Matrix Pencil for Positioning in Wireless Ad-hoc Sensor Network

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Abstract. Wireless ad-hoc sensor networks (WASN) are attracting research interest recently because of their various applications in distributed monitoring, communication, and computing. In this paper, we concentrate on the range error problem of WASN positioning algorithms. A new matrix pencil based time of arrival (TOA) positioning method (MPP) is proposed for WASN. The new scheme employs matrix pencil for multipath time-delay estimation, and triangulation for absolute positioning. Simulations in a square-room scenario show that the positioning performance is generally robust to multipath effect and the positioning error is limited to around one meter.

1 Introduction

Wireless ad-hoc sensor networks (WASN) are being developed for use in various applications, ranging from monitoring of the environmental characteristics, to home networking, medical applications and smart battlefields. Many such services provided by WASN rely heavily on the acquisition of sensor nodes' position information. In addition, position assisted routing [8] and minimum energy [7] schemes have also been proposed for general ad-hoc networks and can greatly enhance the throughput performance and lifetime of WASN. Thus, developing a distributed practical positioning algorithm is probably the most important and challenging task in WASN. The term, "practical", suggests that such algorithms should be versatile for diverse environments, rapidly deployable, of low energy consumption, and low cost.

A fundamental problem in WASN positioning is range error, which is defined according to different measurement methods. In RSSI [5] (Received Signal Strength Indicator) and TOA or TDOA (Time of Arrival / Time Difference of Arrival) [3], range measurement is the associated distance estimate, and positioning can be carried through triangulation if more than three anchors are available [6]. In AOA (angle of arrival), it is the angle estimation. Among these measurement methods, AOA requires costly antenna arrays at each node, and is hard to implement. Although RSSI suffers significant range error (as high as over 50%) due to multi-path fading, it is deemed as the primary candidate for WASN range measurement and is well researched [2, 4, 9]. Various propagation methods were proposed in [4] to achieve a crude localization. A two-phase positioning algorithm was proposed in [2] to compensate high range error. However, these methods either suffer from low accuracy or high communication and computation load. Based on RSSI, RADAR [9] was proposed for indoor localization and tracking. This method, however, is centralized and depends on a considerable preplanning effort, which is not appropriate for rapidly deployable WASN.

In TOA approaches, traditional code acquisition and tracking radiolocation, [19], meets significant problems, since the short distances between sensor nodes can demand an unacceptably wide system bandwidth for measurement. TDOA also suffers from this problem. An alternative is the combination of RF and acoustic signals [3, 20], where time-of-flight of acoustic signal is used to calculate the distance. Experiments of such systems as AHLos [3] demonstrated fairly accurate positioning. However, acoustic signals are usually temperature dependent and require an unobstructed line of sight. Furthermore, the maximum range of acoustic signals is usually small, around 10 meters.

Instead of code tracking and acoustic signals, a proposed TOA approach [1] is based on applying high-resolution techniques, such as MUSIC [18] and matrix pencil algorithms [15, 16], on channel response function. MUSIC suffers some severe limitations when compared with matrix pencil, which includes covariance matrix estimation and the assumption of white noise. Both these two assumptions are usually not available in the estimated channel response. In [1], the authors compared MUSIC and direct matrix pencil for multipath delay estimation, and matrix pencil was found much better in terms of both accuracy and computation cost.

In this paper, a matrix pencil based positioning (MPP) algorithm is proposed for WASN, when the transmission time of RF signal (TTR) is assumed to be available at receiver. Based on simulations with a square-room multipath model, we demonstrate that the range measurement error of MPP is much lower than RSSI. By least squares (LS) triangulation [6], the localization error can be generally limited to as low as one meter. When transmission ranges in simulations are considered, the positioning error percent is around 5%. The TTR assumption of our approach may be realized by MAC (Medium Access Control) layer designs, where RF signals can be transmitted on fixed time slots. In the following, Section 2 describes the channel model and the estimation method. The MPP algorithm is then proposed and analyzed in Section 3. We describe our indoor simulation model and present the simulation results of MPP in Section 4. Finally, conclusions are drawn in Section 5.

