Relay Selection for Low-Complexity Coded Cooperation

Josephine P. K. Chu*, Raviraj S. Adve* and Andrew W. Eckford†

*Dept. of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada
†Dept. of Computer Science and Engineering, York University, Toronto, Ontario, Canada

E-mail: {chuj,rsadve}@comm.utoronto.ca, aeckford@yorku.ca

Abstract—This paper explores relay selection and selection diversity for coded cooperation in wireless sensor networks, with complexity constraints for the sensor nodes. In previous work, a relaying scheme based on repeat-and-accumulate (RA) codes was introduced, where the relay is assumed incapable of decoding the received signal’s error-correcting code, and simply uses demodulated bits to form codewords. However, in a network setting with multiple potential relays where relays do not decode the source transmission, it is not obvious how to select the best relay. The optimal choice involves the source-relay, relay-destination, and source-destination channels. In this paper optimal relay selection is discussed, and it is shown that the mutual information of the equivalent relay channel is a good selection heuristic. It is also shown that an alternative, intuitively appealing, relay selection strategy (max-min selection) actually results in a serious performance loss, emphasizing the importance of the relay selection problem.

I. INTRODUCTION

With the growing importance of sensor networks and an increasing interest in developing mesh networks to augment wireless systems with centralized base-stations, the number of studies on networks utilizing relays has increased dramatically. Figure 1 depicts a typical three-node relay channel, where S, R and D are the source, relay and destination nodes respectively. In relay channels, it is assumed that the relay has no data of its own to send and its sole purpose is to assist the source node. One of the earliest works on relay channels is [1], which presents the capacity of a degraded relay channel. The concept of cooperative diversity, where mobile stations form pairs or groups to achieve spatial diversity by sharing antennas, was first presented in [2], [3] and further developed in [4], [5] where two simple yet popular schemes, decode-and-forward and amplify-and-forward, were introduced. Since then, many coding schemes have been introduced to exploit the performance improvement provided by cooperative diversity. Some well-known schemes include coded-cooperative diversity [6], distributed turbo codes [7] and dynamic decode-and-forward [8]. These schemes provide excellent performance in fading channels, but like many of the schemes available, the relays only assist if the information bits from the source are decoded correctly. Otherwise the received bits from the source node are discarded.

In [9], we presented an alternative approach wherein a relay demodulates (but does not decode) the source transmission and forwards parity bits from its hard decisions to the destination. The relay, therefore, does not perform any decoding and always transmits some information for the source. The destination uses belief propagation in the decoding process to account for the quality of source-relay (S-R) channel. The scheme is shown to provide full diversity order (order-2 in the 3-node network of Fig. 1). In [10] and [11], this scheme is extended to low-density generator matrix (LDGM) and repeat-and-accumulate (RA) codes respectively. LDGM and RA codes are chosen because of the simplicity of implementation. These cooperative codes are more flexible than the simple parity check code of [9], as the rate of the code from both the source and the relay can be easily adjusted. Most importantly, it allows for the notion of fractional cooperation, where a relay helps “as much as it can” - depending on its own constraints it could transmit only a part of the source codeword to the destination. These schemes provide an exceedingly simple mechanism for cooperation with great flexibility and energy efficiency. A relay node can choose a level of cooperation depending on its own status, e.g., a relay node in a battery operated sensor network could use its battery level to determine the energy it is willing to “sacrifice” to cooperate.

The analyses in [9]–[11] focus on the 3-node network of Fig. 1. Real world networks are clearly larger, comprising many nodes, and, in theory, more than one relay can be employed to increase the available diversity order of a source-destination transmission. As synchronization is not required with the cooperative LDGM and RA codes, using multiple antennas implies the relays need to transmit their codewords to the destination using multiplexing schemes such as time-division multiple access (TDMA) or frequency-division multiple access (FDMA) in order to avoid conflict. This would, however, greatly reduce network capacity and increase power usage. One way to achieve full diversity without sacrificing network capacity or power efficiency is to select one best relay out of the available pool to transmit to the destination [12]–
Relay selection also has the advantage of significantly simplifying code design. In these works and in networks using coded cooperation, relay selection is relatively straightforward: the optimal relay is the one that decodes correctly and has the best relay-destination (R-D) channel [12], [13] or, in the case of amplify-and-forward, has the best source-relay-destination compound channel [14].

