Accounting for the Effects of Mutual Coupling in Adaptive Antennas

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Abstract

This paper presents a method to account for the effects of mutual coupling in least squared error adaptive algorithms. The mutual coupling is quantized using the Method of Moments. The Method of Moments admittance matrix is used to compensate for the mutual coupling.

I. INTRODUCTION

Least Squared Error (LSE) adaptive algorithms [1], [2], [3] have been proposed to overcome the drawbacks of classical, statistical, adaptive algorithms. They adaptively minimize the total interference power at the output of the receiver while maintaining array gain in the direction of the signal. The least squares techniques use data from the range cell of interest only and hence bypass the requirement of secondary data. Further, not estimating a covariance matrix leads to enormous savings in required real time computations.

The LSE techniques show some promise to overcome the drawbacks of traditional statistical methods. However, all adaptive algorithms assume the receiving antenna array is composed of independent, isotropic, point sensors. This implies that the sensor only passively samples the incident fields spatially. But, the elements of the array must physically be some kind of antenna. The elements not only spatially sample, but also re-radiate the incident fields causing mutual coupling between the array elements. When mutual coupling is taken into account statistical and LSE algorithms fail [4], [5], [6].

Another assumption of the adaptive algorithms is that the array operates in a physical environment where nothing impedes the reception of the signals and interference. However, the array is often in the presence of scatterers, e.g. an airborne radar is in the presence of the fuselage of the aircraft. These near field scatterers much like the array elements themselves, affect the signal reception by re-radiating the incident fields.

The Method of Moments (MOM) [7] is an established electromagnetic analysis technique that is ideally suited to evaluate the mutual coupling analysis. The work of [5] has presented a simple formulation, based on the MOM, to eliminate the effects of mutual coupling on the LSE algorithm of [1]. The formulation there uses the MOM admittance matrix, with multiple unknowns per element, to relate the voltages measured at the ports of the antenna with selected entries of the MOM voltage vector. The key to the formulation is reducing the larger MOM admittance matrix to a smaller square matrix of order equal to the number of elements in the array. This is in contrast to [4] where only one MOM unknown is used per element.

The major drawback with the formulation of [5] is that it is valid only for a $z-$directed linear dipole array. Another drawback is that it cannot account for the effects of scatterers in the vicinity of the array. This is because, to take the scatterers into account, the Method of Moments solution requires additional columns in the equations relating the measured voltages with the MOM voltages. Therefore, in the presence of scatterers the square matrix equation cannot be formed.

This paper introduces a technique that is applicable in general. The proposed algorithm uses the admittance matrix and the measured voltages to estimate the entire MOM voltage vector. This is done by using the minimum norm solution of an underdetermined system of equations. The estimated MOM voltages are used in conjunction with Matrix Pencil to estimate the directions of arrival of the signal and interference. The signal is then recovered by maximizing the gain of the antenna in the look direction while, simultaneously, placing pattern nulls in the directions of the interference. Therefore, the mutual coupling is not eliminated, but rather accounted for in each step of the adaptive process.

A. Array and Signal Models

In this paper, the receiving antenna is modeled as a linear array of N_e straight, perfectly conducting, identical dipole elements, equispaced along the $x-$axis,. The wires are thin and $z-$directed. The wires are assumed to be point loaded at the center. The near field scatterers are also assumed to be thin, short circuited, $z-$directed wire dipoles. This assumption allows for a simplified MOM analysis of the electromagnetics of the problem.

The desired signal is modeled as a source of incident energy arriving from a given direction in space. For the radar problem, this may be thought of as a point scatterer that has reflected a transmitted beam from a specific angle. As space is scanned for targets, large regions will not contain any targets. In such a case, the "desired signal" is identically zero. The thermal noise is modeled to be white and Gaussian. Deliberate jamming is treated like a farfield point source of incident energy. The jammer location in space is unknown.

The other interference source considered here is clutter. The clutter is assumed to arrive at the same Doppler frequency as the signal. The clutter is modeled as arriving from a ridge - a area of space as opposed to a point source. Within the area, the continuous clutter ridge is approximated as many weak point sources spaced very close to each other. Each clutter source has random amplitude found from a uniform random variable. The limits of random variable set the total clutter power.

