CLUTTER DOPPLER RANGE DEPENDENCY IN ORTHOGONAL-WAVEFORM-DIVERSITY-PROCESSING FOR DISTRIBUTED APERTURE RADARS

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Abstract

Distributed aperture radars represent an interesting solution for target detection in environments affected by clutter. Due to the large distances between array elements, both target and interfering sources are in the near field of the antenna array. Recent works have demonstrated the benefits of combining frequency diversity and space time adaptive processing for distributed aperture radars. Using orthogonal signaling the receivers can treat the incoming signals independently, solving several bistatic problems instead of the initial multistatic problem. However, a well known problem in bistatic radar is the dependency of the clutter Doppler center on range. We analyze the benefits of joint use of waveform diversity and adaptive techniques to counteract the non-stationarity of the clutter Doppler.

1 Introduction

Recent works have shown the benefit of the joint use of distributed aperture radars and waveform diversity [1, 2]. The large baseline of the distributed aperture radar results in improved angular resolution compared to the resolution of a monolithic system, at cost of grating lobes or high sidelobes. The system under consideration is a very sparse array of subapertures placed thousands of wavelengths apart. Each subaperture of the array transmits a unique waveform, orthogonal to the signals transmitted by the others; to achieve time orthogonality we use pulses that do not overlap in the time domain. Each aperture receives all the transmitted signals, but, due to the orthogonality assumption, each signal can be treated independent of the others. Waveform diversity is achieved using multiple signals characterized by different pulse durations.

A very important issue arising from the work in [1] and [2] is that, due to the very long baseline, both signals and interference sources are not in the far field of the antenna array. For this configuration, the spatial steering vector depends not only on signal *angle* of arrival but also on the *distance* between receiver and target. To take in account this range dependency, some works model the steering vector as a function of the curvature radius of the wave [3], modifying the phase shift contributed to each antenna element. However, as outlined in [1], to take in account the waveform diversity, instead of using phase shifts to model the delay of wave propagation through the array, the processing scheme requires true time delay between the widely distributed antennas. The interference is modeled as a sum of several low power interference sources, each with a range dependent contribution. A well known problem in bistatic radar [4] is that clutter Doppler center is range dependent due to the relative motion between antennas and interference source. This dependency significantly degrades the achievable performance of the receiver and must be taken in account for effective clutter suppression.

The primary goal of this paper is to characterize the impact of clutter non-stationarity due to the bistatic problem in the context of waveform diversity. This paper represents an initial effort in the area of bistatic and multistatic space-time adaptive processing (STAP) applied to distributed apertures. Based on previous results, in this paper, we use a new waveform diversity model that involves the pulse duration [5] instead of the frequency diversity proposed in [1]. In this paper we analyze the performance improvement achievable using specific techniques to counteract clutter Doppler non-stationarity.

The paper is organized as follows. In Section 2 we present the problem at hand and develop the system and interference models in the case of interest. In Section 3 we outline the clutter Doppler range dependency. Section 4 presents results of simulations illustrating the efficacy of our proposed approach. In Section 5 we present the conclusions and outline the possible future research to improve on our present results.

2 System Model

The system under consideration is a ground based distributed aperture radar attempting to detect low flying targets. For distributed arrays the steering vector depends on both the signal angle of arrival (like in a far field source model) and on the distance, due to the near field source model. In fact, given an antenna array of aperture D, operating at wavelength λ, the distance r to the far field must be satisfy [3]

$$\begin{array}{rcl} r & \gg & D, \\ r & \gg & \lambda, \\ r & \gg & 2D^2/\lambda. \end{array} \tag{1}$$

Using typical values for distributed radars, D=200m and λ=0.03m the far field distance begins at a distance of approximately 2700km. It is evident that for many practical applications both signals and interference source might not be in the far field. In this case the steering vector depends on both angle and range.

