Relay Selection and Max-Min Resource Allocation for Multi-Source OFDM-Based Mesh Networks

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***Abstract*—We consider a multi-source mesh network of static access points wherein sources use decode-and-forward to cooperate with each other. All transmissions use orthogonal frequency division multiplexing (OFDM). Our objective is to maximize the minimum achievable rate across all flows. We find a tight upper bound on the performance of the subcarrier-based cooperation and show that selecting a single relay for each subcarrier is optimal for almost all subcarriers. The solution to the related optimization problem simultaneously solves the relay, power, and subcarrier assignment problems. Second, unlike previous works, we also consider relay selection for the entire OFDM block. This addresses the fact that, in addition to the synchronization problems caused, it is likely impractical for a relay to only decode a subset of subcarriers. We propose three selection-based cooperation schemes to relay the entire OFDM block with varying complexity. Simulation results show that under the COST-231 channel model, the performance of the simplest scheme almost exactly tracks that of an exhaustive search.**

I. Introduction

Cooperative diversity is a class of spatial diversity techniques made possible through relaying. The nodes of a distributed communication network share resources to achieve the benefits of multiple-input multiple-output systems with only a single antenna at each transmitter/receiver. Cooperation can also help address large-scale fading. Two popular relaying schemes are decode-and-forward (DF - the relay decodes and re-encodes the source data) and amplify-and-forward (AF - the relay amplifies and retransmits its received signal).

The initial work [1]–[3] has led to much research activity in this area. The most relevant literature here is the work on selection which showed that pairing a source with a single "best relay" minimizes the overhead due to relaying and avoids the need for synchronization across relays [4], [5]. In multi-source networks, selection also makes better use of limited power than distributed space time codes [5]. Selection has since been extended to several relaying scenarios [6], [7].

On the separate track, orthogonal frequency division multiplexing (OFDM) has been shown to be a promising, and increasingly popular, technique to mitigate the impact of multipath fading. In OFDM, data is transmitted in parallel over multiple (N) frequency subcarriers. The key here is the ability to create the transmit signal using a N-point inverse Fast Fourier Transform (IFFT) of the data symbols. Furthermore, because each subcarrier experiences a different channel realization, resource allocation can significantly enhance performance [8]–[11].

Recently, the combination of OFDM and cooperation diversity has attracted intense interest. Specifically, selection of the right relaying node and dynamic allocation of subcarrier and power are considered critical. Li and Liu studied the capacity of OFDM-based relay networks for both AF and DF strategies [12] and the problem of maximizing the sum rate with fairness constraints in a multiple-source multiple-relay network using a graph theoretical approach [13]. Fairness is imposed by limiting the number of sources a single relay can help. Ng and Yu [14] used a utility maximization framework to choose the optimal relay strategy and resource allocation in cellular networks. Dai et al. [15] provided outage analysis of two different relaying strategies in OFDM-based multi-hop networks and show that potentially choosing a different optimal partner for each subcarrier achieves full diversity order while there is no diversity gain if selecting the relay which has the highest combined signal-to-noise ratio (SNR). The work in [16] allows for subcarrier permutation in a cooperative network in context of max-min fairness. Weng et al. [17] proposed a resource allocation scheme in which a group of nodes can share their redundant resources with others.

All these works deal with the OFDM transmission on a per-subcarrier basis, i.e., as if each were an *independent* transmission. This is, unfortunately, not true. Given the importance of time and frequency synchronization in OFDM, it is unrealistic to expect different relays to relay individual subcarriers independently. Furthermore, in OFDM, the raw data is channel encoded before data modulation and the IFFT; DF requires decoding all subcarriers. Most of the subcarrier-based selection is, therefore, theoretically optimal, but impractical.

In this paper, we consider max-min optimization for a multi-source multi-destination cooperative OFDM-based static mesh network of access points (APs). Each node is a source as well as a potential relay for other nodes. We begin with per-subcarrier selection. It has been claimed that selection is the *exact* power allocation solution [18]. Building on the work in [19], we show that this is true for *most, not all*, the subcarriers. Using the Karush-Kuhn-Tucker (KKT) conditions, we characterize an upper bound to the original problem which leads to the joint relay and power allocation for each subcarrier. This solution also leads to a simple, heuristic that is also a lower bound. Simulations show that the two bounds are indistinguishable when using the COST-231 [20] channel model. We then deal with selection for an entire OFDM block.