In our case of coded cooperation based on “demodulate and forward”, no method is available yet to select the optimal relay. Instead, as we will show, relay selection is a function of the code used. Our contribution in this paper is to demonstrate the features of optimal relay selection for practical coded cooperation schemes, and to show that the mutual information of the equivalent relay channel is a good heuristic to approximate the optimal selection method. Furthermore, we show the importance of a good relay selection method by introducing a max-min selection technique, which is intuitively pleasing but results in a very poor choice in practice.

This paper is organized as follows. Section II introduces the system model for the relay channel problem. Section III provides some background information on RA codes, and various parameters associated with the cooperative version of the code. In Section IV, the relay selection schemes based on max-min S-R-D channel and maximum mutual information of the equivalent relay channel are introduced, followed by simulation results in Section V to illustrate the performance of each scheme. Finally, we draw some conclusions from the simulation results in Section VI and point the way forward.

II. SYSTEM MODEL

The system model used, also known as the classical relay channel, is illustrated in Fig. 1. S, R and D are the source, relay and destination nodes respectively. The sole purpose of the relay node R is to assist S in transmitting information to D. The system model assumes: (i) the channels between the three nodes are quasi-static Rayleigh fading channels and a block fading model is used; (ii) at any instant each node can only either be transmitting or receiving; (iii) perfect channel state information (CSI) is available at all receiving nodes, and the instantaneous S-R signal-to-noise ratio (SNR) is available at D; and (iv) the transmit CSI of the R-D channel is available at the relay. In a network setting, the source has multiple relays to choose from.

The transmission in divided into two phases. In the first phase, S “broadcasts” coded symbols. The discrete-time signals received at time index k by R and D respectively are

\[ y_{SR}[k] = h_{SR} \sqrt{E_s} c_s[k] + n_{R}[k], \]

\[ y_{SD}[k] = h_{SD} \sqrt{E_s} c_s[k] + n_{D}[k], \quad k = 1, \ldots, l_{D} \tag{2} \]

where \( c_s[k] \in \{-1, 1\} \) are the transmitted bits in a codeword of blocklength \( l_s \), \( h_{SR} \) and \( h_{SD} \) are fading channel coefficients on the S-R and S-D channels respectively, \( E_s \) is the transmitted symbol energy, and \( n_{R}[k] \) and \( n_{D}[k] \) are independent complex white Gaussian noise with variance \( N_{0,R} \) and \( N_{0,D} \) respectively. The average received SNR of the S-D is

\[ \gamma_{SD} = E[\gamma_{SD}] = E[|h_{SD}|^2]E_s/N_{0,D} \]

where \( E[\cdot] \) represents statistical expectation. The average received SNR of the S-R and R-D channel, \( \gamma_{SR} \) and \( \gamma_{RD} \), are defined in a similar manner.

At the relay, the transmitted symbols are estimated, and codewords are then formed based on the estimated bits. In the second phase, the relay symbols are transmitted by R and received by D. The received signal is given by

\[ y_{RD}[k] = h_{RD} \sqrt{E_s} c_r[k] + n_{D'}[k], \quad k = 1, \ldots, l_r \tag{3} \]

where \( h_{RD} \) is fading channel coefficient on the R-D channel and \( l_r \) is the blocklength of the relay codeword \( c_r \).