2 Channel Model and Estimation

Propagation of RF signals is through a quasi-fading channel with complex additive white Gaussian noise, and multipath effect of a maximum propagation delay D. Then, the channel response function can then be written as,

$$h(t) = \sum_{m=1}^{M} \alpha_m \delta\left(t - \tau_m\right), \tag{1}$$

and in the frequency domain,

$$H(\omega) = \sum_{m=1}^{M} \alpha_m e^{-j\omega\tau_m}, \qquad (2)$$

where M is the number of multipaths, τ_m and α_m are the associated time delay and gain respectively.

Assuming that $X(\omega)$ and $Y(\omega)$ denote the frequency representations of transmitted and received signal respectively, we have,

$$Y(\omega) = X(\omega)H(\omega) + N(\omega), \qquad (3)$$

where $N(\omega)$ denotes the additive Gaussian white noise.

Most advanced channel estimation algorithms, as [10], are based on the symbol rate or chip rate in CDMA system. However, these methods fail in WASN settings where nodes are densely deployed. In WASN, most multipath delays τ_m are less than one symbol/chip time T_S, and the maximum propagation delay D is usually of the order of T_S. As an example, the typical bandwidth of IEEE 802.11 wireless LAN is $F_B = 11MHz$, thus $T_S = 1/F_B = 0.09\mu s$. A typical distance between two nodes in WASN is around 10 meters, with the associated LOS (Line of Sight) delay $\tau_{LOS} = 0.03\mu s$. The maximum multipath delay will then be $D \approx T_S$. This observation suggests a simple FFT-based channel estimation scheme.

Consider transmitting a training sequence $Tr(n)$ of length $K \cdot L$, designed as,

$$\begin{cases} Tr(lK+1) = 1, & l = 0 \dots L-1 \\ Tr(lK+i) = 0, & l = 0 \dots L-1, i = 2 \dots K \end{cases}. \qquad (4)$$

At the receiving node, the RF training signal is divided into L segments, where each contain one '1' and $K-1$ '0's. K should be chosen such that $(K-1) \cdot T_S > D$. In the described settings above, K is practically small and can be set to 2. Then let $Y_l(\omega_n)$ denote the FFT of the lth such segment, and let $G(\omega)$ denote the transmit filter response. According to (3), we get,

$$\begin{cases} Y_l(\omega_n) = H(\omega_n) \cdot G(\omega_n) + N_l(\omega_n) \\ \omega_n = 2\pi n \cdot F_B/N - C, n = 0 \dots N-1 \end{cases}, \qquad (5)$$

where N is the number of FFT points, and C is a constant shift. The channel estimation can be obtained by,

$$\begin{aligned} \widehat{H}(n) &= \tfrac{1}{L} \textstyle\sum_{l=1}^{L} \frac{Y_l(\omega_n)}{G(\omega_n)} \\ &= H(\omega_n) + \tfrac{1}{L} \textstyle\sum_{l=1}^{L} \frac{N_l(\omega_n)}{G(\omega_n)}, \\ &= H(\omega_n) + v_n \end{aligned} \qquad (6)$$

where v_n denotes noise component in channel estimation and can be suppressed by increasing the training length L.

By equations (2), (5), and (6), we have,

$$\begin{cases} \widehat{H}(n) = \sum_{m=1}^{M} \beta_m e^{-jn \cdot (2\pi F_B \cdot \tau_m / N)} + v_n \\ \beta_m = \alpha_m \cdot e^{jC\tau_m}, n = 0 \dots N-1 \end{cases}. \qquad (7)$$

3 MPP Algorithm

Once channel estimates $\left\{\widehat{H}(n)|n=0\ldots N-1\right\}$ are obtained via (6), MPP can be performed for positioning. We define a group of $L_2 \times L_1$ matrices $\mathbf{X}_n$as,

$$\mathbf{X}_n = \begin{bmatrix} \widehat{H}(n) & \widehat{H}(n+1) & \ldots & \widehat{H}(n+L_1-1) \\ \widehat{H}(n+1) & \widehat{H}(n+2) & \ldots & \widehat{H}(n+L_1) \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{H}(n+L_2-1) & \widehat{H}(n+L_2) & \ldots & \widehat{H}(n+L_1+L_2-2) \end{bmatrix}, \quad (8)$$
$$n=0\ldots N-L_1-L_2+1$$

