Discrete Suppression with $\Sigma\Delta$-STAP

Byung Wook Jung, Joohwan Chun
Division of Electrical Engineering
Department of Electrical and
Computer Engineering
Korea Advanced Institute of
Science and Technology
Daejeon, Korea

Ravi Adve
Department of Electrical and
Computer Engineering
University of Toronto
Toronto, Canada

Jong Hoon Chun
Samsung Thales Co. Ltd.
Yongin-City
Kyungki-Do, Korea

***Abstract*— This paper presents the hybrid algorithm of $\Sigma\Delta$-STAP and direct data domain(D^3) which is robust to the discrete interferer. $\Sigma\Delta$-STAP provides high performance result with the relatively low complexity of calculation. It also requires small number of training sample to estimate covariance matrix. However this algorithm is vulnerable to the discrete interferer which is target-like interferer with high SNR. This paper shows that $\Sigma\Delta$-STAP can suppress discrete interferer signal in conjunction with direct data domain(D^3) method which is an effective method to eliminate non-correlated interference such as discrete interferer in the non-homogeneous environment.**

I. Introduction

Space-Time Adaptive Processing(STAP) is well known for interference suppression such as clutter, electric counter measure(ECM) mixed with receiver noise [1]- [3]. STAP is optimum in the sense of maximizing signal-to-interference-plus-ratio(SINR). In addition, STAP maximizes probability of detection for a given false alarm probability and minimizes output power subject to an unity constraint in the target direction [3]. However, even though STAP has a high performance on suppressing interference, *optimum* STAP[1] algorithm has a problem with computation load and sample support to estimate covariance matrix since ideal covariance matrix with the unwanted signal can not be obtained in the real world [3]. Besides, existing STAP algorithms have a problem with the non-homogeneous environment which makes hard to detect the target. The presence of the discrete interference deceives radar into detecting a wrong target.

To overcome these computation problems, many algorithms have been proposed [3]- [9]. One example is $\Sigma\Delta$-STAP which is a very unique and efficient algorithm proposed in [7] [8]. This algorithm reduces computation load dramatically with high performance result in the sense of modified sample matrix inversion(MSMI) [8]. However, if there exists a discrete interference, it is hard to detect the target with $\Sigma\Delta$-STAP within the primary range cell. It causes a false alarm at the different angle/Doppler domain from that of target. D^3 method can be used to solve this problem. [10] shows that discrete interferer in the primary range cell can be removed by using direct data domain(D^3) method. D^3 method is originally combined with joint domain localized(JDL) algorithm which is another computation efficient STAP algorithm. However, no paper has been published about removing discrete interferer of $\Sigma\Delta$-STAP. The solution of suppressing of the discrete interferer will be introduceed in conjunction with $\Sigma\Delta$-STAP. This paper will also provide the theoretical background of $\Sigma\Delta$-STAP and D^3 method.

The remainder of this paper is organized as follows. $\Sigma\Delta$-STAP and D^3 Algorithm are reviewed in section II and section III, respectively. In section IV, hybrid algorithm is proposed, and the simulation results are given in section V. Section VI concludes the paper.

[1]it is also called by *fully adapted* STAP to distinguish itself from the other sub-optimum STAP algorithms

II. $\Sigma\Delta$-STAP

These are many systems using sum and different beams which has low side lobe due to the advance in antenna technology. Because of this fact, $\Sigma\Delta$-STAP can be applied to the clutter suppression without the major change in the front-end electronics [8]. Another advantage of $\Sigma\Delta$-STAP is that it needs less training samples than other STAP algorithms. For example, $K \geq 2NM$ samples are needed to achieve performance within 3dB of optimum, where N is number of spatial channel and M is number of pulses in a coherent processing interval(CPI) [5]. 3×3 JDL in [6] needs 18 training samples while only 12 training samples are needed in $\Sigma\Delta$-STAP with one difference channel. In addition, $\Sigma\Delta$-STAP has an advantage on the size of covariance over JDL algorithm. At the same situation, JDL has a covariance matrix of size 9×9 while the size of the covariance matrix of $\Sigma\Delta$-STAP is 6×6.