II. ANALYSIS OF THE ARRAY

To properly understand how the mutual coupling between the elements of the array affects the antenna behavior, and to quantize the mutual coupling, we have to analyze the response of the antenna to an incident field. In this work we use the Method of Moments [7] to analyze the antenna. The Method of Moments numerically solves the linear integral equation relating the incident field impinging on the antenna (considered the known) with the currents on the antenna (considered the unknown). The integral equation is then reduced to solving a matrix equation. As we will show, the elements of the matrix quantize the mutual coupling between the antenna elements.

There are assumed to be N_w total wires in the array and scatterer system. The array elements are of length L and radius a, with $a \ll L$. The array is in the presence of an arbitrary linearly polarized time harmonic incident field $\mathbf{E^{inc}}$. The incident field induces a current $\mathbf{J_s}$ and charge σ_s on the surface of the wires. The induced current re-radiates to produce a scattered field $\mathbf{E^s}$ to satisfy the Maxwell's equations and the boundary conditions of the problem.

Since the wires are thin, the following assumptions can be made [8]:

1. The current flows only in the direction of the wire axes (here the z-direction).
2. The surface current ($\mathbf{J_s}$) and charge (σ_s) densities on the wire can be approximated by line currents (I) and charge (ρ) on the wire axes (they lie in the $y = 0$ plane).
3. The boundary condition can be applied to the axial component of $\mathbf{E}$ on the wire axes i.e. the boundary condition is applied to E_z on the wire axes.

Using these assumptions, and the boundary condition that the total electric field on the axis of the wires must be identically zero, the integral equation that characterizes the behavior of the antenna array is

$$E_z^{inc}(z) = j\omega\mu_0 \int_{axes} I(z') \frac{e^{-jkR}}{4\pi R} dz' - \frac{1}{j\omega\epsilon_0} \frac{\partial}{\partial z} \int_{axes} \frac{\partial I(z')}{\partial z'} \frac{e^{-jkR}}{4\pi R} dz' \quad z \in axes \quad (1)$$

We solve this equation for the currents using the Method of Moments (MOM). The procedure is to expand the currents in a series of convenient basis functions. To reduce the integral equation to a matrix equation, we test the resulting expansion with a set of weighting functions. The basis functions used are the piecewise sinusoids as described by Strait et.al. in [9]. The weighting functions are the same piecewise sinusoids i.e. a Galerkin formulation has been used. This formulation has been chosen because it yields analytic expressions for the elements of the matrix and hence eliminating the need for numerical integration.

The resulting matrix equation can be written as

$$[V] = [Z][I] \Rightarrow [I] = [Y][V] \quad (2)$$

where $[I]$ is the MOM current vector with the coefficients of the expansion of the current in the above basis. $[Z]$ is the MOM impedance matrix. $[Y]$ is the MOM admittance matrix, the inverse of the impedance matrix. The matrices are of order $N \times N$, where N is the number of unknowns used in the MOM formulation. The entries of $[V]$ and $[Z]$ are given by

$$V_i = \int_{z_{q-1,m}}^{z_{q+1,m}} f_{q,m}(z) E_z^{inc}(z) dz \quad (3)$$

$$= \quad \frac{E_0 e^{jkx_m \cos\phi \sin\theta}}{k \sin(k\Delta z) \sin^2\theta} 2 e^{jkz_{q,m}\cos\theta} \times [\cos(k\Delta z \cos\theta) - \cos(k\Delta z)] \quad (4)$$

where, θ, ϕ is the elevation and azimuth direction of arrival of the incident field.

$$Z_{i,l} = \int_{z_{q-1,m}}^{z_{q+1,m}} f_{q,m}(z) \left\{ \left[j\omega\mu_0 \int_{z_{p-1,n}}^{z_{p+1,n}} f_{p,n}(z') \frac{e^{-jkR}}{4\pi R} dz' - \frac{1}{j\omega\epsilon_0} \frac{\partial}{\partial z} \int_{z_{p-1,n}}^{z_{p+1,n}} \frac{df_{p,n}(z')}{dz'} \frac{e^{-jkR}}{4\pi R} dz' \right] \right\} dz \quad (5)$$

where, $i = [(m-1)P + q]$. $l = [(n-1)P + p]$ and $f_{q,m}$ is the q-th basis function on the m-th element.