where L_1 and L_2 satisfy,

$$\begin{array}{l} L_1, L_2 \geq M \\ L_1+L_2 \leq N \end{array}. \quad (9)$$

In a noise free environment, the rank of $\mathbf{X}_n$ is M, i.e. $v_n=0$, however, it is greater than M when noise components are considered. To reduce the effect of noise, the M-truncated SVD (Singular Value Decomposition) of $\mathbf{X}_n$ is obtained by,

$$\widetilde{\mathbf{X}}_n = \mathbf{U}_n \cdot \mathbf{\Sigma}_n \cdot \mathbf{V}_n^H, \quad (10)$$

where $\mathbf{\Sigma}_n$ is the M-by-M diagonal matrix of the M principal singular values of $\mathbf{X}_n$. The $\mathbf{U}_n$ consists of the M principal left singular vectors of $\mathbf{X}_n$, and the $\mathbf{V}_n$ consists of the M principal right singular vectors of $\mathbf{X}_n$. Based on relation (7), it was shown in [16] that the M multipath delays can be estimated by the eigenvalues of any M-by-M $\mathbf{Q}$ matrices,

$$\begin{cases} \mathbf{Q}_n = \mathbf{\Sigma}_{n+1}^{-1}(\mathbf{U}_{n+1}^H \mathbf{U}_n \mathbf{\Sigma}_n \mathbf{V}_n^H \mathbf{V}_{n+1}) \\ n=0\ldots N-L_1-L_2 \end{cases}. \quad (11)$$

Thus, $N-L_1-L_2+1$ $\mathbf{Q}$ matrices can be averaged to reduce noise interference. The delay estimation is obtained by,

$$\widehat{\tau_m} = \frac{\ln(z_m)\cdot N}{-j2\pi F_B}, \quad (12)$$

where $\{z_m|m=1\ldots M\}$ are the M eigenvalues of,

$$\mathbf{Q}_{avg} = \frac{1}{N-L_1-L_2+1}\sum_{n=0}^{N-L_1-L_2}\mathbf{Q}_n. \quad (13)$$

Since the LOS path corresponds to the shortest delay, distance between two nodes is decided by,

$$\widehat{d} = c\cdot\min\left\{\widehat{\tau_m}, m=1\ldots M\right\}, \quad (14)$$

where c is the speed of EM waves, $3\times10^8 m/sec$. Once distances to more than three anchor nodes are available, the desired absolute position can be estimated by LS triangulation localization [6].

An important related question is how to decide coefficients M, and L_1, L_2 at sensor nodes. Decision on M involves a well-researched area of estimating the sinusoids number [17]. However, most these approaches assume white noise components, which are generally not true in our scenario. Alternatively, a simple way is adopted to estimate M. Assume that the maximum singular value of $\mathbf{X}_n$ is ϑ_n , and M_n denotes the number of singular values greater than $\delta \cdot \vartheta_n$, where δ is a threshold coefficient related to channel condition. M is decided by averaging M_n over $n = 0 \ldots N - L_1 - L_2 + 1$.

L_1, L_2 are then chosen under condition (9). However, analyzing the optimal choice of L_1, L_2 is a difficult problem. In [15], one similar such coefficient was treated as a free parameter. In our scenario, it is more complex, since two parameters are introduced. Basically, choosing smaller values for both L_1 and L_2 will increase the number of $\mathbf{Q}$ matrices and suppress noise degradations, however, it will also make $\mathbf{X}_n$ more ill-conditioned. We simply point out the tradeoff here and leave it for future research.