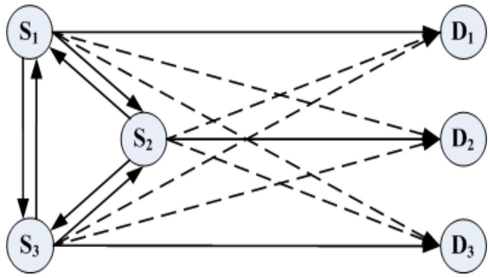

Fig. 1. Cooperative multi-source multi-destination network

We propose a simple selection scheme but with performance close to exhaustive search and not much different from per-subcarrier selection.

The rest of the paper is structured as follows. Section II, develops the system model for the multiple-source OFDM-based network under consideration. In Section III the issue of resource allocation on a per-subcarrier basis is investigated in some detail. Section IV deals with selection based on an entire OFDM block. Section V presents the results of simulations that illustrate the workings of the theory presented. Section VI concludes this paper.

II. System Model

The system under consideration is an outdoor static mesh network of APs. Some of the nodes are physically connected to the internet backbone and are a gateway for other APs. The nodes are installed at some height and have a Rician channel, with a line-of-sight component, to neighboring nodes (potential relays), but a Rayleigh channel to the destination. Each node in the network acts as both a source as well as a potential relay for other sources. In addition, each node has its own destination which is not within the set of K source nodes. The system model is illustrated in Fig. 1.

All transmissions use OFDM within their own frequency band, i.e., simultaneous transmissions do not interfere and, even with a half-duplex constraint, a node can receive the transmissions from *other* sources while transmitting. Because each user experiences a different channel realization on each subcarrier, power and subcarrier allocation enhances system performance. For each subcarrier, the channel between a node i and destination j is modeled as a flat and slowly-fading Rayleigh channel. We assume all inter-node channels vary slowly enough for channel state information to be fed back with limited overhead, making resource allocation possible. All APs are attached to a power supply and transmit with constant and maximum total power of P Joules/symbol. We consider the DF protocol wherein each relay receives, decodes and re-encodes the information with the same codebook as the transmitter, and forwards it to the destination.

We place a half duplex constraint and communication happens in two phases. In Fig. 1, the solid arrows indicate the first, data sharing, stage. The dashed arrows represent the second phase wherein the sources relay for one another. Each source transmits its data using N subcarriers and receives information from other sources. During the second time slot, only one node relays each subcarrier of a source. Finally, the destination node combines messages received in the two phases to decode the original information.

Consider for now a system where a relay is chosen per-subcarrier. On subcarrier n, with relay l helping, the rate at which source k can transmit is:

$$I_{d_k}^{(n)} = \max_{l} \min \left\{ I_{s_k r_l}^{(n)}, I_{s_k r_l d_k}^{(n)} \right\}, \tag{1}$$

$$I_{s_k r_l}^{(n)} = \frac{1}{2} \log_2 \left(1 + snr_{kl}^{(n)} \right), \tag{2}$$

$$I_{s_k r_l d_k}^{(n)} = \frac{1}{2} \log_2 \left(1 + snr_{0k}^{(n)} + snr_{lk}^{(n)} \right), \tag{3}$$

where, $snr_{kl}^{(n)} = P|h_{kl}^{(n)}|^2/(N * N_0)$ is the received signal-to-noise ratio (SNR) of the n^{th} subcarrier at relay l from source k while $snr_{0k}^{(n)} = P|h_{0k}^{(n)}|^2/(N * N_0)$ and $snr_{lk}^{(n)} = p_{lk}^{(n)}|h_{lk}^{(n)}|^2/N_0$ are the received SNR of the n^{th} subcarrier of source k through the direct and relaying paths, respectively. In these expressions, N_0 is the power spectral density of the white receiver noise, $h_{kl}^{(n)}$ is the channel gain between node k and relay l on the n^{th} subcarrier; $h_{0k}^{(n)}$ and $h_{lk}^{(n)}$ are the channel gains of source-destination and relay-destination on the n^{th} subcarrier, respectively. The factor of $(1/2)$ accounts for the fact that communication happens in two phases. $I_{s_k r_l}^{(n)}$ is the rate at which source k can communicate with relay l, while $I_{s_k r_l d_k}^{(n)}$ is the rate at which it transmits information to its destination node with the help of relay l on the n^{th} subcarrier.