Note that the entries of the voltage vector are directly related to the incident field and are hence free of the effects of mutual coupling. The entries of the impedance matrix are the interaction between the field due to the current source $f_{p,n}$ at the location corresponding to the basis function $f_{q,m}$. Therefore, by their very nature, the entries of the impedance matrix are a measure of the mutual coupling between the sections of the array.

Using the MOM admittance matrix and the voltage vector, we can show that the voltages measured at the ports of the array are given by

$$[V]^{meas} = [Z_L][Y_{port}][V] \quad (6)$$

where, $[Y_{port}]$ is the $N_e \times N$ matrix of the rows of $[Y]$ that correspond to the ports of the elements. $[Z_L]$ is the diagonal matrix with the port loads as its entries.

III. EFFECT OF MUTUAL COUPLING

The effects of mutual coupling are illustrated using the LSE algorithm of Sarkar and Sangruji [1]. The least squared error algorithm presented by [1] automatically steer nulls in the direction of interference while simultaneously maintaining the gain of the antenna in the given look direction. The effects of mutual coupling are illustrated comparing the following two scenarios.

An array receives a target signal from a known direction. The signal reception is corrupted by three jammers. In the first scenario, the receiving algorithm is applied to the hypothetical case where mutual coupling is absent. The receiving array is assumed to be a linear array of isotropic, point sensors. The array voltages due to the signal and jammers is given by

$$V_i = S e^{(i-1)jkdu_0} + \sum_{m=1}^{M} J_m e^{(i-1)jkdu_m} \qquad i = 1, \ldots, N_e. \quad (7)$$

where, S is the complex intensity of the signal and J_m is the complex amplitude of the $m-$th interference source. The M interfering sources arrive from the direction corresponding to $u_m = \phi_m$, $m = 1, \ldots, M$. These voltages V_i are used as input to the least squared error algorithm to recover the signal while nulling the jammers.

In the second scenario, the mutual coupling is taken into account. The antenna is analyzed using the Method of Moments. The intensities of the signal and interference and their directions of arrival, in conjunction with equation (3), are used to calculate the Method of Moments voltage vector. Equation (6) is used to find the voltages that are measured in the presence of mutual coupling. These measured voltages, without any correction, are input to the signal recovery program. An attempt is made to recover the signal intensity using the same adaptive algorithm.

A seven element array is composed of wires of length $\lambda/2$ and radius $\lambda/200$ and spaced $\lambda/2$ apart. Each wire is loaded at the center with a 50Ω load. The MOM analysis uses 7 unknowns per wire. This array receives a signal of intensity $1.0V/m$ from direction $\phi = 45^o$ and two jammers of intensity $1.0V/m$ and $1.5V/m$ from directions $\phi = 60^o$ and $\phi = 30^o$ respectively. The third jammer arrives from $\phi = 75^o$ and its intensity is varied from $1.0V/m$ (0 dB) to $1000.0V/m$ (60 dB) in steps of $5V/m$. For each intensity, the voltages in the absence of mutual coupling are calculated using equation (7) and used to recover the signal. Further, the voltages in the presence of mutual coupling are found using equation (6) and used to recover the signal. The two cases are compared in Figure 1.

Figure 1 shows that, in the absence of mutual coupling, the adaptive nulling algorithm is accurate and can null a strong (60 dB) jammer. However, when mutual coupling is taken into account, the jammer is not nulled and the reconstructed voltage is approximately linear with respect to jammer intensity. This is because the jammer has not been nulled and the residual jammer component completely overwhelms the signal.

IV. ACCOUNTING FOR MUTUAL COUPLING

Reference [5] presents a simple method, based on the MOM admittance matrix, to eliminate the effects of mutual coupling on adaptive algorithms. However, the method presented there is valid for $z-$directed wire arrays only. Further, the method is not applicable when the array is in the presence of near field scatterers. Here we present a method that is applicable in general. The method accounts for the mutual coupling rather than eliminating it.

