Node Selection and Two-Hop Power Allocation in Multi-Source Cooperative Mesh Networks

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Abstract-This paper considers relay selection and power allocation in a two-hop multi-source multi-destination mesh network wherein fixed relay nodes use the decode-and-forward protocol. The jointly optimal solution is of exponential complexity. Introducing a set of time-sharing factors into the objective function, and relaxing the selection constraint, provides an upper bound to the original problem. We also provide a heuristic method to impose selection on each source-destination pair. Second, we propose a decentralized selection scheme in which each individual source-destination pair chooses its best relay independently followed by power allocation. Simulation results reveals that the performance of the decentralized selection scheme almost exactly tracks that of an upper bound for both max sum rate as well as max min rate metrics. The key difference from previous works on selection is that, throughout, we account for the source-relay channel and the need for the relay to decode the source.

I. INTRODUCTION

Cooperative diversity is a relatively new class of spatial diversity techniques made possible by retransmitting the information of a source through geographically distributed relay nodes in the system. By sharing the network resources, the nodes in a distributed communication network can harness the benefits of multiple-input multiple-output (MIMO) systems with only a single antenna at each transmitter/receiver. The works in [1]–[3] has led to much research activity in the area of cooperative communications.

In [2], [3] Laneman et al. introduced different cooperation modes including decode-and-forward (DF) wherein each relay decodes, re-encodes the source data with the same codebook, and retransmits it to the destination. They also showed that distributed space-time code achieves full spatial diversity order in the number of potential relays in the system and has higher spectral efficiency than the repetition-based schemes. However, from a practical point of view, due to the need for symbol level synchronization over distributed nodes in the system, this scheme is likely impractical. The work in [4], [5] showed that *selection*, wherein a single "best" relay helps the source, provides all the benefits of cooperation while minimizing the overhead.

Selection cooperation in a single-source single-destination network has been well studied for the DF protocol [4]–[6], but is not as well investigated for multiple sources where finding the best relay for each source becomes a combinatorial problem. In such scenarios, one specific relay node can be chosen by multiple sources. Thus, it should share its available power amongst all source nodes which have selected that node as a relay. Without considering power allocation, authors in [5] proposed low-complexity sub-optimal schemes for relay selection. In [7], the authors showed that with K users and J dedicated relays, selection cooperation is the optimal relaying strategy for at least K - J + 1 of the users and system performance is indistinguishably close to optimal if selection is imposed on all K users. Therefore, given its many benefits, selection appears to be the best choice when multiple relays are available. However, the work in [7] assumes all relays can decode, an assumption valid for their cellular network. Unfortunately, this assumption is not valid in the mesh networks considered here.

The importance of considering the source-relay channel in resource allocation problems has been dealt with in numerous studies in literature especially for orthogonal frequency division multiplexing/multiple access (OFDM/OFDMA). Wang et al. [8] studied the resource allocation problem to maximize the user rate in a three terminal OFDM-based network. The works in [9]–[11] impose a joint power constraint on the source and relays and solve the resource allocation problem to maximize the sum rate or the minimum rate across users. In our work we impose selection with a per-node power constraint. As we will see, there is also a key difference in our approach making our respective optimization problems concave. In [12], without considering power allocation, a fairness-aware graph-based relay and subcarrier allocation approach is proposed which maximizes the sum rate in the network.

In [13] Ng et al. constructed a utility maximization framework for solving both the resource allocation problem as well as choosing an appropriate cooperation strategy in a cellular network after a brute force search over a fixed set of rates. The total utility of the network is decomposed as a sum of utilities of each individual data stream; however, this scheme cannot be applied to max-min problems. Furthermore, the DF cooperation strategy is used whenever the source-relay channel is stronger than the compound sourcerelay- destination communication link. Weng et al. in [14] proposed a resource allocation and cooperation strategy for an OFDMA-based network wherein relay nodes have their own data to transmit. The cooperation strategy is chosen only by comparing the source-destination and source-relay channels. The decomposition method of [13] is used to minimize the power subject to data rate constraints on each source node. The authors of [15] proposed a resource allocation scheme for two-hop transmisson with relays selected *a priori*.