Let $\mathbf{x}_\Sigma$ be $N_t \times 1$ sum-channel data and $\mathbf{x}_\Delta$ be $N_{ps}N_t \times 1$ stacked delta-channel data before degree of freedom(DOF) reduction. $N_t \times 1$ temporal steering vector of a chosen Doppler bin is defined as $\mathbf{s}_t$. N_{ps} is the processor's spatial DOF which equals to the number of Δ-beams with the absence of redundant Δ-beams and N_t is the number of pulses in CPI. With $N_t \times (N_{pt}+1)$ temporal DOF reduction matrix $\mathbf{Q}$ where N_{pt} is the processor's temporal DOF, data after DOF reduction can be expressed as follows

$$\tilde{\mathbf{x}}_\Sigma = \mathbf{Q}^H \mathbf{x}_\Sigma, \qquad (N_{pt}+1)\times 1 \tag{1}$$

$$\tilde{\mathbf{x}}_\Delta = \left[\mathbf{I}(N_{ps}) \otimes \mathbf{Q}\right]^H \mathbf{x}_\Delta, \qquad N_{ps}(N_{pt}+1)\times 1 \tag{2}$$

$$\tilde{\mathbf{s}}_t = \mathbf{Q}^H \mathbf{s}_t, \qquad (N_{pt}+1)\times 1 \tag{3}$$

where $(\cdot)^H$ represent Hermitian(conjugate transpose), $\mathbf{I}(N_{ps})$ the identity matrix of order N_{ps}.

Equation (1) and (2) can be put together as follows

$$\tilde{\mathbf{x}} = \begin{bmatrix} \tilde{\mathbf{x}}_\Sigma \\ \tilde{\mathbf{x}}_\Delta \end{bmatrix} = \begin{array}{l} [\mathbf{I}(N_{ps}+1)\otimes \mathbf{Q}]^H \begin{bmatrix} \mathbf{x}_\Sigma \\ \mathbf{x}_\Delta \end{bmatrix} \\ ,(N_{pt}+1)(N_{ps}+1)\times 1 \end{array} \tag{4}$$

Equation (4) can be re-written as

$$\tilde{\mathbf{x}} = \mathbf{Q}_{\Sigma\Delta}^H \mathbf{x}, \qquad (N_{ps}+1)(N_{pt}+1)\times 1 \tag{5}$$

if $\mathbf{x}$ and $\mathbf{Q}_{\Sigma\Delta}$ are defined as

$$\mathbf{x} \equiv \begin{bmatrix} \mathbf{x}_\Sigma \\ \mathbf{x}_\Delta \end{bmatrix}, \qquad N_t(N_{ps}+1)\times 1 \tag{6}$$

$$\mathbf{Q}_{\Sigma\Delta} \equiv \begin{array}{l} [\mathbf{I}(N_{ps}+1)\otimes \mathbf{Q}] \\ ,(N_{ps}+1)N_t \times (N_{ps}+1)(N_{pt}+1) \end{array} \tag{7}$$

where $\otimes$ is Kronecker product.

Correlation matrix $\mathbf{R}$ is defined as follows

$$\mathbf{R} = E\left\{\mathbf{x}\mathbf{x}^H\right\} \tag{8}$$

After DOF reduction, low-dimension correlation matrix can be found as

$$\tilde{\mathbf{R}} = \mathbf{Q}_{\Sigma\Delta}^H \ \mathbf{R} \ \mathbf{Q}_{\Sigma\Delta} \tag{9}$$

Since the ideal correlation matrix can not be obtained in the real world, estimate of $\mathbf{R}$ can be used instead and is defined as $\hat{\mathbf{R}}$.

$$\hat{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{x}_k \mathbf{x}_k^H \tag{10}$$

with $\mathbf{x}_k, k = 1, 2, \ldots, K$ being data samples from nearby range cells.

From the general frame work of the STAP, filtering weight vector can be found as follows.

$$\hat{\tilde{\mathbf{w}}} = \tilde{\mathbf{R}}^{-1}\tilde{\mathbf{s}}, \qquad (N_{ps}+1)(N_{pt}+1)\times 1 \tag{11}$$

where $\tilde{\mathbf{s}}$ is defined as follows

$$\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{\mathbf{s}}_t \\ \mathbf{0} \end{bmatrix}, \qquad (N_{ps}+1)(N_{pt}+1)\times 1 \tag{12}$$

Therefore MSMI can be obtained

$$\eta_{MSMI} = \frac{\left|\hat{\tilde{\mathbf{w}}}^H \tilde{\mathbf{x}}\right|^2}{\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{s}}} \begin{array}{c} \mathbf{H}_1 \\ > \\ < \\ \mathbf{H}_0 \end{array} \eta_0 \tag{13}$$

III. D^3 Algorithm

D^3 algorithm does not employ data from outside the primary range cell under test and no covariance matrix estimation is performed. Therefore it is very useful in a severely non-homogeneous environment which statistics between range cells varies rapidly.

