

Time-Orthogonal-Waveform-Space-Time Adaptive Processing for Distributed Aperture Radars

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Abstract—Distributed aperture radars represent an interesting solution for target detection in environments affected by ground clutter. Due to the large distances between array elements, both target and interfering sources are in the near field of the antenna array. As a consequence the characterization of both the target and the clutter is complicated, combining bistatic and monostatic configurations. Using orthogonal signaling the receivers can treat the incoming signals independently solving separately bistatic problems instead of the initial multistatic problem. Recent works have demonstrated the benefits of the use of frequency diversity space time adaptive processing for distributed aperture radars. This paper modifies the waveform diversity signal model, resorting to a time orthogonal signaling scheme, which does not present the coherence loss exhibited by frequency diversity.

I. INTRODUCTION

Recent works have shown the benefits of the joint use of distributed aperture radars and waveform diversity [2], [3]. The large baseline of the distributed aperture radar results in improved angular resolution compared to the resolution of a monolithic system, at cost of grating lobes or high sidelobes. The phrase “waveform diversity” has now come to include distributed communication networks, distributed space-time coding, and distributed target detection [1]. Our focus here is target detection using a distributed radar. In this regard, we focus on the furthering the development of space-time adaptive processing (STAP) algorithms for distributed apertures.

The system under consideration is a very sparse array of sub-apertures placed thousands of wavelengths apart. Each sub-aperture of the array transmits an unique waveform, orthogonal to the signals transmitted by the others; to achieve time orthogonality we use pulses that do not overlap in the time domain. Each aperture receives all the transmitted signals, but, due to the orthogonality hypothesis, each signal can be treated independent of

the others. Waveform diversity is achieved using multiple signals characterized by different pulse durations.

An important issue arising from the work in [2] and [3] is that, due to the very long baseline, both signals and interference sources are not in the far field of the antenna array. For this configuration, the spatial steering vector depends not only on signal *angle* of arrival but also on the *distance* between receiver and target. To take in account this range dependency, some works model the steering vector as a function of the curvature radius of the wave [4], modifying the phase shift contributing to each antenna element. However, as outlined in [2], to take in account the waveform diversity, instead of using phase shifts to model the delay of wave propagation through the array, the processing scheme requires true time delays between the widely distributed antennas. Moreover, the interference is modeled as a sum of several low power interference sources, each with a range dependent contribution. Previous works have developed the model required to generate simulated data [2], [3] to develop and test signal processing algorithms.

Previous work such as described above has focused on *frequency diversity* to enable orthogonal transmissions from each element in the distributed array. However, frequency diversity raises the difficult issue of coherent processing across a wide frequency range. This paper proposes a system using an alternative approach, using time orthogonal waveforms, with differing pulse durations, to achieve diversity. Waveform diversity using varying FM rates was proposed in [5] in the context of target tracking. In addition, the distributed radar problem is inherently multistatic with multiple radars illuminating the area of interest, and also receiving and potentially processing all these transmissions. A true development of STAP for distributed apertures will therefore include both monostatic [6] and bistatic configurations [7], [8].

The goal of this paper is to develop a new model for waveform diversity for distributed aperture radars with time-orthogonal waveforms. In this regard, this paper represents a continuation in the research about waveform diversity for distributed aperture radars and also an effort on the bistatic and multistatic STAP applied to distributed aperture. The time orthogonal waveforms, just like with frequency diversity, allows for independent processing of each transmit-receive combination. A companion paper focuses on the case of time-overlapping transmissions with differing pulse widths [9]. Based on previous results, in this paper, we introduce a new waveform diversity model that involves the pulse duration instead of the frequency diversity proposed in [2].

The paper is organized as follows. In Section II we develop the system and interference model in the case of interest. In Section III we report the results of numerical simulations using the quoted model. In Section IV we present the conclusions and outline the future possible works to improve our results.

II. SYSTEM MODEL

The system under consideration is a ground based distributed aperture radar attempting to detect low flying targets. For distributed arrays the steering vector depends on both the signal angle of arrival (like in a far field source model) and on the distance, due to the near field source model. In fact, given an antenna array of aperture D , operating at wavelength λ , the distance r to the far field must satisfy [4]

$$\begin{aligned} r &\gg D, \\ r &\gg \lambda, \\ r &\gg 2D^2/\lambda. \end{aligned} \quad (1)$$

Using typical values for distributed radars, $D=200\text{m}$ and $\lambda=0.03\text{m}$, the far field distance begins at a distance of approximately 2700km. It is evident that for many practical applications both signals and interference source might not be in the far field. In this case the steering vector depends on both angle and range.