The focus of this paper is on a static mesh network of access points (APs) with multiple sources and fixed-relay nodes, wherein each source can communicate with its corresponding destination only through the *selected* relays. The main objective of this paper is to provide an optimization framework to jointly assign a relay and power to each source node to maximize two performance metrics: the sum rate and minimum rate across all source nodes while accounting for the source-relay and relay-destination channels. Neither relay selection nor power allocation is done *a priori*. Unlike previous works, we impose a per-node power constraint.

We begin by formulating the optimization problem to allocate resources to each flow. By introducing time-sharing coefficients, we transform this combinatorial problem into a standard convex optimization problem which leads to an upper bound for the original resource allocation scheme with the selection constraint. We then deal with imposing selection scheme on each flow using a heuristic method. Finally, we propose a distributed relay selection scheme with low complexity which has close-to-optimal performance. Once the relay has been assigned to each source node, power allocation across source nodes can be applied.

The rest of the paper is organized as follows. Section II, describes the system model under consideration in this paper. In section III, the node selection and power allocation in two-hop mesh networks is investigated for both performance metrics in some detail. In section IV, distributed relay selection scheme is introduced. Simulation results are illustrated in section V. Finally, section VI wraps up this paper.

II. SYSTEM MODEL

The system under consideration is a static mesh network of APs wherein K source nodes are assisted by J fixed relay nodes. Each source node has its own destination which is not in the set of source and relay nodes. Since nodes are static, we assume the inter-node channels vary slowly enough that feedback of the channel state information (CSI) to some centralized unit is possible without significant overhead.

Each of the source nodes uses an orthogonal channel, over which source-to-relay and relay-to-destination communications take place. Furthermore, it is assumed that a source node can transmit data only via relay nodes and direct path is blocked due to the source-destination distance or obstacles. All inter-node paths are modeled as block-fading Rayleigh channels. The system model is illustrated in Fig. 1. All APs are attached to the power supply and can transmit data with maximum power of P watts/symbol. Relaying is based on DF and only relay nodes which fully decode a received data stream can be chosen to forward it to the corresponding destination. The system uses time division duplex and all communications happen in two phases. In Fig. 1 the solid arrows show the data sharing stage while dashed arrows represent the second phase wherein *only one node is assigned to each source node*.



Fig. 1. Two-Hop multi-source multi-destination mesh network

Consider, for now, a system where a relay is chosen for each source node. As a result, the maximum achievable rate at which source k can communicate with its destination with relay l helping is:

$$I_{d_k} = \min\{I_{s_k r_l}, I_{r_l d_k}\},$$
 (1)

$$I_{s_k r_l} = \frac{1}{2} \log_2 \left(1 + \operatorname{SNR}_{kl} \right), \qquad (2)$$

$$I_{r_l d_k} = \frac{1}{2} \log_2 \left(1 + \operatorname{SNR}_{lk} \right), \tag{3}$$

where, $\text{SNR}_{kl} = P|h_{kl}|^2/N_0$ is the received signal-to-noise ratio (SNR) at relay *l* due to the transmission from source *k* while $\text{SNR}_{lk} = \alpha_{lk}P|h_{lk}|^2/N_0$ is the received SNR at the k^{th} destination due to the transmission from relay *l*. In these expressions, α_{lk} is the fraction of power that relay *l* allocates to source *k*, h_{kl} is the channel gain between node *k* and relay *l* while h_{lk} is the channel gain of the relay-destination path; N_0 is the power spectral density of the white noise at the receiver. $I_{s_kr_l}$ is the rate at which source *k* can communicate with relay *l*, while $I_{r_ld_k}$ is the rate at which relay *l* can transmit data to the destination. Therefore, (1) ensures that the maximum overall rate is the rate at which both relay and destination can decode source information.