Define $\mathbf{X}_\Delta^a$ and $\mathbf{X}$ as follows

$$\mathbf{X}_\Delta^a = \left[\mathbf{x}_{\Delta_1}, \mathbf{x}_{\Delta_2}, \ldots, \mathbf{x}_{\Delta_{N_{ps}}}\right], N_t \times N_{ps} \tag{14}$$

$$\mathbf{X} = [\ \mathbf{x}_\Sigma \ , \ \ \mathbf{X}_\Delta^a \ ], \qquad N_t \times (N_{ps}+1) \tag{15}$$

where $\mathbf{x}_{\Delta_k}, k = 1, 2, \ldots, N_{ps}$ is the data of kth delta-channel. $\mathbf{X}$ is a matrix version of (6) filling each column with the data of each delta-channel.

$(N_{ps}+1)\times(N_t-1)$ matrix $\mathbf{B}$ can be defined with the phase factor $z = e^{j2\pi\varpi}$, where ϖ is normalized Doppler frequency, as follows

$$\mathbf{B} = \begin{bmatrix} \mathbf{X}_{00} - z^{-1}\mathbf{X}_{10} & \mathbf{X}_{10} - z^{-1}\mathbf{X}_{20} & \ldots & \mathbf{X}_{(N_t-2)0} - z^{-1}\mathbf{X}_{(N_t-1)0} \\ \mathbf{X}_{01} - z^{-1}\mathbf{X}_{11} & \mathbf{X}_{11} - z^{-1}\mathbf{X}_{21} & \ldots & \mathbf{X}_{(N_t-2)1} - z^{-1}\mathbf{X}_{(N_t-1)1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{X}_{0N_{ps}} - z^{-1}\mathbf{X}_{1N_{ps}} & \mathbf{X}_{1N_{ps}} - z^{-1}\mathbf{X}_{2N_{ps}} & \ldots & \mathbf{X}_{(N_t-2)N_{ps}} - z^{-1}\mathbf{X}_{(N_t-1)N_{ps}} \end{bmatrix} \tag{16}$$

where $\mathbf{X}_{nm}$ is nth row and mth column element of $\mathbf{X}$.

In the obtained signals after subtraction, there are no target signal and contain only interference signals. The basic idea of D^3 algorithm is to obtain the weight vector which makes this interference term minimize while maintaining gain in the target direction. Consider the following two scalar functions of a weight vector $\mathbf{w}_t$

$$G_{\mathbf{w}_t} = \left|\mathbf{w}_t \mathbf{b}_{(0:M-2)}\right|^2 = \mathbf{w}_t^H \mathbf{b}_{(0:M-2)} \mathbf{b}_{((0:M-2)}^H \mathbf{w}_t \tag{17}$$

$$I_{\mathbf{w}_t} = \left\|\mathbf{B}^* \mathbf{w}_t\right\|^2 = \mathbf{w}_t^H \mathbf{B}^T \mathbf{B}^* \mathbf{w}_t \tag{18}$$

$$R_{\mathbf{w}_t} = G_{\mathbf{w}_t} - \kappa^2 I_{\mathbf{w}_t} \tag{19}$$

where $\|\cdot\|$ represents the two-norm of a vector, $(\cdot)^*$ the complex conjugate, $\mathbf{b}_{(0:M-2)}$ the first $(M-1)$ entries of the temporal steering vector, and κ the emphasis parameter.

The term $G_{\mathbf{w}_t}$ in (17) represents the gain of weight vector $\mathbf{w}_t$ at the look Doppler frequency f_t while $I_{\mathbf{w}_t}$ in (18) represents the residual interference power after the data is filtered by the same weights. Hence, $R_{\mathbf{w}_t}$ in (19) represents the difference between the gain of the antenna at the look Doppler and the residual interference power. The D^3 algorithm finds the weights which maximize the difference. Mathematically

$$\begin{aligned} \max_{\|\mathbf{w}_t\|_2=1} R_{\mathbf{w}_t} &= \max_{\|\mathbf{w}_t\|_2=1} \left[G_{\mathbf{w}_t} - \kappa^2 I_{\mathbf{w}_t}\right] \\ &= \max_{\|\mathbf{w}_t\|_2=1} \mathbf{w}_t^H \left[\mathbf{b}_{(0:M-2)} \mathbf{b}_{((0:M-2)}^H - \kappa^2 \mathbf{B}^T \mathbf{B}^*\right] \mathbf{w}_t \end{aligned} \tag{20}$$

where as $\kappa \to 0$ the D_3 weight vector approaches the non-adaptive steering vector used in pulse-Doppler processing.