The proposed receiving algorithm can be broken into

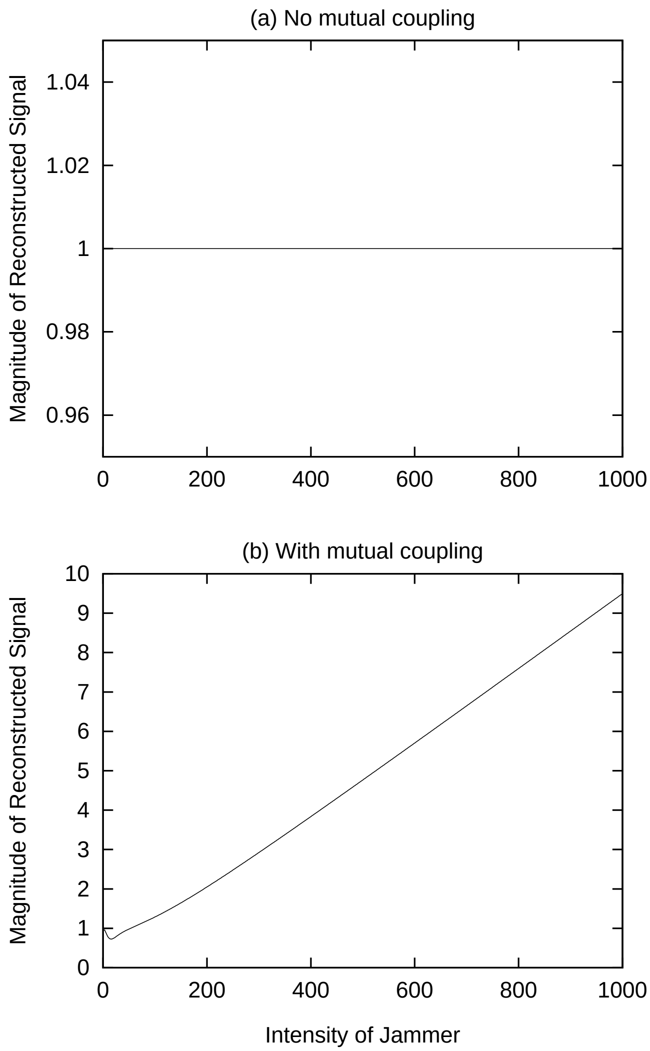

Figure 1: Signal recovery in the absence and presence of mutual coupling.

three steps.

1. Using the measured voltages to estimate the entire MOM voltage vector.
2. Use the MOM voltage vector to estimate the directions of arrival of the interference.
3. Use the direction of arrival estimates to suppress interference and maximize gain in the direction of the signal

Consider the underdetermined matrix equation (6). This equation can be used to find the minimum norm solution for the MOM voltage vector $[V]$.

$$[\tilde{V}] = [C]^H \left([C][C^H]\right)^{-1} [V]_{meas} \tag{8}$$

where, $[C] = [Z_L][Y_{port}]$ is the $N_e \times N$ matrix relating the measured voltages with the MOM voltage vector. Since, the effects of near field scatterers can be incorporated in $[Y_{port}]$, this equation is valid for any given array, even if it is in the presence of near field scatterers.

The MOM voltages are estimated using an underdetermined system of equations, and so cannot be directly used for signal recovery. We use the MOM voltages corresponding to the array ports to estimate the directions of arrival of the interference. Using equation (4), these MOM voltages corresponding to the array ports can be written as

$$V_i = \sum_{m=1}^{M} A_m z_m^i + n_i \tag{9}$$

The exponents are directly related to the directions of the incident fields.

$$\phi_m = \cos^{-1}\left\{\frac{\Im\left[\ln(z_m)\right]}{d}\right\} \tag{10}$$

where, ϕ_m is the azimuth direction of arrival of the $m-$th interference source and d is the distance between two elements. The Matrix Pencil [10] is a signal processing technique to estimate the parameters of a sum of complex exponentials in the presence of noise. Given the entries of the estimated MOM voltages vector that correspond to the array ports V_i, the Matrix Pencil is used to estimate z_m and hence ϕ_m i.e. an estimate of the directions of arrival of the interference is obtained.

Once the directions of the interference is estimated, beam pattern nulls are placed in the direction of the interference. Simultaneously, the gain of the antenna is maximized in the direction of the signal. We use a procedure similar to that of [11]. However, the problem of solving a new Method of Moments problem is bypassed by analytically evaluating the far field patterns of the antenna.