III. NODE SELECTION AND POWER ALLOCATION

As described above, in a two-hop mesh network framework, without a source-destination channel, the rate limiting factor is either the source-relay or relay-destination channel. In keeping with many benefits of selection cooperation, we enforce the condition that each source transmits its information via a single relay node. However, in a practical system model, the number of sources, K, is much higher than the number of relays, J. Thus, it is likely that multiple source nodes are to be supported by a single relay. In such a situation, each relay has to distribute its available power amongst those source nodes. This section develops the joint node selection and power allocation problem in the described network to maximize two performance metrics.

A. Max Sum Rate

The sum rate metric measures the maximum achievable throughput that all source nodes can deliver to their own destinations. Therefore, the formal optimization problem is:

$$\max_{\alpha_{lk}} \sum_{k=1}^{K} I_{d_k},\tag{4}$$

s.t.
$$C_1 : \alpha_{lk}.\alpha_{jk} = 0, \forall k, l \neq j,$$
 (5)

$$C_2: \sum_{k=1}^{K} \alpha_{lk} = 1, \forall l, \tag{6}$$

$$C_3: \alpha_{lk} \ge 0, \forall l, k. \tag{7}$$

 $I_{d_k} = \max \{\min \{I_{s_k r_1}, I_{r_1 d_k}\}, ..., \min \{I_{s_k r_J}, I_{r_J d_k}\}\},$ constraint C_1 enforces selection cooperation on each source node, and C_2 limits the available power of each relay. C_3 states that each relay allocates non-negative fraction of its total available power to source nodes which have chosen that node as a relay. As can be observed from (5), each source node can only receive power from one relay.

Since each relay must split its available power amongst all source nodes which it supports, the individually optimal relay allocation scheme may not be globally optimal. Hence, the problem formulated in (4)-(7) is a combinatorial optimization problem with exponential complexity in the number of communication links in the network. In order to make the problem tractable, we introduce KJ indicator variables, ρ_{lk} , to the objective function. Therefore, the new optimization problem is:

$$\max_{\substack{\alpha_{lk} \in [0,1]\\\rho_{lk} \in \{0,1\}}} \sum_{l=1}^{J} \sum_{k=1}^{K} \rho_{lk} \min \left\{ I_{s_k r_l}, I_{r_l d_k} \right\},$$
(8)

s.t.
$$C_1 : \sum_{l=1}^{J} \rho_{lk} = 1, \forall k,$$
 (9)

$$C_2: \sum_{k=1}^{K} \rho_{lk} \alpha_{lk} = 1, \forall l, \tag{10}$$

From this modified optimization problem, for any set of α_{lk} satisfying (5)-(7), we have:

$$\rho_{lk} = \begin{cases} 1, & \alpha_{lk} \neq 0\\ 0, & \alpha_{lk} = 0. \end{cases}$$
(11)

Equations (9)-(11) enforce each source to be relayed with only one relay node in the system. Thus, (9)-(11) are equivalent to (5) in the original optimization problem. However, because ρ_{lk} can only take integer values, the problem is still mixedinteger optimization problem. Our next step is to relax the corresponding constraint and allow each source to be helped by more than one relay. Thus, relay indicators can take any value on the convex hull of the original discrete set. Consequently, the resulting sum rate from this modified problem is an upper bound on the original sum-rate optimization problem formulated in (4)-(7).

The term ρ_{lk} can now be interpreted as the *fraction* of time that the data of the source k can be relayed by relay l. Since each source transmits with its maximum available power at the first stage and central resource allocation unit has the full CSI, $I_{s_k r_l}$ is constant. Although $I_{r_l d_k}$ is individually concave in α_{lk} and ρ_{lk} , it is easy to show that the second term in the objective function, $\rho_{lk}I_{r_ld_k}$, is not *jointly concave* in (ρ_{lk}, α_{lk}) . Using the technique introduced in [16], we set $r_{lk} = \rho_{lk}\alpha_{lk}$. This is a key difference from the work in [10], [11] which did not take the coupling constraint between ρ and r into account. The new optimization problem in terms of r_{lk} and ρ_{lk} is:

$$\max_{\substack{\rho_{lk} \in [0,1]\\r_{lk} \in [0,\rho_{lk}]}} \sum_{l=1}^{J} \sum_{k=1}^{K} \min\left\{ \rho_{lk} I_{s_k r_l}, \frac{\rho_{lk}}{2} \log_2(1 + \frac{r_{lk} |h_{lk}|^2}{\rho_{lk} N_0}) \right\}$$
(12)

s.t.
$$C_1 : \sum_{l=1}^{J} \rho_{lk} = 1, \forall k,$$
 (13)

$$C_2: \sum_{k=1}^{K} r_{lk} = 1, \forall l.$$
(14)

Proposition 1: The objective function in (12) is concave in (ρ, r) .

Proof: As can be seen from (12), the achievable rate of each source node consists of two terms where the first term is affine in ρ_{lk} .

The second term has the form $g(\rho_{lk}, r_{lk}) = \rho_{lk} f(\frac{r_{lk}}{\rho_{lk}})$, which is jointly concave in (ρ_{lk}, r_{lk}) . In particular, $\nabla^2 g \leq 0$, i.e., the Hessian evaluated within the optimization region is a negative semi-definite matrix. Furthermore, from [17] it is known that a point-wise minimum as well as the nonnegative summation of a set of concave functions are also concave functions. Hence, the underlying objective function is concave in (ρ, r) .

Finally, by introducing KJ new variables, ζ_k^l , the optimization problem can be stated in the epigraph form as:

$$\max_{\substack{\zeta_{l}^{k}\\\rho_{lk}\in[0,1]\\r_{lk}\in[0,\rho_{lk}]}} \sum_{l=1}^{J} \sum_{k=1}^{K} \zeta_{l}^{k}$$
(15)

s.t.
$$C_1 : \sum_{l=1}^{J} \rho_{lk} = 1, \forall k,$$
 (16)

$$C_2: \sum_{k=1}^{K} r_{lk} = 1, \forall l,$$
(17)

$$C_3: \rho_{lk} I_{s_k r_l} \ge \zeta_l^k, \quad \forall l, k,$$
(18)

$$C_4: \frac{\rho_{lk}}{2} \log_2\left(1 + \frac{r_{lk}|h_{lk}|^2}{\rho_{lk}N_0}\right) \ge \zeta_l^k, \ \forall l, k.$$
(19)

The optimization problem formulated in (15)-(19) is a standard convex optimization problem which can be solved using efficient iterative algorithms [17]. The relaxed convex problem does not necessarily ensure selection, since relay indicators can take any rational number between 0 and 1 and a specific transmission can receive power from multiple relay nodes. Hence, the solution is an upper bound on the original problem.

A simple heuristic method to impose selection is to assign to each source node the relay that provides the maximum achievable rate. Mathematically, selection can be applied as:

$$r_k = r_m$$
, $m = \arg\max_{l} \min\{I_{s_k r_l}, I_{r_l d_k}\}$

where r_k is the relay assigned to node $k, l \in \{1, ..., J\}$. Thus, the modified power allocation matrix, $[r'_{lk}]$ can be constructed as follows:

$$r'_{mk} = r_{mk}, \ r'_{lk} = 0, \ \forall l \neq m, \ \forall k \in \{1, ..., K\}.$$

Since this solution satisfies all constraints of the original problem in (4)-(7), this heuristic scheme also provides a *lower bound* on the achievable sum rate. Moreover, the power freed up by the selection step can be reused by waterfilling over other source nodes which are helped by each individual relays. As we show in Section V, this scheme with power rearrangement provides an achievable sum rate extremely close to the upper bound.