On the other hand, if κ is chosen to be large, the role of the gain term $G_{\mathbf{w}_t}$ is negligible and the weight vector is dependent on the interference terms only. Using the method of Lagrange multipliers, weight vector is the eigenvector corresponding to the maximum eigenvalue of the matrix $\left[\mathbf{b}_{(0:M-2)}\mathbf{b}^H_{((0:M-2)} - \kappa^2\mathbf{B}^T\mathbf{B}^*\right]$. Finally, D^3 weight can be found

$$\mathbf{w}_{D3} = \begin{bmatrix} \mathbf{w}_t \\ 0 \end{bmatrix} \tag{21}$$

The zero is appended to represent the lost of DOF.

As can be seen above, since no secondary data are required in D^3 algorithm suppression on the non-homogeneous interference in the primary range cell can be performed fast and effective. However, effects of correlated interference still exists because of lack of statistical information on those such as jammer, clutter.

IV. Hybrid Algorithm

This section presents the hybrid algorithm which is a combination of $\Sigma\Delta$-STAP and D^3. As explained in previous sections, STAP algorithm has a ability of suppressing correlated interferences while D^3 method has a great advantage in removing non-correlated interferences such as discrete interferer in the primary range cell in the non-homogeneous environment. Therefore, this algorithm consists of two stages. At the first stage, discrete interferer signal is removed using only primary cell data. And correlated interference signals are suppressed at the second stage using $\Sigma\Delta$-STAP which performs a statistical processing.

A. STAGE 1 : Non-Correlated Interferer Suppression

DOF reduction matrix $\mathbf{Q}$ in $\Sigma\Delta$-STAP is the discrete Fourier transform(DFT) of the size $N_t \times (N_{pt}+1)$ under the assumption of no zero-padding and a uniform pulse train [8]. Instead of using DFT matrix, DOF reduction matrix is made using D^3 method. Each column of $\mathbf{Q}$ is made of D^3 weight with different Doppler bins as similar manner in [10].

$$\bar{\mathbf{Q}} = \left[\mathbf{w}_{D3}(f_0), \mathbf{w}_{D3}(f_1), \ldots, \mathbf{w}_{D3}(f_{N_{pt}})\right] \tag{22}$$

where $f_k, k = 0, 1, \ldots, N_{pt}$ is the Doppler frequency of the corresponding Doppler bin.

Therefore overall temporal DOF reduction matrix in Eq. (7) can be re-defined as follows

$$\bar{\mathbf{Q}}_{\Sigma\Delta} = \left[\mathbf{I}(N_{ps}+1) \otimes \bar{\mathbf{Q}}\right] \tag{23}$$

B. STAGE 2 : Correlated Interferer Suppression

With given DOF reduction matrix $\bar{\mathbf{Q}}_{\Sigma\Delta}$ from Stage 1, reduced dimension snapshot $\bar{\mathbf{x}}$ and covariance matrix $\bar{\mathbf{R}}$ can be expressed as follows

$$\bar{\mathbf{x}} = \bar{\mathbf{Q}}^H_{\Sigma\Delta}\ \mathbf{x} \tag{24}$$

$$\bar{\mathbf{R}} = \bar{\mathbf{Q}}^H_{\Sigma\Delta}\ \hat{\mathbf{R}}\ \bar{\mathbf{Q}}_{\Sigma\Delta} \tag{25}$$

Weight vector $\bar{\mathbf{w}}$ is obtained similar to original $\Sigma\Delta$-STAP case using (11).