4 Simulations and Results

Simulations are performed to test the positioning accuracy of MPP in WSAN. In obtaining the channel model coefficients in (2), the simulation environmental model is set as a square room, with $(M-1)$ scatterers uniform-randomly distributed in it. Thus, including LOS, there are altogether M multipaths in one channel. Assume that $\{\mathbf{p}_m | m = 1 \ldots M-1\}$ denote the $(M-1)$ scatterers' coordinates, and $\mathbf{rx}, \mathbf{tx}$ denote the coordinates of receive node and transmit node respectively. Then, the associated time-delays with $M-1$ scattered paths are,

$$\tau_m = \frac{\{\|\mathbf{p}_m - \mathbf{rx}\| + \|\mathbf{p}_m - \mathbf{tx}\|\}}{c}, m = 1 \ldots M-1, \tag{15}$$

while the LOS delay is,

$$\tau_M = \tau_{LOS} = \frac{\|\mathbf{tx} - \mathbf{rx}\|}{c}. \tag{16}$$

Being consistent with some indoor propagation experimental results [12, 13], the coefficients α_m are modeled as following,

$$\alpha_m = u_m \cdot \tau_{LOS}/\tau_m, m = 1 \ldots M, \tag{17}$$

where $\{u_m | m = 1 \ldots M\}$ are independent zero-mean unit-variance complex Gaussian random variables.

In our simulations, all nodes are assumed to be within the transmission range of each other. System bandwidth is set the same as IEEE 802.11, $F_B = 11MHz$. The transmitting filter uses a raised cosine pulse. The training length L in (5) is set as 1000, and K varies according to different room sizes. For all cases, $K \leq 4$, thus the total length of training sequence is less than 4000. White Gaussian noise with variance σ_n^2 is added to RF signals on receiver. Peak signal to noise ratio is thus defined as,

$$SNR = \frac{|\alpha_{LOS}|^2}{\sigma_n^2}. \tag{18}$$

The number of FFT points is set to $N = 25$. To simplify our problem, L_1 is set to $M + 2$, and L_2 is chosen as $N - L_1$. Thus, only one $\mathbf{Q}$ matrix is obtained, which also suggests our simulation results can be further improved. The M-decision threshold δ is set as 0.01.

Range measurement error percents are simulated in different rooms when either M or SNR varies. It is defined as,

$$RangeErrorPercent = \left| \frac{\|\mathbf{rx} - \mathbf{tx}\| - \widehat{d}}{\|\mathbf{rx} - \mathbf{tx}\|} \right| \times 100. \tag{19}$$

Transmit and receive nodes are randomly deployed in the room. The average of 1000 Monte-Carlo runs is plotted. In Fig.1, M is fixed as 3 and SNR varies from $-4dB$ to $20dB$. In Fig.2, SNR is fixed as $10dB$, while M varies from 1 to 7. In both figures, simulations in larger rooms show better performance. Larger rooms are more likely to have widespread multipath delays, which prevent ill-conditioning of the $\mathbf{Q}$ matrix. Another observation is that generally smaller M suggests better performance, especially, $M = 1$ results in far lower error percent than others. The reasons can be found by Cramer-Rao bound analysis [14]. However, the degradation with increasing M is not steep. At $10dB$ SNR, the range error is still within 30%, even with $M = 8$. The fluctuations of these curves can be due to our non-optimal choice of L_1 and L_2. Compared with RSSI, which may exhibit a range error as large as 50% [5], our results show a significant improvement.

Furthermore, positioning errors are simulated with different size rooms and different multipath number M, when the number of anchor nodes varies. SNR is fixed at $10dB$. All nodes are randomly deployed over the room. A simple strategy similar to [11] is employed to discover and discard NLOS measurements, and 10 MPP iterations are averaged for each positioning. Results of 200 Monte-Carlo runs are then averaged and plotted in Fig.3-Fig.5 for different size rooms. The results show robustness to multipath number M, when $M > 1$. However, when $M = 1$, much better performance is achieved due to much smaller range error. When enough anchors are available, the positioning error is around one meter. Considering different transmission ranges in different rooms, the positioning error percent is within 5%. In an extreme case with only three anchors, it is around 10%. When positioning error percents are considered, larger rooms still have better performance, which is due to the same reason as in range error percent.