The overall transmission rate of user k is the sum over all N subcarriers:

$$R_k = \sum_{n=1}^{N} I_{d_k}^{(n)}. \tag{4}$$

Note that the source distributes its power equally while $p_{lk}^{(n)}$ is the power that the l^{th} relay allocates to the n^{th} subcarrier of node k. Therefore, (1) states that the maximum transmission rate is the rate at which both relay and destination can decode information. Like most other works in this area, in order to make the problem tractable, we assume that all inter-source channels are strong enough that

$$I_{s_k r_l}^{(n)} \geq I_{s_k r_l d_k}^{(n)} \;\; \Rightarrow \;\; I_{d_k}^{(n)} = I_{s_k r_l d_k}^{(n)} \;\; \forall k, n. \tag{5}$$

This is a crucial assumption justified by the modeling of source-relay channels as Rician while the source-destination and relay-destination channels are Rayleigh.

III. Subcarrier-Based Resource Allocation

As described above, all source nodes can decode each others' data and, from (5), the rate limiting factor is the compound source-relay-destination channel. This section develops the optimal relay selection and power allocation scheme to achieve max-min fairness, i.e., to maximize the minimum rate across all sources. As far as possible, this metric leads to an equal rate for all source nodes. In keeping with its many benefits, we impose selection in the second, relaying, phase. Therefore, the optimization problem we wish to solve is:

$$\max_{\left\{p_{lk}^{(n)}\right\}} \min_k R_k, \tag{6}$$

$$s.t.\ \ C_1 : p_{l_1 k}^{(n)} \times p_{l_2 k}^{(n)} = 0,\ \ \forall k, n \text{ and } l_1 \neq l_2, \tag{7}$$

$$C_2 : p_{lk}^{(n)} \geq 0,\ \ \forall l, k, n,\ \ \ C_3 : \sum_{k=1}^{K} \sum_{n} p_{lk}^{(n)} \leq P,\ \ \forall l. \tag{8}$$

Constraint C_1 enforces selection by allowing only one node to devote power to each subcarrier. Constraints C_2 and C_3 state that power must be non-negative and that the total available power of the l^{th} relay is limited to P Joules/symbol.

Due to the selection constraint, (6)-(8) is an, essentially intractable, mixed-integer programming optimization problem. One proposed solution [9], [11] separates the power allocation and selection problems. First, subcarriers are selected assuming equal power allocation; then, power is distributed based on this selection. However, with K sources and N subcarriers, there are K^2N relay assignments to be checked. Therefore, even this scheme is infeasible for realistic values of N and K. We build on an alternative approach developed in [19] to form an approximate solution that is also an upper bound[1].

A. Approximate Solution and Upper Bound

Other than the integer constraint in (7), the constraints in the original problem of (6)-(8) are affine. To find an approximate solution we ignore the constraint in (7). Hence, the solution to the new problem is an upper bound to the subcarrier based (UBSB) resource allocation problem of (6)-(8). The revised formulation, stated here in the *epigraph form*, is a concave maximization problem with efficient solvers available [21]:

$$\max_{\left\{t, p_{lk}^{(n)}\right\}} t \tag{9}$$

$$C_1 : \sum_{n=1}^{N} \frac{1}{2} \log_2 \left(1 + snr_{0k}^{(n)} + \sum_{\substack{l=1 \\ l \neq k}}^{K} snr_{lk}^{(n)} \right) - t \geq 0,\ \ \forall k, \tag{10}$$

$$C_2 : p_{lk}^{(n)} \geq 0,\ \ \forall l, k, n,\ \ \ C_3 : \sum_{\substack{k=1 \\ k \neq l}}^{K} \sum_{n} p_{lk}^{(n)} = P,\ \ \forall l. \tag{11}$$

The solution to this optimization problem is characterized by the KKT conditions [21]; the Lagrangian is given by:

$$\begin{aligned}
&\mathcal{L}\left(p_{lk}^{(n)}, \alpha_k, \mu_l, \lambda_{lkn}\right) = t \\
&+ \sum_{k=1}^{K} \alpha_k \left(\sum_{n=1}^{N} \frac{1}{2} \log_2 \left(1 + snr_{0k}^{(n)} + \sum_{\substack{l=1 \\ l \neq k}}^{K} snr_{lk}^{(n)} \right) - t \right) \\
&+ \sum_{l=1}^{K} \mu_l \left(P - \sum_{\substack{k=1 \\ k \neq l}}^{K} \sum_{n} p_{lk}^{(n)} \right) + \sum_{l=1}^{K} \sum_{\substack{k=1 \\ k \neq l}}^{K} \sum_{n} \lambda_{lkn} p_{lk}^{(n)},
\end{aligned} \tag{12}$$