This leads to enormous savings in required real time computations. Using such a procedure yields a set of weights $\{w_i, \ i = 1, \ldots, N_w\}$. The weights multiplied with the voltages measured across the loads yield the maximum signal reception and maximum interference rejection [6].

V. Numerical Examples

Example 1. No scatterers.

The first example chosen is a 21 element array receiving a signal of complex intensity $(1.0, 0.0)$ from the direction $\phi = 80^o$. The reception of the signal is corrupted by three jammers and clutter. The signal to noise ratio (SNR) ratio was 13dB. The signal, jammer and clutter intensities and directions of arrival are given in Table 1. The clutter in the given azimuth range is modeled as many clutter sources spaced 0.1 degree apart. The CNR is the total clutter to noise ratio.

	Intensity (V/m)	DOA(ϕ)
Signal	(1.0,0.0)	80^o
Jammer # 1	(0.0,100.0)	100^o
Jammer # 2	(10.0,0.0)	60^o
Jammer # 3	(0.0,1.0)	45^o
Clutter #1	50 dB CNR	100.0^o - 110.0^o
Clutter #2	40 dB CNR	130.0^o - 140.0^o

Table 1: Jammer and clutter intensities and directions of arrival. Example 1.

To test the performance of the receiving algorithm, 40 independent simulations were carried out. The results of the simulations are summarized in the Table 2.

Number of samples	40
True Value	(1.0,0.0) V/m
Mean of 40 estimates	(1.00902,0.024044) V/m
Bias of estimate	(0.00902,0.024044) V/m
Variance of estimates	0.010105
Output SINR	19.679 dB

Table 2: Results of 40 simulations. Example 1.

The reason the reconstruction is so good can be visualized from the beam patterns of the weighted array. Figure 2 shows the weighted beam pattern for a sample simulation. Notice the deep nulls at the location of the jammers and the broad nulls in the direction of the clutter.

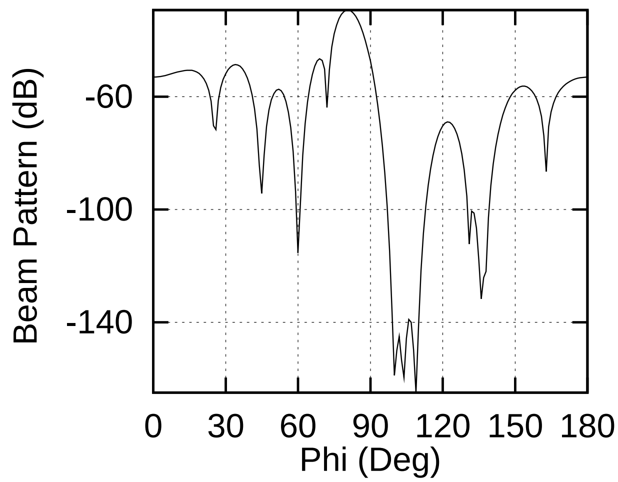

Figure 2: Sample beam pattern. Example 1.

Example 2. Randomly spaced scatterers

The second example presents the response of the same array in the presence near field scatterers. The location of the scatterers is given in Table 3.

Length	Radius	x	y	z	# of Unknowns
0.40	0.004	2.9	-4.0	1.0	5
0.80	0.005	4.0	0.0	1.3	11
0.80	0.008	9.8	-3.4	0.4	11
0.90	0.005	10.5	-2.0	-1.3	11
0.30	0.005	7.8	-1.3	0.0	3

Table 3: Geometry and locations of near field scatterers. Example 3.

The target is absent, i.e. the signal is absent. However, the array receives two strong jammers and one weak jammer. Further, one strong clutter ridge interferes with the array reception. The signal is assumed to arrive from $\phi = 110^o$. The jammer and clutter intensities and directions of arrival are detailed in Table 4. The nominal SNR is 13dB.