III. REPEAT-AND-ACCUMULATE CODES

In [15], a class of simple turbo-like codes, called repeat-and-accumulate (RA) codes, was introduced. In RA codes, the information bits are first repeated \( q \) times, then interleaved, and finally fed into a truncated rate-1 recursive convolutional encoder with transfer function \( 1/(1 + D) \). The input \( u \) and the output \( w \) of the encoder can described by

\[ w[1] = u[1] \]

\[ w[k] = u[k] \oplus w[k-1], \quad k = 2, \ldots, l \]

where \( u[k] \in \{0,1\} \) and \( l \) is the blocklength. The rate of RA code can be changed easily by puncturing the parity bits. In our scheme, systematic RA codes are used, where the binary codeword \( d \) is the concatenation of the \( d_i \), the vector of information bits and \( |d_p| \), the vector of punctured parity bits from the output of the encoder, i.e., \( d = [d_i, d_p] \). The transmitted BPSK symbol vector \( c = [c_i, c_p] \) can then be obtained by using the mapping function \( \sigma(\cdot) \) on \( d \) where \( \sigma(0) = 1 \) and \( \sigma(1) = -1 \).

A. Cooperative RA Code

Let \( m_{i,s} \) and \( m_{p,s} \) be the length of \( c_i \) and \( c_p \) from the source node respectively. In our system, the source node first broadcasts a codeword \( c_s \) with rate \( r_s = m_{i,s}/(m_{i,s} + m_{p,s}) \) in the first phase. The relay will choose a fraction of the bits from the source codeword \( c_s \), and use them as information bits to form a new codeword \( c_r \). Let \( \epsilon_{i,s} \) and \( \epsilon_{p,s} \) be the fraction of source information and parity bits that are used to form the relay codeword. Hence, when the relay has no information of its own to transmit,

\[ m_{i,r} = \epsilon_{i,s} m_{i,s} + \epsilon_{p,s} m_{p,s}, \]

where \( m_{i,r} \) is the number of relay “information bits”. The codeword formed by the relay has code rate \( r_r = m_{i,r}/(m_{i,r} + m_{p,r}) \), where \( m_{p,r} \) is the number of parity bits in the relay codeword \( c_r \). The cooperative RA can also be represented by a factor graph, as illustrated in Fig. 2. In the figure, the squares represent factor nodes, and the circles represent variable nodes. The nodes labeled \( v_{i,r} \), \( v_{p,r} \), \( v_{r,s} \) and \( v_{r,p} \) are the source and relay information and parity bits respectively. The shaded nodes represent bits that are discarded in the puncturing process and are therefore not transmitted.

It is shown in [16] that as long as \( q \geq 3 \), and the channel SNR is above a given threshold \( \gamma_0 \), as the blocklength of the
code $l \to \infty$, the average maximum-likelihood word error probability approaches 0 for the ensemble of rate $1/q$. Hence our implementation of the cooperative RA code uses $q = 3$.

B. Decoding RA Codes

As the variable and check degrees of RA codes are quite small, decoding of the cooperative RA codes can be performed using the sum-product algorithm (SPA). Background information on the SPA can be found in [17]. When messages are passed between source and relay codewords, the errors that occur from the relay’s estimation of source bits must be taken into account. More details on the calculation of the messages being passed between the source and relay bits can be found in [10].

IV. RELAY SELECTION

A. Optimal relay selection

An optimal relay selection scheme selects the relay that minimizes the bit error rate (BER) in overall decoding. As such, it is necessary to know the probability of symbol error in decoding for every possible co-ordinate of $\gamma_{SD}$, $\gamma_{SR}$, and $\gamma_{RD}$.

Using density evolution [18], we have plotted the contours of various BERs of cooperative RA code with $r_s = r_r = 1/2$ in Fig. 3. These contours are as expected—the BER cannot go below some value when either the S-R or R-D channel qualities are poor. In the plot, the solid and dash-dot lines represent results for $\gamma_{SD} = -4$ dB and $\gamma_{SD} = -6$ dB respectively. Optimally, some network entity would use such a figure to determine the best relay given knowledge of $\gamma_{SD}$ and $\gamma_{SR}$ and $\gamma_{RD}$ for every available relay.