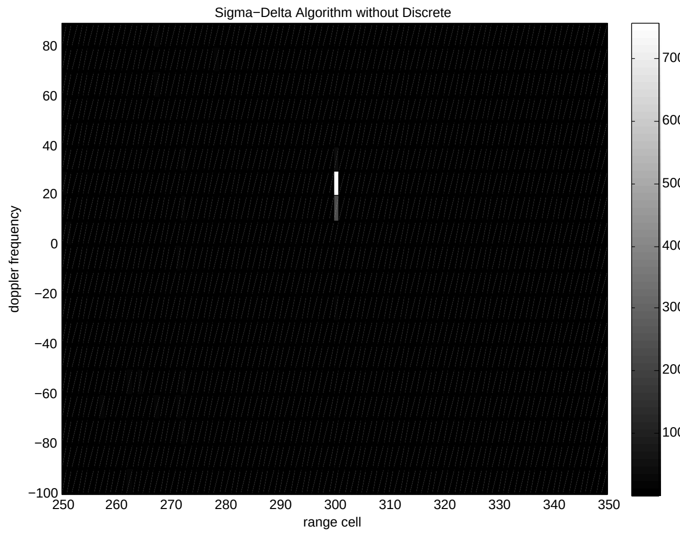

Fig. 1. MSMI : $\Sigma\Delta$-STAP without Dicrete interferer

As can be seen from above discussion, since each range bin has a different $\bar{\mathbf{Q}}_{\Sigma\Delta}$, weight vector needs to be calculated at each range bin different from other STAP algorithm. Therefore computation load of hybrid algorithm is higher than that of $\Sigma\Delta$-STAP. However, if it is compared to two-step hybrid algorithm in [10], the computation load is lighter because of the size of covariance matrix.

V. Simulation Reaults

This section provides simulation results. Simulation parameters are shown in Table I. Fig. 1 shows the MSMI result of $\Sigma\Delta$-STAP without discrete interferer. Target is inserted in the range bin number 300 and Doppler frequency is 20Hz. Target can be seen clearly in this figure. Next, discrete interferer with Doppler frequency 40Hz is inserted at the same range bin where the target presents. As can be seen in Fig. 2, the presence of discrete interferer causes false alarm because the power of discrete interferer is relatively higher than that of the target. This fact can deceive the radar because statistical characteristic of the discrete interferer is same as that of the target. As a result STAP processor produces weight vector which maximizes SNR of discrete interferer. Consequently, false alarm at Doppler frequency 40Hz appears. Hybrid algorithm presented this paper can solve this problem caused by discrete interferer. Instead of using DFT matrix as a DOF reduction matrix $\mathbf{Q}$, hybrid algorithm uses D^3 weights as a column of $\bar{\mathbf{Q}}$ removing unwanted signals. Fig. 3 shows the MSMI result of the hybrid algorithm. Since the D^3 method maximizes the gain in the look Doppler, discrete interferer signals which have different Doppler frequency are suppressed. Due to the characteristic of D^3 method, correlated signals are suppressed at the second stage. Comparing Fig's 2 and 3, it can be seen that discrete interferer is removed apparently and only the target is detected in Fig. 3.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Operating Frequency	450MHz
Peak Transmit Power	200KW
Instantaneous BW	4MHz
System Loss	4dB
PRF	200Hz
Pulse Width	200μs
Platform Altitude	9Km
Platform Velocity	50 m/s
N_t	20
N_{pt}	4
N_{ps}	3
Target SNR	0dB
Discrete Interferer SNR	20dB
Target Doppler Frequency	20Hz
Discrete Interferer Doppler Frequency	40Hz
Target Range Cell Number	300
Discrete Interferer Range Cell Number	300