Accounting for waveform diversity and the dependence of the steering vector on range the processing scheme requires the use of true time delays. In the following, we develop the model for signal and interference source. The system is composed of N elements that are both receivers and transmitters. Fig. 1 presents an example with 3 transmitting elements and 2 pulses per element. The sensors are located in the $x-y$ plane at the points $(x_n, y_n), n = 1, \ldots, N$ and transmit a coherent stream of M linear FM pulses, with common frequency f, common pulse repetition interval (PRI) T_r, common bandwidth B but different pulse durations, i.e., the slope of instantaneous frequency varies among the N transmitted signals. To achieve the orthogonality and waveform diversity the pulses have different durations and do not overlap in the time domain. All N elements receive and process all N incoming

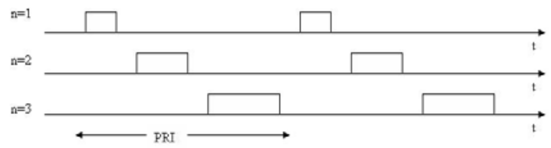

Figure 1: Time orthogonal signals with different pulse duration and common PRI

signals, i.e., if M pulses are used in a coherent pulse interval (CPI), the return signal over time, space and waveform can be written as a N^2M-length vector.

Due to the orthogonality of the signals, the receiver processes each incoming signal independently from the others and uses true time delay to focus on a look-point (X_t, Y_t, Z_t). Denote as $D_n = \sqrt{(X_t - x_n)^2 + (Y_t - y_n)^2 + (Z_t - z_n)^2}$ the distance between the look point (X_t, Y_t, Z_t) and the n^{th} element. The true time delay used by the receiver n is [1]

$$\Delta T_n = \frac{\max_i\{D_i\} - D_n}{c}, \tag{2}$$

where c is the speed of light. By using the true time delays, the normalized response at the N elements due to the N signals is just a vector of ones, i.e., the space-time steering vector, **s**, is given by

$$\begin{aligned} \mathbf{s} &= \mathbf{s}_t \otimes \mathbf{s}_{sf}, && (3)\\ \mathbf{s}_t &= \left[1, e^{j2\pi f_d T_r}, \ldots, e^{j(M-1)\times 2\pi f_d T_r}\right]^T, && (4)\\ \mathbf{s}_{sf} &= [1, 1, 1, \ldots, 1]^T, && (5) \end{aligned}$$

where $\otimes$ denotes the Kronecker product, f_d is the target Doppler frequency, $\mathbf{s}_t$ is the M-length temporal steering vector as in [6] and $\mathbf{s}_{sf}$ is the N^2-length space-waveform steering vector of ones.

As in [6], the interference here is modelled as the sum of many low power sources. The signal, transmitted by the n^{th} element, over M pulses with pulse shape $u_{pn}(t)$ is given by

$$s_n(t) = u_n(t)e^{j2\pi ft+\psi}; u_n(t) = \sum_{m=0}^{M-1} u_{pn}(t - mT_r), \tag{6}$$

where ψ is a random phase shift. The choice of same PRI ensures the same temporal configuration over the pulse number m. The received signal at the element i corresponding to the n^{th} transmitted signal due to the l^{th} artifact located at the point (x^l, y^l, z^l) is

$$\tilde{r}_i^n(t) = A_n^l u_n(t - \tau_{inl})e^{j2\pi(f+f_{dcn}^l)(t-\tau_{inl})}, \tag{7}$$

where A_n^l is the complex amplitude with the random phase (ψ is therein incorporated), f_{dcn}^l is the Doppler frequency of the interference source and

$$\tau_{inl} = \frac{\sqrt{(x_i - x^l)^2 + (y_i - y^l)^2 + (z_i - z^l)^2}}{c}, + \frac{\sqrt{(x_n - x^l)^2 + (y_n - y^l)^2 + (z_n - z^l)^2}}{c},$$

is the delay from n^{th} transmitter to l^{th} interference source plus the delay from the last one to the i^{th} receiver element. After down conversion, delay and matched filtering, the received signal becomes

$$x_i^n(t) = \sum_{m=0}^{M-1} A_n^l e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l mT_r} \chi_n(t - mT_r - \tau_{inl} - \Delta T_i, f_{dcn}^l), \quad (8)$$

where $\chi_n(\tau, f)$ is the ambiguity function of the pulse shape $u_{pn}(t)$ evaluated at the time delay τ and the Doppler f_{dcn}^l. Sampling this signal every $t = kT_s$ corresponding to each range bin and using $\chi_n(mT_r, f) \simeq 0, m \neq 0$,

$$x_i^n(kT_s) = \sum_{m=0}^{M-1} A_n^l e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l mT_r} \chi_n(kT_s - mT_r - \tau_{inl} - \Delta T_i, f_{dcn}^l). \quad (9)$$