Clearly such an optimal approach is highly impractical—the function form of the contours are difficult to characterize, and while density evolution is far more computationally efficient than simulations, obtaining a contour plot for every possible value of $\gamma_{SD}$ is clearly impossible.

B. Max-Min S-R-D Channel

As an alternative to the optimal scheme, we present here a simple heuristic. The max-min criterion effectively assumes that the overall BER is limited by the worse of the S-R and R-D channels (since the S-D channel is common to all the relays). This is equivalent to approximating the contours in Fig 3 as a rectangle, with two lines at the SNR levels where the BER saturates.

For each relay $i$ with S-R and R-D channel SNR $\gamma_{SR,i}$ and $\gamma_{RD,i}$, we find its associated parameter $\xi_i$ where

$$\xi_i = \min(\gamma_{SR,i}, \gamma_{RD,i})$$

Then the relay with the largest $\xi_i$ is chosen to relay the source bits, or equivalently,

$$R_{\text{max-min}} = \arg \max_{i \in \mathcal{R}} \xi_i$$

where $\mathcal{R}$ represents the set of all available relays to assist the source in transmitting its data and $R_{\text{max-min}}$ is the relay with the max-min S-R-D channel and is used to relay the bits.

C. Maximum Mutual Information

The heuristic in Section IV-B is simple and intuitively appealing; however, as shown in Sec. V, provides very poor performance. It is presented here mainly to illustrate the importance of the relay selection problem and the need for more rigorous, if practical, approaches. The heuristic presented here uses the mutual information in the source-relay-destination compound channel. This is based on the notion that a “good” code is close to capacity achieving and the mutual information is a good measure of the information theoretic quality of any channel.

Let $p_{SR} = 0.5 \text{erfc}(\sqrt{\gamma_{SR}})$ be the probability of bit error between the S-R channel, where erfc($\cdot$) is the complementary error function. Since hard decisions are formed on the received bits, the S-R channel can be modelled as a binary symmetric channel (BSC)

$$c_r[i] = \begin{cases} c_s[\phi(i)] & \text{with prob. } 1 - p_{SR}, \\ 1 - c_s[\phi(i)] & \text{with prob. } p_{SR}. \end{cases}$$

where $\phi(i)$ describes the mapping that the $\phi(i)th$ bit of $c_s$ is relayed by the $ith$ bit of $c_r$. 
In the following analysis, we assume that \( \phi(i) = i \). For simplicity, we will omit the index \( i \) from the following equations in this section. The joint distribution of the source bits, the relay bits and the signals received at the destination and relay is given by

\[
p(c_s, c_r, y_{SD}, y_{RD}) = p(y_{SD}|c_s)p(y_{RD}|c_r)p(c_r)p(c_s)
\]

as \( y_{SD} \) and \( (c_r, y_{RD}) \) are independent given \( c_s \). The mutual information of the relay channel is given by [19]

\[
I(C_s; Y_{SD}, Y_{RD}) = H(Y_{SD}, Y_{RD}) - H(Y_{SD}, Y_{RD}|C_s)
\]

\[
= H(Y_{SD}, Y_{RD}) - (H(Y_{SD}|C_s) + H(Y_{RD}|C_s))
\]

where \( H(\cdot) \) is the entropy. After some expansion and simplification, we have the following probability density functions

\[
p(y_{SD}, y_{RD}) = \sum_{c_r} \sum_{c_s} p(c_r, c_s, y_{SD}, y_{RD})
\]

\[
= 0.5 \left[ f_1(t_1, \gamma_{SD})(p_{SR}f_0(t_2, \gamma_{RD})
+ (1 - p_{SR})f_1(t_2, \gamma_{RD}))
+ f_0(t_1, \gamma_{SD})(p_{SR}f_1(t_2, \gamma_{RD})
+ (1 - p_{SR})f_0(t_2, \gamma_{RD})) \right]
\]