5 Conclusions and Future work

In this paper, we have proposed a new positioning algorithm for wireless ad-hoc sensor network, MPP. The new method depends on matrix pencil approach

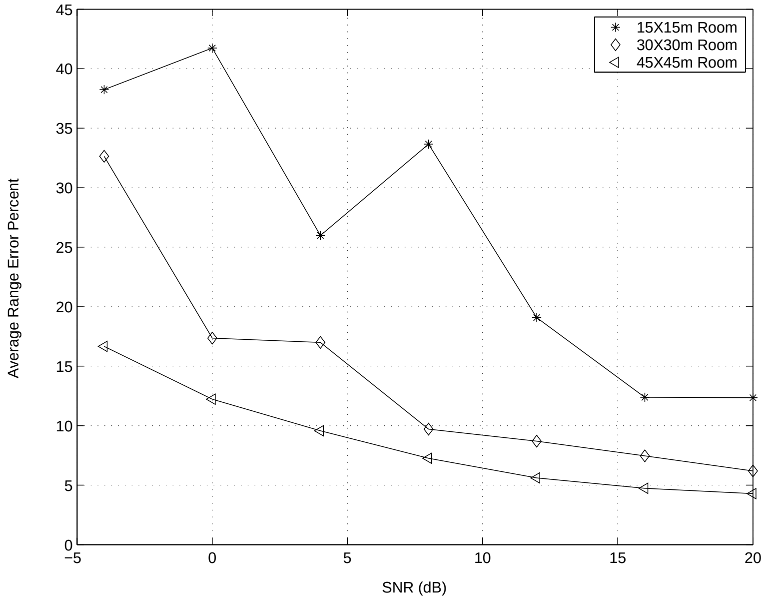

Fig. 1. $M = 3$, range error percent simulations in square rooms

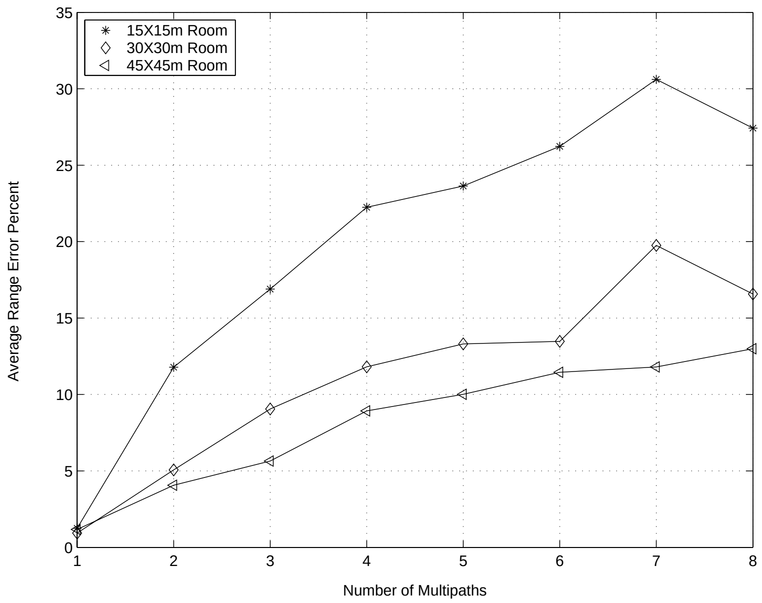

Fig. 2. $SNR = 10dB$, range error percent simulations in square rooms

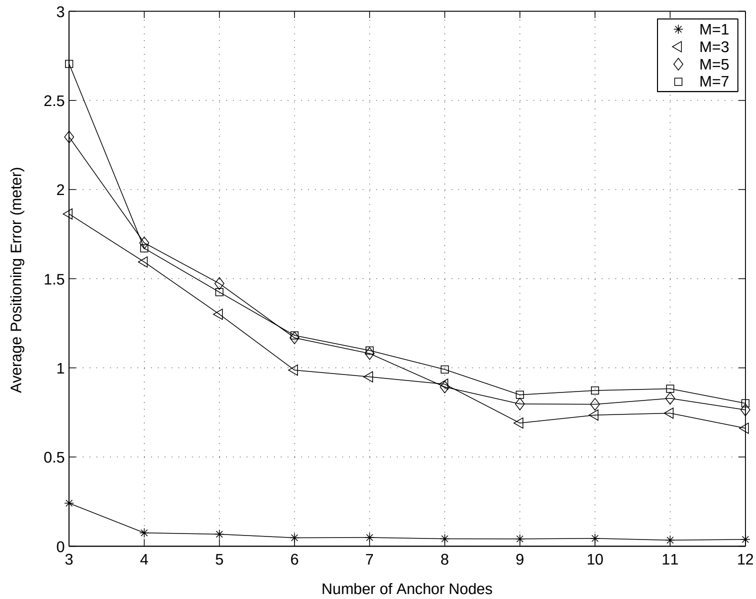

Fig. 3. Mean positioning error simulations in $15 \times 15m$ Room

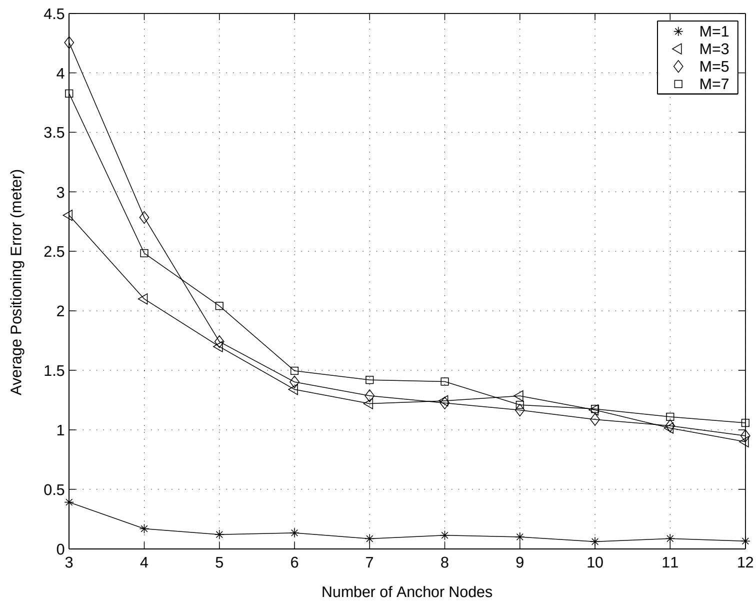

Fig. 4. Mean positioning error simulations in $30 \times 30m$ Room

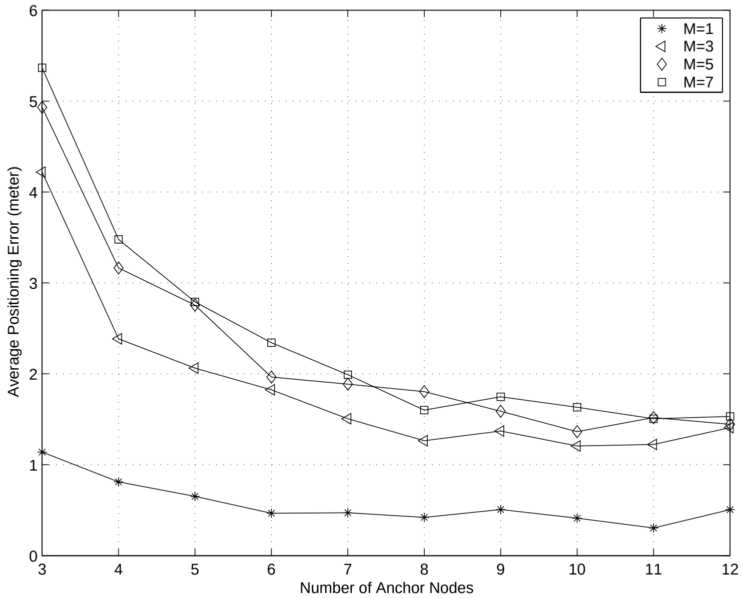

Fig. 5. Mean positioning error simulations in $45 \times 45m$ Room

to perform TOA estimation and on triangulation to obtain positioning. Compared with RSSI, the TOA range measurement by MPP is much more accurate, thus complex compensation algorithms are avoided. Compared with AHLos like methods, our approach avoids the disadvantages of acoustic signal and can be more robust to diverse environments.

Future works should consider an optimal decision theory of coefficients L_1 and L_2. Also, the implementation of MPP depends on the important assumption that the transmission time is known. More research on this TTR assumption in WASN is needed before it can be practically implemented.

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