Finally, given N_c interfering sources located at points $(x^l, y^l, z^l), l = 1, \dots, N_c$, the received signal at i^{th} receiver on the m^{th} pulse due to n^{th} signal is

$$x_i^n(kT_s, m) = \sum_{l=0}^{N_c-1} A_n^l e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l mT_r} \chi(kT_s - mT_r - \tau_{inl} - \Delta T_i, f_{dcn}^l). \quad (10)$$

We can now implement a space-time-adaptive-processing (STAP) involving the modified sample matrix inversion (MSMI) [7] statistic for target detection. As usual, we estimate the interference covariance matrix from secondary data. Due to the orthogonality the covariance matrix is diagonal

$$\hat{\mathbf{R}} = \begin{bmatrix} \hat{\mathbf{R}}_1 & 0 & \dots & 0 \\ 0 & \hat{\mathbf{R}}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{\mathbf{R}}_N \end{bmatrix} \quad (11)$$

where

$$\hat{\mathbf{R}}_n = \frac{1}{K} \sum_{k=1}^{K} \mathbf{y}_{nk} \mathbf{y}_{nk}^H$$

is the n^{th} block of the matrix in (11) and is relative to the n^{th} transmission. The vectors $\mathbf{y}_{nk}, n = 1, \dots, N, k = 1, \dots, K$ are the secondary data collected for the n^{th} transmission; they include the additive white Gaussian noise beyond the clutter. The superscript $(\cdot)^H$ represents the Hermitian or conjugate transpose. Using the above defined matrices we can calculate the weight vectors for each bistatic problem

$$\mathbf{w}_n = \hat{\mathbf{R}}_n^{-1} \mathbf{s}_n \quad (12)$$

involving the space-time steering vector $\mathbf{s}_n$; these are the space-time steering vector of each transmission, related to the steering vector in (3) by

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_N \end{bmatrix}. \quad (13)$$

Finally, the coherent output statistic is

$$\text{MSMI} = \frac{\left| \sum_{n=1}^{N} \mathbf{w}_n^H \mathbf{y}_n \right|^2}{\left| \sum_{n=1}^{N} \mathbf{w}_n^H \mathbf{s}_n \right|}. \quad (14)$$

where $\mathbf{y}_n$ is the received signal. Note that the statistic assumes coherence across all the transmissions. This is possible because, unlike the frequency diversity case of [1], all transmissions share a common center frequency.

3 Clutter Non-stationarity

One of the advantages of using orthogonal signaling is the independent treatment of each signal at the receiver. Furthermore, this assumption changes the multistatic initial problem into many individual bistatic problems and we can use results from bistatic radar theory for our treatment. In particular, the non-stationary nature of ground-based clutter in bistatic airborne radar is well known [4]; in this configuration the motion of either transmitter and receiver causes the range dependency of clutter Doppler. In our system the motion is due to the artifact, while both transmitter and receiver are ground-based and we have to reconsider the bistatic problem in the context of waveform diversity. Fig. 2 shows the geometry of the system, where Ar is the artifact, $\mathbf{v}$ is the artifact velocity, θ_T, θ_R and θ_v are, respectively, the elevation angles of the transmitter-artifact path, the receiver-artifact path and the artifact velocity, φ_T, φ_R and φ_v are, respectively, the azimuth angles of the transmitter-artifact path, the receiver-artifact path and the artifact velocity. The clutter Doppler frequency due to the motion is

$$f_D = f_{Dx} + f_{Dy} + f_{Dz} \quad (15)$$

where

$$f_{Dx} = \frac{\mathbf{v} \cos\theta_v \cos\varphi_v [\cos\theta_t \cos\varphi_t + \cos\theta_r \cos\varphi_r]}{\lambda} \quad (16)$$

$$f_{Dy} = \frac{\mathbf{v} \cos\theta_v \sin\varphi_v [\cos\theta_t \sin\varphi_t + \cos\theta_r \sin\varphi_r]}{\lambda} \quad (17)$$

$$f_{Dz} = \frac{\mathbf{v} \sin\theta_v [\sin\theta_t + \sin\varphi_r]}{\lambda}. \quad (18)$$