In order to account for waveform diversity and the dependence of the steering vector on range, the processing scheme requires the use of true time delays. In the following, we develop the model for the signal and the interference source. Actually the computation of the steering vector requires accounting for these issues.

A. System model and steering vector

The system is composed of N elements that are both receivers and transmitters. To achieve orthogonality and waveform diversity the pulses have different durations and do not overlap in the time domain; Fig. 1 presents an example with 3 transmitting elements and 2 pulses per element. The elements share a common pulse repetition interval (PRI). The sensors are located in the $x-y$ plane at the points $(x_n, y_n), n = 1, \dots, N$ and transmit a coherent

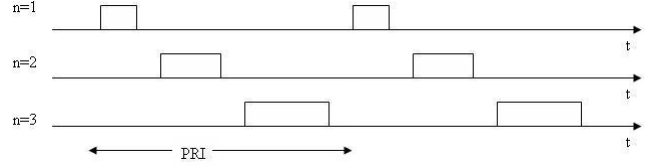


Fig. 1. Time orthogonal signals with different pulse duration and common PRI

stream of M linear FM pulses, with common center frequency f , common pulse repetition interval (PRI) T_r , common bandwidth B but different pulse durations, i.e., the slope of instantaneous frequency varies among the N transmitted signals. All N elements receive and process all N incoming signals, i.e., if M pulses are used in a coherent pulse interval (CPI), the overall return signal over time, space and waveform can be written as a N^2M -length vector.

Due to the orthogonality of the signals, the receiver processes each incoming signal separately from each other and uses true time delay to focus on a look-point (X_t, Y_t, Z_t) . Denote as $D_n = \sqrt{(X_t - x_n)^2 + (Y_t - y_n)^2 + (Z_t - z_n)^2}$ the distance between the look point (X_t, Y_t, Z_t) and the n^{th} element. The true time delay used by the receiver is [2]

$$\Delta T_n = \frac{\max_i \{D_i\} - D_n}{c} \quad (2)$$

where c is the speed of light. By using the true time delays, the normalized response at the N elements due to the N signals is just a vector of ones, i.e., the space-time steering vector, \mathbf{s} , is given by

$$\mathbf{s} = \mathbf{s}_t \otimes \mathbf{s}_{sf}, \quad (3)$$

$$\mathbf{s}_t = \left[1, e^{j2\pi f_d T_r}, \dots, e^{j(M-1)2\pi f_d T_r} \right]^T, \quad (4)$$

$$\mathbf{s}_{sf} = [1, 1, 1, \dots, 1]^T, \quad (5)$$

where \otimes denotes the Kronecker product, f_d is the target Doppler frequency, \mathbf{s}_t is the M -length temporal steering vector and \mathbf{s}_{sf} is the N^2 -length space-waveform steering vector of ones.

B. Clutter model

As in [6], the interference here is modeled as the sum of many low power sources. The signal, transmitted by the n^{th} element, over M pulses with pulse shape $u_{pn}(t)$ is given by

$$s_n(t) = u_n(t)e^{j(2\pi ft + \psi)} \quad (6)$$

with pulse shape

$$u_n(t) = \sum_{m=0}^{M-1} u_{pn}(t - mT_r) \quad (7)$$

where ψ is a random phase shift. The choice of same PRI ensures the same temporal configuration over the pulse number m . The received signal at the element i^{th}

corresponding to the n^{th} transmitted signal due to the l^{th} artifact located at the point (x^l, y^l, z^l) is

$$\tilde{r}_i^n(t) = A_n^l u_n(t - \tau_{inl}) e^{j2\pi(f + f_{dcn}^l)(t - \tau_{inl})}, \quad (8)$$

where A_n^l is the complex amplitude with the random phase (ψ is therein incorporated), f_{dcn}^l is the Doppler frequency of the interference source and

$$\begin{aligned} \tau_{inl} &= \frac{\sqrt{(x_i - x^l)^2 + (y_i - y^l)^2 + (z_i - z^l)^2}}{c} \\ &+ \frac{\sqrt{(x_n - x^l)^2 + (y_n - y^l)^2 + (z_n - z^l)^2}}{c}, \end{aligned}$$

is the delay from the n^{th} transmitter to the l^{th} interference source plus the delay from the last one to the i^{th} receiver element. After down conversion and delay, it becomes

$$\tilde{r}_i^n(t) = A_n^l u_n(t - \tau_{inl} - \Delta T_i) e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l (t - \tau_{inl} - \Delta T_i)}. \quad (9)$$