where the α_k, μ_l, and λ_{lkn} are the Lagrange multipliers associated with rate, total power, and positive power constraints, respectively. For the sake of clarity, assume that $K = 3$. Any solution for the power that nodes l_1 and l_2 allocate to the j^{th} subcarrier of source k_1 satisfies KKT conditions, which are:

$$\frac{\partial \mathcal{L}\left(p_{lk}^{(n)}, \alpha_k, \mu_l, \lambda_{lkn}\right)}{\partial p_{l_1 k_1}^{(j)}} = \frac{\partial \mathcal{L}\left(p_{lk}^{(n)}, \alpha_k, \mu_l, \lambda_{lkn}\right)}{\partial p_{l_2 k_1}^{(j)}} = 0, \tag{13}$$

$$p_{lk}^{(n)} \geq 0\ \ \alpha_k \geq 0,\ \ \mu_l \geq 0,\ \ \lambda_{lkn} \geq 0,\ \ \forall l, k, n,$$

$$\alpha_k (R_k - \xi) = 0,\ \ \lambda_{lkn} p_{lk}^{(n)} = 0,\ \ \forall l, k, n.$$

Now suppose that both relays, l_1 and l_2, allocate some power to the j^{th} subcarrier of the specified source. Using the KKT conditions and the fact that $\lambda_{l_1 k_1 j} = \lambda_{l_2 k_1 j} = 0$:

$$\frac{\mu_1}{\mu_2} = \frac{\mid h_{l_1 k_1}^{(j)} \mid^2}{\mid h_{l_2 k_1}^{(j)} \mid^2}. \tag{14}$$

Similarly, if the power that these two relays allocate to the i^{th} subcarrier of the same source are not zero, using the same KKT conditions, one can conclude that:

$$\frac{\mu_1}{\mu_2} = \frac{\mid h_{l_1 k_1}^{(i)} \mid^2}{\mid h_{l_2 k_1}^{(i)} \mid^2}. \tag{15}$$

Equations (14) and (15) cannot be simultaneously satisfied since channel gains are continuous random variables. Thus, unlike in [18], *at most one subcarrier* of each source *can* be helped by more than one node in the system.

Now, let us evaluate all the possible relay selections in the network with four source nodes, where the j^{th} subcarrier of source k_1 is relayed by all other nodes in the system. The KKT conditions state that:

$$\frac{\mu_1}{\mid h_{l_1 k_1}^{(j)} \mid^2} = \frac{\mu_2}{\mid h_{l_2 k_1}^{(j)} \mid^2} = \frac{\mu_3}{\mid h_{l_3 k_1}^{(j)} \mid^2}. \tag{16}$$

Now suppose that the i^{th} subcarrier of the same source is relayed through both nodes l_1 and l_2. Then,

$$\frac{\mu_1}{\mid h_{l_1 k_1}^{(i)} \mid^2} = \frac{\mu_2}{\mid h_{l_2 k_1}^{(i)} \mid^2}. \tag{17}$$

As a result, along with (16), we have $\mid h_{l_1 k_1}^{(j)} \mid^2 / \mid h_{l_2 k_1}^{(j)} \mid^2 = \mid h_{l_1 k_1}^{(i)} \mid^2 / \mid h_{l_2 k_1}^{(i)} \mid^2$, which is a zero-probability event.

Now assume that none of the subcarriers can be helped with all three relays. As an example, consider the case in which the j^{th} subcarrier is relayed via node l_1 and l_2 and the i^{th} subcarrier can be helped by node l_1 and l_3 in the system. Applying the same KKT conditions, it follows that:

$$\frac{\mu_1}{\mid h_{l_1 k_1}^{(j)} \mid^2} = \frac{\mu_2}{\mid h_{l_2 k_1}^{(j)} \mid^2} \ \ \text{and}\ \ \frac{\mu_1}{\mid h_{l_1 k_1}^{(i)} \mid^2} = \frac{\mu_3}{\mid h_{l_3 k_1}^{(i)} \mid^2}. \tag{18}$$