B. Max Min Rate

In this section, we consider the second performance metric: maximizing the minimum rate across users. This metric assures that the source node with the poorest channel receives most of the resources of the network. Thus, as far as possible, max-min resource allocation leads to an equal rate for all source nodes. Succinctly, the optimization problem is:

$$\max_{\alpha_{lk}} \min_{k} I_k, \tag{20}$$

s.t.
$$C_1: \alpha_{lk}.\alpha_{jk} = 0, \forall k, l \neq j,$$
 (21)

$$C_2: \sum_{k=1}^{N} \alpha_{lk} = 1, \forall l, \tag{22}$$

$$C_3: \alpha_{lk} \ge 0, \forall l, k.$$
(23)

Following the same procedure that has been used in the previous part, it is easy to show that the max-min resource allocation problem is:

$$\max_{\substack{\rho_{lk} \in [0,1]\\ r_{lk} \in [0,\rho_{lk}]}} \min_{k} \sum_{l=1}^{J} \min\left\{\rho_{lk} I_{s_k r_l}, \frac{\rho_{lk}}{2} \log_2\left(1 + \frac{r_{lk} |h_{lk}|^2}{\rho_{lk} N_0}\right)\right\}$$
(24)

s.t.
$$C_1 : \sum_{l=1}^{J} \rho_{lk} = 1, \forall k,$$
 (25)

$$C_2: \sum_{k=1}^{K} r_{lk} = 1, \forall l,$$
(26)

wherein based on Proposition 1, the objective function stated in (24) is *jointly concave* in (ρ, r) . Applying the epigraph form of convex problems, the alternative optimization problem is:

$$\max_{\substack{\zeta, t_l^k\\\rho_{lk} \in [0,1]\\r_{lk} \in [0,\rho_{lk}]}} \zeta \tag{27}$$

s.t.
$$C_1: \sum_{l=1}^{J} t_l^k \ge \zeta, \forall k$$
 (28)

$$C_2 : \sum_{l=1}^{J} \rho_{lk} = 1, \forall k,$$
(29)

$$C_3 : \sum_{k=1}^{K} r_{lk} = 1, \forall l,$$
(30)

$$C_4: \rho_{lk}I_{s_kr_l} \ge t_l^k \ \forall l, k, \tag{31}$$

$$C_5: \frac{\rho_{lk}}{2} \log_2(1 + \frac{r_{lk}|h_{lk}|^2}{\rho_{lk}N_0}) \ge t_l^k \ \forall l, k.$$
(32)

The problem formulated in (27)-(32) is a standard convex optimization problem, which can be solved efficiently using interior-point algorithm. Using the same approach that we have introduced for max sum resource allocation, the close to upper bound achievable min rate can be found for this metric.

IV. DECENTRALIZED RESOURCE ALLOCATION

The optimization problem and solution detailed so far is to jointly assign relay node and allocate power to each source in the network. This solution requires a centralized solution with a resource allocation unit which has complete CSI. In this section we develop a simplified decentralized scheme, wherein each source selects its best relay as if it were *the only source* in the network, i.e., assuming that the selected relay can allocate all its power only to that specific source. In particular:

$$r_{k} = r_{m} \text{ if}$$

$$m = \arg\max_{l} \left\{ \min\left\{ I_{s_{k}r_{l}}, \frac{1}{2}\log_{2}(1 + \frac{P|h_{lk}|^{2}}{N_{0}}) \right\} \right\}, \quad (33)$$

where r_k is the relay assigned to node k, and $l \in \{1, ..., J\}$. Given that each individual source node has been assigned a relay, J waterfilling problems need to be solved in order to maximize the sum rate or minimum rate across source nodes. This approach has some similarities to the "simple selection" scheme of [5], wherein each source-destination pair selects its best relay based only on relay-destination channel gain. However, as we will see, the performance is superior; in Section V, we will show that, in fact, the performance of the distributed scheme closely tracks that of an optimum algorithm.

V. SIMULATION RESULTS

This section presents the results of simulations for the proposed relay selection and power allocation algorithm in a two-hop mesh network to compare their performance under the sum and max-min rate metrics. We consider two different network geometries; in the first scenario, all inter-node channels are modeled as independent and identically distributed (i.i.d.). The second network setup is more realistic with nodes



Fig. 2. Achievable sum rate of different resource allocation strategies in "I.I.D Channel" scenario with J = 2.

are randomly distributed in space. Each channel comprises of two components: a small-scale Rayleigh fading component and a large-scale path loss component characterized by the node locations. Therefore, internode channels have uneven average power. We simulate two mesh networks with J=2 relay nodes and K=3 or 4 source nodes.