Applying matched filtering on this signal, the received signal finally becomes

$$\begin{aligned} x_i^n(t) &= \sum_{m=0}^{M-1} A_n^l e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l m T_r} \\ &\chi_n(t - m T_r - \tau_{inl} - \Delta T_i, f_{dcn}^l), \end{aligned} \quad (10)$$

where $\chi_n(\tau, f)$ is the ambiguity function of the pulse shape $u_{pn}(t)$ evaluated at the time delay $\tau = t - m T_r - \tau_{inl} - \Delta T_i$ and the Doppler f_{dcn}^l . Sampling this signal every $t = k T_s$ corresponding to each range bin and using $\chi_n(m T_r, f) \simeq 0, m \neq 0$,

$$\begin{aligned} x_i^n(k T_s) &= \sum_{m=0}^{M-1} A_n^l e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l m T_r} \\ &\chi_n(k T_s - m T_r - \tau_{inl} - \Delta T_i, f_{dcn}^l). \end{aligned} \quad (11)$$

Finally, given N_c interfering sources located at points $(x^l, y^l, z^l), l = 1, \dots, N_c$, the received signal at i^{th} receiver on the m^{th} pulse due to n^{th} signal is

$$\begin{aligned} x_i^n(k T_s, m) &= \sum_{l=0}^{N_c-1} A_n^l e^{-j2\pi f \tau_{inl}} e^{j2\pi f_{dcn}^l m T_r} \\ &\chi(k T_s - m T_r - \tau_{inl} - \Delta T_i, f_{dcn}^l). \end{aligned} \quad (12)$$

C. Space Time Adaptive Processing

We can now implement a space-time-adaptive-processing (STAP) involving the modified sample matrix inversion (MSMI) [10] statistic for target detection. As usual, we estimate the interference covariance matrix from secondary data. Due to the time orthogonality the covariance matrix is diagonal

$$\hat{\mathbf{R}} = \begin{bmatrix} \hat{\mathbf{R}}_1 & 0 & \dots & 0 \\ 0 & \hat{\mathbf{R}}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{\mathbf{R}}_N \end{bmatrix} \quad (13)$$

where

$$\hat{\mathbf{R}}_n = \frac{1}{K} \sum_{k=1}^K \mathbf{y}_{nk} \mathbf{y}_{nk}^H$$

is the n^{th} block of the matrix in (13) and is relative to the n^{th} transmission. The vectors $\mathbf{y}_{nk}, n = 1, \dots, N, k = 1, \dots, K$ are the secondary data collected relative to the n^{th} transmission; they include the additive white gaussian noise beyond the clutter. The superscript $(\cdot)^H$ represents the Hermitian or conjugate transpose. Using the above defined matrices we can calculate the weight vectors for each bistatic problem

$$\mathbf{w}_n = \hat{\mathbf{R}}_n^{-1} \mathbf{s}_n \quad (14)$$

involving the space-time steering vector \mathbf{s}_n ; these are the space-time steering vector of each transmission, related to the steering vector in (3) by

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_N \end{bmatrix}. \quad (15)$$

Finally, the coherent output statistic is

$$\text{MSMI} = \frac{\left| \sum_{n=1}^N \mathbf{w}_n^H \mathbf{y}_n \right|^2}{\sum_{n=1}^N \mathbf{w}_n^H \mathbf{s}_n} \quad (16)$$

where \mathbf{y}_n is the received signal. Note that the statistic assumes coherence across all the transmissions. This is possible because, unlike the frequency diversity case of [2], all transmissions share a common center frequency.

III. NUMERICAL SIMULATIONS

In this section we present the results of numerical simulations using the model developed in the Section II. The experiments use the common parameters shown in table I.

Parameter	Value	Parameter	Value
N	9	M	3
T_{MIN}	10 μ s	T_{MAX}	100 μ s
B	10MHz	f	10GHz
PRI	5 T_{MAX}	INR	50dB
Target Velocity	50m/s	Target SNR	10dB
X_t	476.9158m	Y_t	-59.9566m
Z_t	200km	N_c	1e5

TABLE I

COMMON PARAMETERS USED IN THE EXPERIMENTS.

In the table T_{MIN} and T_{MAX} represent the minimum and maximum pulse duration respectively. The difference between pulse durations of the the N transmissions is $(T_{MAX} - T_{MIN})/N$. The array elements are uniformly distributed in the $x - y$ plane on a square $200m \times 200m$ grid. INR is the Interference-to-Noise Ratio.