Now, the n^{th} subcarrier can be helped by node l_2 and l_3 only if $\frac{|h_{l_2 k_1}^{(j)}|^2}{|h_{l_1 k_1}^{(j)}|^2 |h_{l_2 k_1}^{(n)}|^2} = \frac{|h_{l_3 k_1}^{(i)}|^2}{|h_{l_1 k_1}^{(i)}|^2 |h_{l_3 k_1}^{(n)}|^2}$, which happens with zero

[1]It is worth emphasizing that while the solution methodology here is similar to that of [19], both our problem formulation and solution are significantly different. The development here, using the epigraph form, leads to effective solutions to OFDM-based relaying and allows us to show that selection is a *sub-optimal* solution to the resource allocation problem.

probability. Therefore, when two subcarriers are relayed with two nodes, *all others* can be helped by at most one node.

Generalizing this to the network with K source nodes, *one concludes that at most $K-2$ subcarriers of each source can be helped by more than one relay and selection is imposed on $(N-K+2)$ subcarriers.* In practice, $N \gg K$ which means that a large fraction of subcarriers meet the selection criterion, i.e., selection is the *approximate, though not optimal solution*, to the relaxed optimization problem in (9)-(11).

B. A Heuristic Algorithm and a Lower Bound

By neglecting the selection constraint, the solution to the problem in (9)-(11) provides an upper bound to that of the original optimization problem in (6)-(8). Here, we use this to develop a heuristic solution to the original problem. We force the (maximum of $K-2$) subcarriers that do not meet the constraint to receive power only from the single relay that achieves a higher data rate. Since this is a solution that meets all the constraints of the original problem, this is also a *lower bound* on the subcarrier based (LBSB) optimization problem. In Section V, we will show that the performance gap between the upper and lower bounds is indistinguishable. As a result, this heuristic approach provides almost the exact solution to the original mixed-integer optimization problem with significantly reduced solution complexity.

C. Optimal Power Allocation

Using (13), the power that relay l_1 allocates to the j^{th} subcarrier of the source k_1 can be characterized as:

$$p_{l_1k_1}^{(j)} = \left[\frac{\alpha_{k_1}}{2\ln 2\mu_{l_1}} - \frac{1 + snr_{0k_1}^{(j)} + \sum_{\substack{l=1 \\ l\neq l_1 \\ l\neq k_1}}^{K} snr_{lk_1}^{(j)}}{\frac{|h_{l_1k_1}^{(j)}|^2}{N_0}} \right]^+ . \quad (19)$$

Equation (19) shows that subcarriers which suffer more noise or can receive more power from other relays will be allocated with less power. Consequently, the solution to the power allocation problem in the multi-source network follows multi-level waterfilling. Practical algorithms to solve different waterfilling problems are provided in [22].

IV. Block-Based Relay Selection

The optimization problem and solution detailed so far is in keeping with existing literature. It allows different subcarriers within an OFDM block to be helped by different relays. This is problematic for two reasons. One, while not explicitly stated, most of the previous work assumes a relay can treat each subcarrier as an independent transmission. In DF-based relaying, the decoding constraint is at the level of a subcarrier, e.g., (1). However, in OFDM, the data is first protected by a channel code, modulated and then a block of N subcarriers is formed. It is not possible to decode information without receiving and decoding an entire OFDM block. Second, practical OFDM systems depend heavily on accurate time and frequency synchronization. This would be extremely difficult in a distributed mesh network.

To the best of our knowledge, there has been no consideration in the existing literature about selection at the level of an entire OFDM block. In a multi-source network, as long as each relay has to divide its available power amongst all allocated sources, the solution to the relay assignment problem is not immediate. Unfortunately, both the optimization formulation and solution of joint selection and power allocation are extremely complicated. Here we separate the problems into selection followed by power allocation (via waterfilling) across subcarriers. As in [5], we propose three relay selection schemes with different levels of complexity and compare the results in terms of the max-min rate.

A. Optimal Relay Selection

In a network with K sources where each source can act as a relay for other nodes, there are $(K-1)^K$ different possible relay assignments. The optimal scheme is exhaustive search over all possible relay selections and pick the one which provides the maximum minimum rate in the system. This is clearly impossible for any reasonable K.