A. i.i.d. Channels

Our first example is a mesh network with i.i.d. channels. The average SNR of all source-relay links is set to 10dB. Results are averaged over 1000 independent channel realizations. Fig. 2 plots the achievable sum rate for different resource allocation strategies as a function of the relay-destination SNR. As seen from the figure, in both networks, the performance of the heuristic method with Power Rearrangement (PR) is close to the upper bound. The simple selection scheme of [5] has the worst performance which validates the fact that under DF relaying protocol, considering only the relay-destination link as a bottleneck of communication rates is not effective. As it is expected, the network with larger number of source nodes has a higher sum rate.

Fig. 3 shows the achievable minimum rate across all source nodes for different relay assignment and power allocation strategies. As it is shown, the performance gap between decentralized selection scheme and heuristic method with PR is indistinguishable. Furthermore, since source nodes must share available resources in the network, a mesh network with the smaller number of sources has a higher minimum rate.

B. Network Performance for a Distributive Scenario

This section provides simulation results for a more realistic scenario wherein nodes are randomly distributed in the network. The communication links are characterized using the COST-231 channel model recommended by IEEE 802.16j



Fig. 3. Max-min rate across all source nodes of different resource allocation strategies in "i.i.d Channel" scenario with J = 2.

TABLE I PARAMETER VALUES IN COST-231

Parameter	Value	Parameter	Value
AP Height	15m	Frequency	3.5 GHz
Building Spacing	50m	Rooftop Height	30m
Destination Height	15m	Road Orientation	90 deg.
Street Width	12m	Noise PSD	-174 dBm

working group [18]. This approach models both small and large scale fading. Parameters chosen for this model are summarized in Table I. The variance of the log-normal fading is set to 10.6dB. We generate random node locations over an square area of 0.04 square kilometers. We fix the transmitted power of each potential node to [26, 28, 30, 32, 34] dBm. 100 node locations are generated randomly in the network space area. Then, for each individual location, 1000 independent channel realizations are characterized based on the COST-231 channel model.

Figures 4 and 5 compare the performance of different relay selection and power allocation algorithms for maximizing the sum rate as well as minimum rate across source nodes, respectively. Not surprisingly, the performance gap between upper bound and decentralized selection scheme increases with increasing network sizes. This is expected, since in networks where the number of sources is larger than relays, power splitting is more likely. However, under both network metrics, the decentralized scheme shows performance close to that of an upper bound while offering computational and implementation advantages.

VI. CONCLUSION

This paper deals with the problem of relay selection and power allocation in a two-hop cooperative mesh network of



Fig. 4. Achievable sum rate of different resource allocation strategies in "Distributive" scenario with J = 2.



Fig. 5. Achievable min rate across all source nodes of different resource allocation strategies in "Distributive" scenario with J = 2.

APs in order to improve two performance metrics; i) sumrate, ii)min-rate across all sources. As it has been shown in numerous works in literature, selection cooperation has considerable advantages in distributed networks, especially minimizing overhead and avoiding synchronization issues.

Unlike most of the previous works, by taking both sourcerelay and relay-destination links into account, we formulate the underlying problem with selection constraint which ensures that not only the destination, but also the selected relay can fully decode the received information. By introducing the time-sharing factors into the objective function and relaxing the selection constraint, an upper bound to the seemingly difficult original problem is characterized. While others have proposed a similar approach for a joint power constraint, their methodology to make the problem jointly concave across the optimization variables is not valid. By imposing a selection constraint to the given relay assignment and power allocation matrix, a close to upper bound heuristic scheme is introduced. A second contribution in this paper is proposing a decentralized selection scheme. While offering computational benefits over other resource allocation algorithms, the performance of decentralized selection scheme almost exactly tracks that of the upper bound.

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