is shown in Fig. 5. We have also shown simulation results for the case where for a given S-D channel, 2 or 3 relays are used to assist in the transmission, and the relay with the least number of bit errors is used. These results with exhaustive

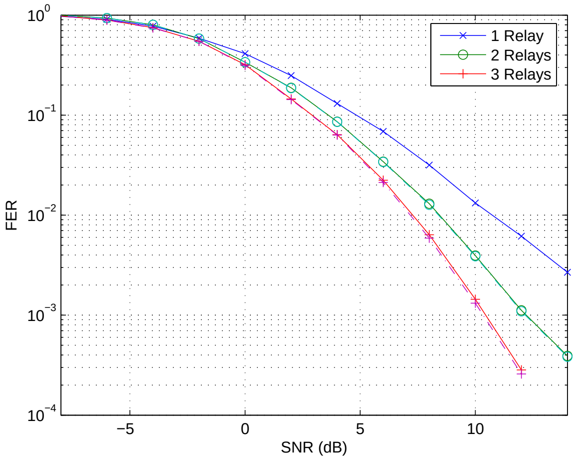

Fig. 5. FER for our relay selection scheme (*solid lines*) and exhaustive search (*dash-dot lines*).

search is used as the perfect case, and provides a lower bound on the FER performance. The simulation results for our relay selection scheme is represented by the *solid lines*, and the exhaustive search represented by the *dash-dot lines*. Note that, as expected, selection cooperation provides full diversity order despite using only one relay. As illustrated in the plot, the FER for exhaustive search and our low complexity relay selection scheme are very similar, showing the relay selection scheme using BP provides excellent performance on minimizing the FER.

VI. Conclusion and Future Work

Relay selection schemes can be employed to increase the diversity order of cooperative networks without sacrificing the power efficiency or transmission rate. In this paper, we have introduced a simple relay selection method for the cooperative coding scheme involving the use of demodulate-and-forward. One advantage the relay selection scheme has over the one presented in [17], where mutual information of the equivalent S-R-D channel is used, is that the calculation can be performed at the relay, hence reducing the communication overhead incurred while informing the nodes of the channel qualities. Furthermore, the use of the BP accounts for the code structure as well. As illustrated in the simulation results, this relay selection scheme using Bhattacharyya parameter of the S-R-D channel provides excellent error performance, and is quite close to the lower bound provided by the exhaustive search.

One drawback of this relay selection scheme is that it can be employed only when "serial decoding" is used at the destination node. On-going work includes the the possibility of applying similar relay selection for systems using "parallel decoding", where source and relay codewords are decoded in parallel, and the soft information are exchanged between the two decoders after each iteration. In addition, the application of this relay selection method to cooperation schemes where partial decoding is performed at the relay will be investigated. By allowing relays to perform partial decoding, a new dimension of flexibility is added to relay selection. Even when the S-R of a relay is not optimal within a pool of available relays, a relay can still be chosen to relay for a source node if its battery level allows it to perform decoding, thereby improving the effective S-R channel.

References

[1] T. M. Cover and A. A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. on Inform. Theory*, vol. IT-25, pp. 572–584, Sept. 1979.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[3] A. Chakrabarti, A. Sabharwal, and B. Aazhang, "Sensitivity of achievable rates for half-duplex relay channel," in *Proc. IEEE Workshop on Sig. Proc. Advances in Wireless Comm.*, June 2005.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, pp. 3037–3063, Sept. 2005.

[5] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. on Inform. Theory*, vol. 46, pp. 388–404, Mar. 2000.

[6] R. Hu and J. Li, "Practical compress-forward in user cooperation: Wyner-ziv cooperation," in *Proc. Int. Symp. Inform. Theory*, pp. 489–493, July 2006.

[7] J. P. K. Chu and R. S. Adve, "Implementation of co-operative diversity using message-passing in wireless sensor networks," in *Proc. IEEE Globecom*, Dec. 2005.

[8] D. Chen and J. N. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Comm.*, vol. 5, pp. 1785–1794, July 2006.

[9] A. W. Eckford, J. P. K. Chu, and R. Adve, "Low complexity cooperative coding for sensor networks using rateless and LDGM codes," in *Proc. IEEE Int. Conf. on Comm. 2006*, June 2006.

[10] A. W. Eckford and R. Adve, "A practical scheme for relaying in sensor. networks using repeat-accumulate codes," in *Proc. Conf. on Inform. Sci. and Systems*, Mar. 2006.

[11] T. R. Oenning and J. Moon, "A low density generator matrix interpretation of parallel concatenated single bit parity codes," *IEEE Trans. Magnetics*, vol. 37, pp. 737–741, Mar. 2001.

[12] D. Divsalar, H. Jin, and R. J. McEliece, "Coding theorems for "turbo-like" codes," in *Proc. Allerton Conf. on Comm., Control, and Computing*, pp. 210–210, Sept. 1998.

[13] R. Dabora and S. D. Servetto, "On the role of estimate-and-forward with time-sharing in cooperative communication." Submitted to IEEE Trans. on Inform. Theory.

[14] A. Bletsas, A. Khisti, D. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE Journal on Selected Areas of Communication*, vol. 24, pp. 659–672, March 2006.

[15] E. Beres and R. S. Adve, "On selection cooperation in distributed networks." Accepted by IEEE Trans. on Wireless Comm.

[16] Y. Zhao, R. Adve, and T. Lim, "Improving amplify-and-forward relay networks: Optimal power allocation versus selection," *IEEE Trans. on Wireless Comm.*, vol. 6, pp. 3114–3123, Aug. 2007.

[17] J. P. K. Chu, R. S. Adve, and A. W. Eckford, "Relay selection for low-complexity coded cooperation," in *Proc. IEEE Globecom*, Nov. 2007.

[18] T. J. Richardson and R. L. Urbanke, "The capacity of low-density parity-check codes under message-passing decoding," *IEEE Trans. Inform. Theory*, vol. 47, pp. 599–618, Feb. 2001.

[19] S. Lin and D. J. Costello, *Error Control Coding: Fundamentals and Applications*. Prentice Hall, second ed., 2004.

[20] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, vol. 47, pp. 498–519, Feb. 2001.

[21] H. Jin and R. J. McEliece, "RA codes achieve awgn channel capacity," in *Proc. 13th Int. Symp. on Applied Algebra, Algebraic Algorithms, and Error-Correcting Codes*, pp. 10–18, Nov. 1999.

[22] R. Liu, P. Spasojević, and E. Soljanin, "Reliable channel regions for good binary codes transmitted over parallel channels," *IEEE Trans. on Inform. Theory*, vol. 52, pp. 1405–1424, April 2006.

[23] M. Ardakani and F. R. Kschischang, "A more accurate one-dimensional analysis and design of irregular LDPC codes," *IEEE Trans. on Comm.*, vol. 52, pp. 2106–2114, Dec. 2004.