\[
p(y_{SD}|c_s) = \begin{cases} f_0(t_1, \gamma_{SD}) & \text{if } c_s = 1; \\
 f_1(t_1, \gamma_{SD}) & \text{if } c_s = -1. \end{cases}
\]

\[
p(y_{RD}|c_s) = \sum_{c_r} p(r_{RD}, c_r|c_s)
\]

\[
= f_0(t_1, \gamma_{RD})p(c_r = 1|c_s)
+ f_1(t_1, \gamma_{RD})p(c_r = -1|c_s)
\]

where

\[
f_0(t, \gamma) = (1/\sqrt{2\pi}) \exp \left\{ -(t - \sqrt{\gamma})^2/2 \right\}
\]

\[
f_1(t, \gamma) = (1/\sqrt{2\pi}) \exp \left\{ -(t + \sqrt{\gamma})^2/2 \right\}
\]

are distributions of a Gaussian random variable with mean \( \sqrt{\gamma} \) and \( -\sqrt{\gamma} \) and variance 1 respectively.

Equations (10), (11), (13) are substituted into (8) to obtain the mutual information. Figure 4 plots the contours for various values of mutual information functions of \( \gamma_{SR} \) and \( \gamma_{RD} \). As shown, these contours have a close resemblance to the RA code contours at low BER in Fig. 3. The differences are probably due to the fact that the codes are not capacity-achieving. We will, however, find equivalent relay channels such that mutual information contours would line up with the RA code BER = 10\(^{-4}\) contours of the equivalent relay channels. These parameters will then be used to perform mutual information calculations to find the best relay.

V. Simulation Results

For the simulation results shown in the following sections, we have set \( m_{i,s} = m_{p,s} = m_{i,r} = m_{p,r} = 2000 \), and \( \epsilon_{i,s} = 1 \) and \( \epsilon_{p,s} = 0 \). In other words, we are using rate-1/2 RA codes at both the source and relay, and only the information bits from the source are used to form a new codeword at the relay. Also, it is assumed that the average received SNR across all the channels are the same.

A. Max-Min S-R-D Channel

We first show the simulation results for the case where the relay with the max-min S-R-D channel is chosen to relay for the source node. Figure 5 illustrates the BER performance when 1, 2, or 3 relays are available to assist the source. As illustrated in the plot, more available relays does not necessarily increase the diversity order.

From Fig. 3, we can see why the performance might be poor: the BER contours are not well approximated by a max-min scheme, for which the contours would be rectangular on the graph. In addition, by using max-min relay selection, we made the assumption that the quality of the S-R and R-D channel has the same effect on the BER performance. The performance is so poor that a full order of diversity is lost over a wide range of SNR. This shows that care must be taken in the relay selection in order to achieve the maximum diversity order. A more intelligent method of relay selection must be used to exploit the larger pool of available relays.

B. Maximum Mutual Information

Here we calculate the mutual information of the equivalent relay channel, as presented in Section IV-C, to assist in the relay selection. These calculations are performed at each relay, and the relay with the maximum mutual information is chosen to assist the source. The simulation results is show in Fig. 6. As shown in the plot, the case where mutual information is used to select the best relay has much better performance than the case where max-min S-R-D channel data is used. The diversity order increases as the number of relays is increased.

Other work, which is excluded for reasons of space, indicates that the shape of the curves in Fig. 3 is sensitive to the code type. For instance, using an LDGM code (as...
This criterion is suggested here to illustrate the fact that an intelligent method of relay selection must be used to achieve the gains available via cooperative diversity. The drawbacks of the mutual information method of relay selection is that it is based on the assumption that the code used achieves capacity. Since the RA codes do not achieve capacity, adjustments must be made such that the mutual information calculation can be used to assist in relay selection. Future work will take into account that codes that are being used when choosing the best relay to assist in transmission. We will also extend this work to study the effects of changing $\epsilon_{s,s}$ and $\epsilon_{p,s}$, i.e., fractional cooperation.

REFERENCES