B. Sequential Relay Selection

In this subsection, we propose sequential relay selection to approximate the results of the optimal relay assignment with less complexity. Based on this scheme, the first node evaluates its achievable rate through selecting its best relay, r_{s_1}. Then, s_2 picks r_j and r_i nodes with the best and second best relay-destination channels. If its best relay has not been assigned to the first node, i.e. $r(s_1) \neq r_j$, it will be allocated to s_2. Otherwise, considering the fact that its best relay distributes its available power amongst both sources, it evaluates the rate of communication over both compound source-relay-destination channels and selects the one with higher rate. This process repeats since one relay has been assigned to each source. In this scheme, $K(K-1)$ waterfilling algorithms have to be solved. Although sequential relay selection is simpler than the optimal relay selection scheme, it is still too complex to be implemented in practice.

C. Decentralized Relay Selection

The decentralized or simple relay selection scheme ignores all other sources. Each source selects its best relay with the assumption that the corresponding relay distributes its power equally over all subcarriers of *only that source*. In particular:

$$\begin{aligned} & r_k = r_m \ \text{ if} \\ & m = \arg\max_j \left(\sum_{n=1}^{N} \log_2 \left(1 + \frac{P \mid h_{r_jk}^{(n)} \mid^2}{N * N_0} \right) \right), \end{aligned} \quad (20)$$

where r_k is the relay assigned to node k, $j \in \{1, ..., K\}$, and $j \neq k$. With each source having selected the relays, the relays allocate power, via waterfilling, to the assigned sources.

Note that since each source picks its best relay independently of all other source nodes, this scheme can be implemented in the decentralized manner. In a network with K source nodes, only K water-filling problems need be solved.

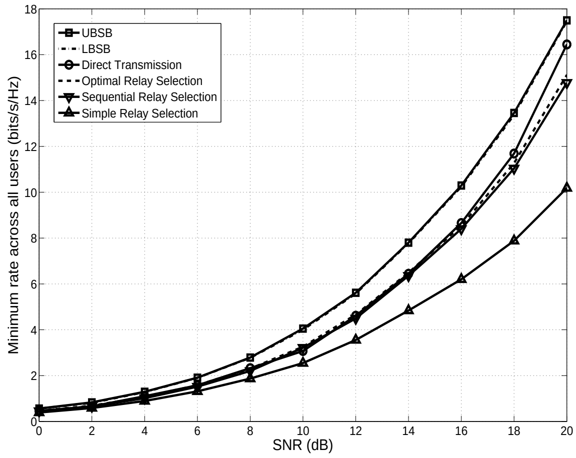

Fig. 2. Minimum rate across all potential sources of different cooperation strategies in "Equal Average Channel" scenario with K=3 and N=16.

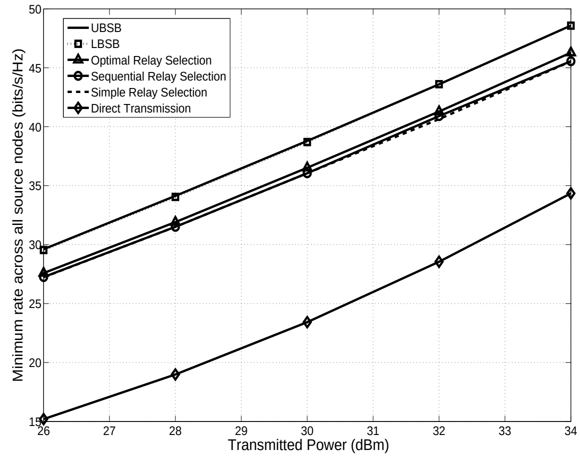

Fig. 3. Minimum rate across all potential sources of different cooperation strategies in "Unequal Average Channel" scenario with K=3 and N=16.

V. Simulation Results

In this section, we present simulation results to evaluate and compare three resource allocation algorithms in different scenarios. They are subcarrier-based relay selection, block-based relay selection, and direct transmission (no cooperation). All inter-node wireless channels are modeled as frequency-selective channels consisting of four resolvable paths. Also, 16 subcarriers are used. We consider two different geometries: in the first scenario, all inter-node channels are independent and have equal average power. The second case is more realistic, where nodes are distributed in the space randomly and channels are characterized by node positions in the network. Therefore, inter-node channels have uneven power. To solve the relaxed optimization problem, we used CVX, a package for specifying and solving convex problems [23], [24].

A. Simulation Results for Equal Average Channels

Our first example uses three APs with all inter-node channels having the same average power. Fig. 2 plots the minimum rate across all users for different values of SNR. As seen in the figure, the upper and lower bounds (the heuristic) are indistinguishable. Furthermore, given the additional flexibility of subcarrier-based cooperation, both outperform block-based resource allocation schemes. Note that at higher values of SNR, direct transmission outperforms all cooperation based protocols. This result validates the fact that cooperation is meaningful only when the relay-destination channel can compensate for the factor of $(1/2)$ due to relaying over two time-slots. Finally, for the block-based selection scheme, the optimal and sequential schemes perform significantly better than the simple selection scheme.