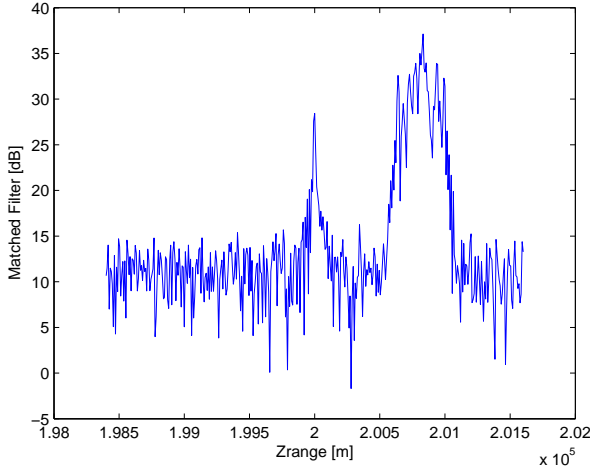


Fig. 2. Matched filter processing along the radial Z-direction. Includes interference.

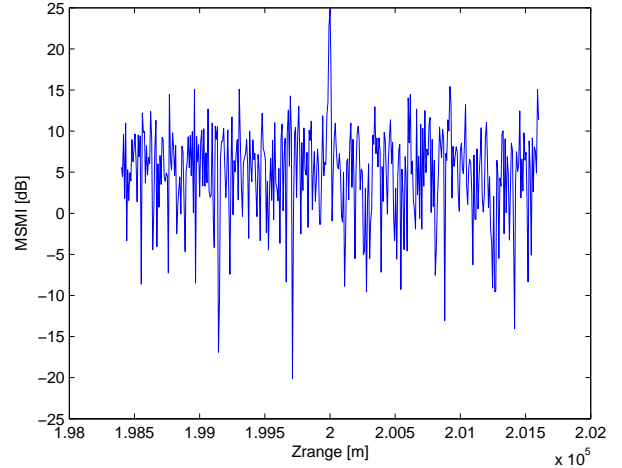


Fig. 4. MSMI statistic along the radial Z-direction. Includes interference.

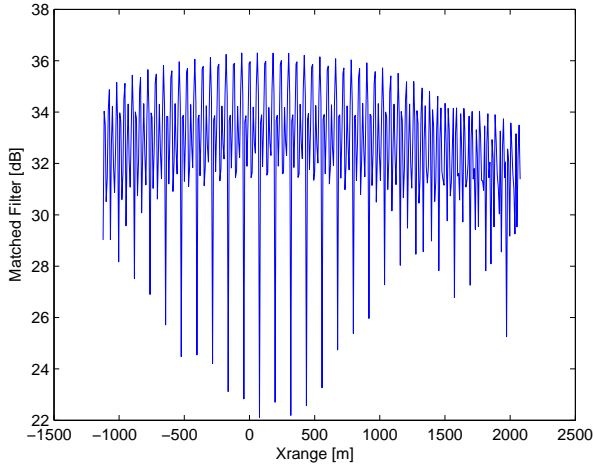


Fig. 3. Matched filter processing along the transverse X-direction. Includes interference.

A. Need for waveform diversity

Results reported in [2] had demonstrated the importance of the use of waveform diversity for distributed aperture radars in order to deal with the problem of grating lobes. Since the steering vectors are range dependent, the beam pattern is a plot of the signal strength versus the transverse coordinate. The range dependency implies a small decay in the grating lobes level further away from the target location X_t . However, this decay is inadequate for target detection. Using frequency diversity proposed in [2] it is possible to eliminate the grating lobes. We expect that using waveform diversity model the grating lobes are smaller than that achievable with frequency diversity model and a clear target identification is preserved.

Figure 2 plots the output of the matched filter along the radial z -direction. The target is at a range of 200km,

in the radial (z) direction. The beam pattern shows low grating lobes for many values of the range, but it is very asymmetric due to the range dependency of the clutter that affects the estimation of the covariance matrix. Figure 3 plots similar results along the transverse x -direction. The target is at a range of 476.9158m in the transverse x -direction. The beam pattern is more regular and symmetric than the radial direction one. This clearly indicates the extent of the interference sources.

Figure 4 plots the modified sample matrix statistic (MSMI) versus the radial z -direction. All interference range cells are used to estimate the interference covariance matrix. The target is very clearly identified, even using only 3 pulses and 9 antenna elements, due to the narrow lobe centered at 200km in range, i.e., at the target range. Figure 5 plots similar results along the transverse x -direction. In this case the target is not clear identifiable and the system shows a performance decay.