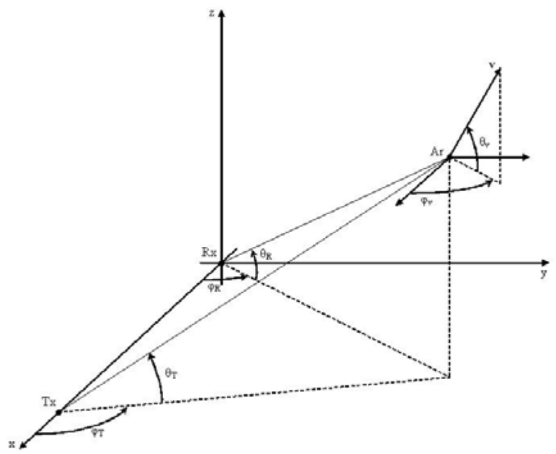

Figure 2: Geometry of bistatic ground radar

As demonstrated in [1] this non-stationarity affects the output of the STAP processor. The basic hypothesis of the space-time-adaptive-processing is the stationarity of the environment. In fact, the secondary data are collected from range gates close to the one under test and are used to estimate the covariance matrix of the interference; the good quality of this estimation depends on the stationarity of the environment. Due to the range dependency of the clutter Doppler frequency, this hypothesis is no longer valid.

To improve the performance of the STAP algorithm over that given in [1] we account for the clutter non-stationarity by modifying the STAP processing [4]. Given the limited sample support available we use the reduced dimension Joint Domain Localized (JDL) algorithm. Using this algorithm we convert the samples from the space-time domain to the angle-Doppler domain. Using the JDL algorithm we can reduce the number of adaptive unknowns from NM in the original domain to $\eta_a \eta_d$, where η_a and η_d are, respectively, the number of angle bins and Doppler bins chosen; Figure 3 presents a pictorial view of the processing scheme, where the bin marked as "Signal" indicates the target location in angle-Doppler domain. As in [8] we can convert the samples using a transformation matrix $\mathbf{T}_n$ for each transmission. In a general case the transformation from the time-space domain in the angle-Doppler domain is an inner product with a space-time steering vector. The transformation matrix is defined as

$$\mathbf{T}_n = \mathbf{s}_t(\mathbf{f}) \otimes \mathbf{s}_{sfn} \tag{19}$$

where $\mathbf{s}_{sfn}$ is the N-length space steering of ones related to the n^{th} transmission and **f** is a vector of Doppler frequencies normalized by the PRF, centered at the Doppler of the target and spaced by $1/M$. For example, if η_a=3 and η_d=3 this matrix is

$$\mathbf{T}_n = [\mathbf{s}_t(f_{-1}), \mathbf{s}_t(f_0), \mathbf{s}_t(f_1)] \otimes [\mathbf{s}_{sfn}, \mathbf{s}_{sfn}, \mathbf{s}_{sfn}]. \tag{20}$$

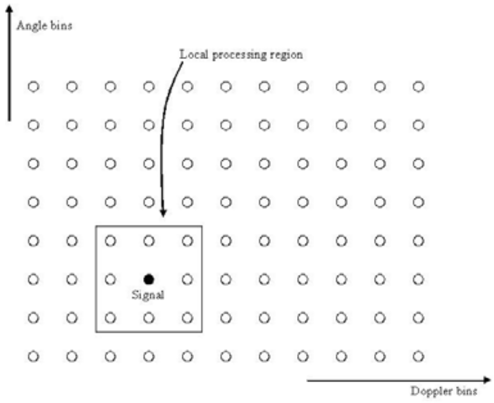

Figure 3: Localised processing region in angle-Doppler domain for $\eta_a = \eta_d$=3.