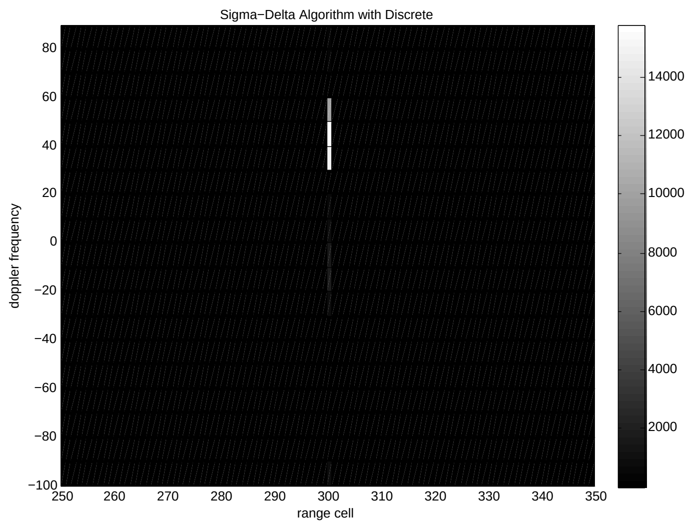

Fig. 2. MSMI : ΣΔ-STAP with Dicrete interferer

VI. CONCLUSION

This paper proposed new ΣΔ-STAP algorithm to suppress discrete interferer using D^3 method. This algorithm consists of two stages : 1)non-correlated interferer suppression part and 2) correlated interferer suppression part. At the non-correlated interferer suppression part, discrete interferer is suppressed by using D^3 method which uses only the primary data cell making the column of the transformation matrix $\bar{\mathbf{Q}}$ of D^3 weight. Residual correlated interferer signal is then suppressed using statistical algorithm which is ΣΔ-STAP after DOF reduction with $\bar{\mathbf{Q}}$. This successive weight calculation process at each range cell makes the computation load high.

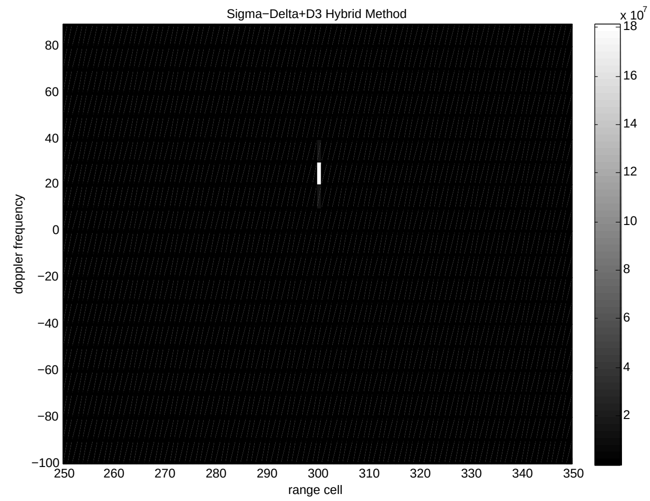

Fig. 3. MSMI : Hybrid algorithm with Discrete inteferer

ACKNOWLEDGMENT

REFERENCES

[1] R. Kelmm, Principles of Space-Time Adaptive Processing, 2ed, London, U.K. : IEE Press, 2002.

[2] J.R. Guerci, Space-lime Adaptive Processing for Radar, Artech House, 2003.

[3] J. Ward, "Space-Time Adaptive Processing for Airborne Radar", Technical Report No. 1015, Lincoln Laboratory, MIT, Dec 1994.

[4] W.L. Melvin, "A STAP Overview",Aerospace and Electronic Systems Magazine, IEEE, volume 19, issue 1, part 2, pp.19-35, Jan 2004.

[5] M.C. Wicks, M. Rangaswamy, R. Adve, T.B. Hale, "Space-time adaptive processing: a knowledge-based perspective for airborne radar", Signal Processing Magazine, IEEE, volume 23, issue 1, pp.51-65, Jan. 2006.

[6] H. Wang, L. Cai, "On Adaptive Spatial-Temporal Processing for Airborne Surveillance Radar Systems", Aerospace and Electronic Systems, IEEE Transactions on, volume 30, issue 3, pp.660-670, July 1994.

[7] R.D. Brown, M.C. Wicks, Y. Zhang, Q. Zhang, H. Wang, "A space-time adaptive processing approach for improved performance and affordability", Radar Conference, Proceedings of the 1996 IEEE National, 1996.

[8] R.D. Brown, R.A. Schneible, M.C.Wicks, H. Wang, Y. Zhang, "STAP for clutter suppression with Sum and Difference Beams", Aerospace and Electronic Systems, IEEE Transactions on, vol 36, issue 2, pp. 634-646, April 2000.

[9] R.S. Adve, T.B. Hale and M.C. Wicks, "Joint Domain Localized Adaptive Processing in Homogeneous and Non-homogeneous Environments. Part I: Homogeneous Environments", IEE Proc. on Radar, Sonar and Navigation, vol. 147, no. 2, pp. 57-65, April 2000.

[10] R.S. Adve, T.B. Hale and M.C. Wicks, "Joint Domain Localized Adaptive Processing in Homogeneous and Non-homogeneous Environments. Part II: Non-Homogeneous Environments", IEE Proc. on Radar, Sonar and Navigation, vol. 147, no. 2, pp. 66-73, April 2000.