IV. CONCLUSIONS AND FUTURE WORKS

This paper develops waveform diversity approach, based on differing FM rates, as an alternative to the frequency diversity approach proposed in [3]. Based on the realization that target and interference are not in the far field of the array, this paper uses a data model accounting for range dependency and waveform diversity based on true time delay. Frequency diversity, using different and orthogonal frequencies, has the problem of the coherence; our approach, based on a single central frequency, avoids this problem making more simple the signaling scheme. The numerical simulations illustrate the importance of the data model and the improved performances achievable. Using the waveform diversity based on the pulse duration the problem of grating lobes in the beam pattern is still present, but the results show their impact is lower than using the frequency diversity scheme and a good detec-

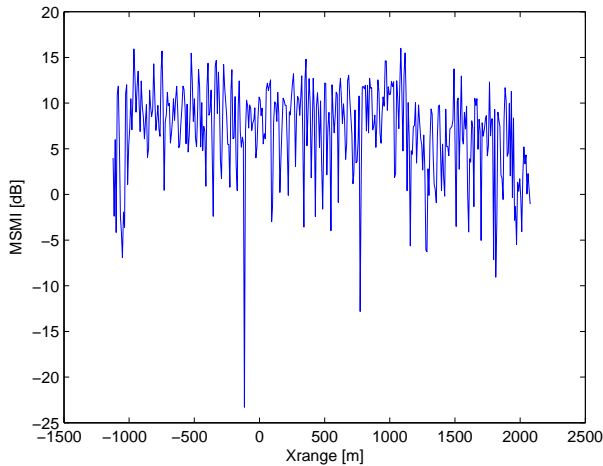


Fig. 5. MSMI statistic along the transverse X-direction. Includes interference.

tion capability is preserved in the transverse direction. In the radial direction the target is not clearly identifiable, exhibiting the need of a new scheme that can improve the detection on this direction.

For future works, an interesting point of view is the possibility of new waveform diversity schemes; using waveform differentiated on more parameters (such for example the PRI and the pulse duration) can even improve the achievable performances making the waveforms used strongly different each others. A companion paper deals with overlapping transmissions. It is also interesting to take in account the range dependency of the clutter that affects the estimation of the covariance matrix; this can allow better performances because of better covariance matrix estimation. The long-term goal is the practical development of space-time adaptive processing schemes for distributed apertures.

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REFERENCES

- [1] M. C. Wicks, "Radar the next generation - sensors as robots", in *Proc. of 2003 IEEE International Radar Conference*, Sept. 2003. Adelaide, Australia.
- [2] L. Applebaum and R.S. Adve, "Adaptive processing with frequency diverse distributed apertures", in *Proc. of the 2nd International Waveform Diversity and Design Conf.*, Jan. 2006. Kauai, HI.
- [3] R. S. Adve, R. A. Schneible, and R. McMillan, "Adaptive space/Frequency Processing for Distributed Aperture Radars", in *Proc. of the 2003 IEEE Radar Conference*, May 2003. Huntsville, AL.
- [4] D. Madurasinghe and L. Teng, "Adaptive Array Processing Near Field Experiment", Tech. Rep. DSTO-TR-0361, Defense Science and Technology Organization, Australia, 1996.
- [5] S. Sira, D. Morell, and A. Papandreou-Suppappola, "Waveform design and scheduling for agile sensors for target tracking", in *Proc. of the 2004 Asilomar Conference on Signals, Systems and Computers*, vol. 1, November 2004.
- [6] J. Ward, "Space-time adaptive processing for airborne radar", Tech. Rep. F19628-95-C-00002, MIT Lincoln Laboratory, December 1994.
- [7] B. Himed, J. H. Michels, and Y. Zhang, "Bistatic STAP performance analysis in radar applications", in *Proc. of the 2001 IEEE Radar Conference*, 2001. Atlanta, GA.
- [8] P. K. Sanyal, R. D. Brown, M. O. Little, R. A. Schneible, and M. C. Wicks, "Space-time adaptive processing bistatic airborne radar", in *Proc. of the 1999 IEEE Radar Conference*, 1999. Waltham, MA.
- [9] E. Lock and R. S. Adve, "Varying fm rates in adaptive processing for distributed radar apertures", in *Proc. of the 3rd International Waveform Diversity and Design Conf.*, June 2007. Pisa, Italy
- [10] L. Cai and H. Wang, "Performance comparisons of modified SMI and GLR algorithms", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 27, No. 2, April 1991.