The relevant transformation is a pre-multiplication with $(NM \times \eta_a \eta_d)$ transformation matrix. So the received signal, the steering vector and the weight vector in the angle-Doppler domain are respectively for the n^{th} transmission

$$\mathbf{y}_{naD} = \mathbf{T}_n^H \mathbf{y}_n \tag{21}$$
$$\mathbf{s}_{naD} = \mathbf{T}_n^H \mathbf{s}_n \tag{22}$$
$$\mathbf{w}_{naD} = \mathbf{T}_n^H \mathbf{w}_n \tag{23}$$

where the subscript *aD* indicates that these vectors relate to the angle-Doppler domain. Using the new vectors we can calculate the decision statistic as in equation (14) with the vectors of the transformed domain.

4 Simulation Results

Results reported in [1] had demonstrated the importance of the use of waveform diversity for distributed aperture radars in order to deal with the problem of grating lobes. Using frequency diversity proposed in [1] it is possible to eliminate the grating lobes. Using waveform diversity model the grating lobes are smaller than that achievable with frequency diversity model and a clear target identification is preserved [5].

Here we are interested in comparing the performances achievable in the time-space domain with that ones in the angle-Doppler domain. The experiments use the parameters shown in Table 1. In the table T_{MIN} and T_{MAX} represent the minimum and maximum pulse duration respectively. The difference between pulse durations of the the N transmissions is $(T_{MAX} - T_{MIN})/N$. The array elements are uniformly distributed in the $x - y$ plane on a square $200m \times 200m$ grid. INR is the Interference-to-Noise Ratio.

Figure 4 compares the performance of the JDL algorithm with the fully traditional space-time approach. As can be seen, the JDL approach provides better results, largely because of the improved estimation of the interference covariance matrix.

Parameter	Value	Parameter	Value
N	9	M	3
T_{MIN}	$10\mu s$	T_{MAX}	$100\mu s$
B	10MHz	f	10GHz
PRI	$5\sum_{i=1}^{N} T_i$	INR	50dB
Target Velocity	50m/s	Target SNR	10dB
X_t	476.9158m	Y_t	-59.9566m
Z_t	200km	N_c	1e5
η_a	3	η_d	3

Table 1: Common parameters used in the experiments.

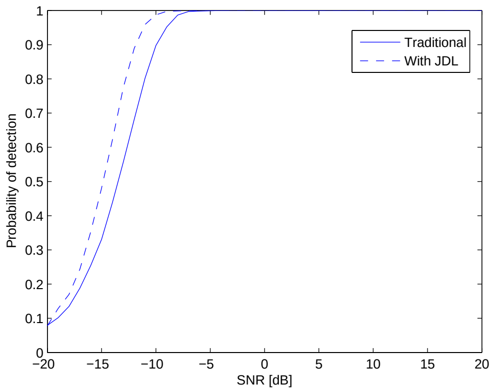

Figure 4: Probability of detection versus SNR applying or not the angle-Doppler transformation.

Due to the little dimensionality difference between the space-time domain and the angle-Doppler domain the improvement is small, less than 2 dB for a detection probability of 0.9.

5 Conclusions

This paper takes in account the range dependency of the clutter Doppler frequency in a multistatic radar due to the relative motion between antenna elements and scatterers. Based on previous results, the signal model uses waveform diversity applied to distributed aperture radars. Based on the realization that both signal and interference sources are not in the far-field of the antenna array, this paper uses a data model accounting for range dependency and waveform diversity based on true time delay. To take in account the range dependency of the clutter Doppler frequency we converted the samples from the space-time domain to the angle-Doppler domain and selected a small number of bins of the joint domain for the decision statistic. The numerical simulation shows the improvement achievable in terms of improved detection probability versus SNR.

For future works, an interesting point of view is the possibility of new waveform diversity schemes; using waveform differentiated on more parameters (such for example the PRI and the pulse duration) can even improve the achievable performances making the waveforms used strongly different each others. Another idea is the analysis of the effects of incorrect knowledge about the target distance. One the fundamental hypotheses is the knowledge of the delay from the i^{th} transmitter to the target plus the delay from the target to the n^{th} transmitter to recover the orthogonality at the receiver.

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