B. Simulation Results for Un-Equal Average Channels

In this subsection, we provide simulation results in the more realistic scenario where nodes are randomly distributed in the network. Thus, inter-node channels have different average

TABLE I
Parameter Values in COST-231

Parameter	Value	Parameter	Value
AP Height	15m	Frequency	3.5 GHz
Building Spacing	50m	Rooftop Height	30m
Destination Height	15m	Road Orientation	90 deg.
Street Width	12m	Noise PSD	-174 dBm

power. The wireless channels are simulated using the COST-231 channel model recommended by IEEE 802.16j working group [20]. This approach models both large and small scale fading. Parameters chosen for this model are summerized in Table I. The variance of the log-normal fading is set to 10.6dB. We generate random node locations over an square area of 0.04 square kilometers. We fix the transmitted power of each potential node to [26, 28, 30, 32, 34] dBm. Results are averaged over both source locations and channel realizations.

Fig. 3 plots the max-min achievable rate across all APs and compares the performance of various resource allocation schemes. From the figure, the performance gap between LBSB and UBSB is, again, negligible. This proves that the heuristic method to find the solution of the original convex optimization problem is almost exact. Furthermore, we compare the performance of block-based schemes. Not surprisingly, the optimal relay selection method outperforms the other two schemes. Simple relay selection closely tracks the sequential relay selection method, but with significantly less complexity. This result indicate that simple relay selection scheme can be implemented in decentralized manner without significant performance loss. Moreover, direct transmission has the worst performance which validates the fact that relaying can enhance the minimum rate of the system in this realistic scenario.

Fig. 4 illustrates the importance of node locations on the performance of different transmission/resource allocation schemes. This example simulates a single source-destination

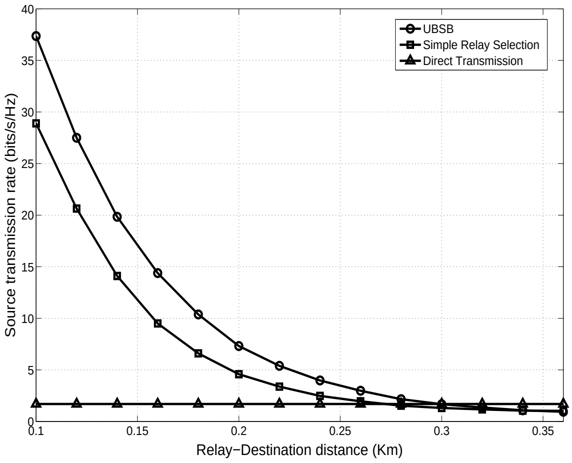

Fig. 4. Source transmission rate in a single source-destination pair network with two nodes act as relays and N=16.

pair with two relay nodes in the system. The source-destination distance is fixed to $0.2\sqrt{2}$ km. Relays are located on both sides of source-destination path. Results are averaged over different channel realizations. Clearly one wants the relay close to the destination; however, note that this may impact on the assumption that the relay can always decode. Simulation results show that relaying schemes outperform direct transmission whenever relays are located between the source and destination nodes. While the upper bound on subcarrier-based selection outperforms block-based selection, the performance loss for this more practical approach is surprisingly small.

VI. Conclusion

This paper investigates resource allocation algorithms for cooperation in a multi-source OFDM-based network. As has been shown earlier, selection cooperation has many advantages in distributed networks, especially minimizing overhead and avoiding issues of synchronization. We set up the underlying problem with a selection constraint on each subcarrier to ensure max-min fairness across all sources. Since this mixed-integer programming problem is computationally complex, we relaxed the selection constraint and formulated a convex optimization problem that provides a tight upper bound. We showed that selection is violated in only $K-2$ of N subcarriers. This in turn leads to a heuristic solution to the original problem and a tight lower bound.

A second contribution in this paper is to formulate block-based selection for a multi-source network. Block-based selection avoids issues of synchronization in OFDM-based networks. We proposed three cooperation schemes with varying complexity. Simulation results reveal that the simplest, distributed, scheme offers computational benefits compare to other proposed schemes, while resulting in negligible performance loss.

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