Again 40 independent simulations are carried out to obtain a mean and variance of the estimate of the signal. The results of the 40 simulations are presented in Table 5. The results are best visualized with a look at a sample beam pattern of the weighted array. Notice the deep nulls at $\phi = 65^o, \phi = 45^o$ and $\phi = 130^o$. Also, the clutter

	Intensity (V/m)	DOA(ϕ)
Signal	(0.0,0.0)	110^o
Jammer # 1	(0.0,100.0)	65^o
Jammer # 2	(10.0,100.0)	45^o
Jammer # 3	(1.0,1.0)	130^o
Clutter	50 dB CNR	80.0^o - 90.0^o

Table 4: Jammer and clutter intensities and directions of arrival. Example 2.

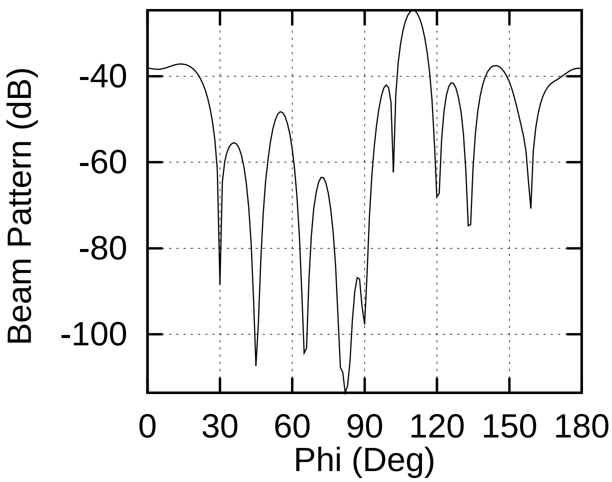

Figure 3: Sample beam pattern. Example 2.

ridge is suppressed by a broad null between $\phi = 80^o$ and $\phi = 90^o$.

Number of samples	40
True Value	(0.0,0.0) V/m
Mean of 40 estimates	(0.048142,0.000829) V/m
Bias of estimate	(0.048142,0.000829) V/m
Variance of estimates	0.01815
Output Interference Power	-16.888 dB

Table 5: Results of 40 simulations. Example 2.

VI. CONCLUSIONS

This paper has presented an adaptive algorithm that accounts for the mutual coupling between the elements of the adaptive array. The mutual coupling, if not taken into account, causes all adaptive algorithms to fail. The technique presented here uses the MOM admittance matrix to quantize the mutual coupling and to find the optimum weights to produce the desired nulling.

References

[1] T.K. Sarkar and N. Sangruji. An adaptive nulling system for a narrow-band signal with a look-direction constraint utilizing the conjugate gradient method. *IEEE Transactions on Antennas and Propogation*, 37(7):940–944, July 1989.

[2] S. Park and T.K. Sarkar. A deterministic eigenvalue approach to space time adaptive processing. In *Proceedings of the IEEE Antennas and Propogation Society International Symposium*, pages 1168–1171, 1996.

[3] R. Schneible. *A Least Squares approach to radar array adaptive nulling.* PhD thesis, Syracuse University, 1996.

[4] I.J. Gupta and A.A. Ksienski. Effect of mutual coupling on the performance of adaptive arrays. *IEEE Transactions on Antennas and Propogation*, 31(5):785–791, September 1983.

[5] R.S. Adve and T.K. Sarkar. Elimination of the effects of mutual coupling in an adaptive nulling system with a look direction constraint. In *Proceedings of the IEEE Antennas and Propogation Society International Symposium*, pages 1164–1167, 1996.

[6] R.S. Adve. *Elimination of the Effects of Mutual Coupling in Adaptive Thin Wire Antennas.* PhD thesis, Syracuse University, December 1996.

[7] R.F. Harrington. *Field Computation by Moment Methods.* Robert E. Kreiger Publishing Co., 1982.

[8] A.R. Djordjevic. et.al. *Analysis Of Wire Antennas and Scatterers: Software and User's Manual.* Artech House.

[9] T.K. Sarkar B.J. Strait and D.C. Kuo. Special programs for analysis of radiation by wire antennas. Technical Report AFCRL-TR-73-0399, Syracuse University, June 1973.

[10] Y. Hua and T.K. Sarkar. Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 38:814–824, May 1990.

[11] T.K. Sarkar and B.J. Strait. Optimization methods for arbitrarily oriented arrays of antennas in any environment. *Radio Science*, 11(12):959–967, Dec. 1976.