Multipath Delay Estimations using Matrix Pencil

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***Abstract*— This paper presents a technique for the recovery of time delays associated with components of a signal in a multipath communication channel. Matrix Pencil is used to recover these delays from the channel frequency response. This algorithm has some key advantages over traditional super-resolution techniques such as MUSIC. Most importantly, Matrix Pencil only requires a single channel estimate and can estimate the delays associated with coherent multipath components.**

I. Introduction

A recent demand on wireless technology has been the ability to accurately locate users in emergency situations, also known as E911. Position location would also enable value added services such as maps, local entertainment options etc. One approach to position location is to triangulate a user based on the time of arrival (TOA) of the user's signal. In a multipath environment, TOA estimation requires only the shortest delay. However, if all delays are known, this information can be used in a RAKE receiver or for timing acquisition.

A proposed approach to TOA estimation has been using super-resolution techniques such as MUltiple SIgnal Classification (MUSIC) [1]. The super-resolution technique is applied after the estimated channel impulse response (CIR) is transformed to the frequency domain. The major drawback with use of traditional techniques is that they start with an estimation of the signal covariance matrix. In our application, this would require several estimates of the same channel, a time consuming and wasteful process. Furthermore, data smoothing is required if any two signal components were correlated, further increasing the computation load while reducing resolution. If the channel was varying slowly, the multipath components would invariably be correlated.

In other applications a relatively new super-resolution technique called Matrix Pencil has been shown to provide accurate parameter estimates [2]–[4]. Here, we show that Matrix Pencil is a practical and superior alternative to the accurate recovery of multipath channel delays. Given a reasonably accurate *single* estimate of the channel, the algorithm provides accurate estimates of the multipath delays. This algorithm can also handle coherent multipaths.

Section II describes the channel model and the method of channel estimation utilized. Section III briefly outlines the Matrix Pencil algorithm. The performance of Matrix Pencil is evaluated and compared to MUSIC in Section IV. The paper ends with some conclusions in Section V.

II. Channel Model and Estimation

Propagation through the channel results in fading, the addition of complex Additive White Gaussian Noise (AWGN), and multipath effects with a maximum propagation delay D. The complex AWGN is modelled with zero mean and unit power. Without any noise considerations, the channel impulse response (CIR), $h(t)$, is modelled as a sum of M delta pulses (multipath components) shifted according to the corresponding time delays. The magnitude of each component is modelled as a Rayleigh distributed random variable. The delays may be modelled as Poisson distributed random variables [5]. The use of a Poisson distribution is appropriate for indoor wireless channels. However, one might argue that in crowded cities, this distribution applies, but in a lower scattering environment. The channel and its frequency domain representation are therefore given by

$$h(t) = \sum_{m=1}^{M} \alpha_m \delta(t - \tau_m), \tag{1}$$

$$H(j\omega) = \sum_{m=1}^{M} \alpha_m e^{-j\omega\tau_m}, \tag{2}$$

where M is the number of multipath signals generated by the channel, α_m is the amplitude of the m-th component and τ_m the associated delay. Eqn. (2) shows that given the channel, the delays may be estimated using a super-resolution technique such as MUSIC.

In practice the channel is unknown and must be estimated. In this paper we use the technique of [6], obtaining a maximum likelihood (ML) estimate of the channel using a training sequence. The training signal, $s(t)$ consists of b bits of data transmitted over the channel. The input and output signals are sampled every Δt. The received data has length $N = D + P$, P being the midamble samples of this received data, where $P > D$ in order to satisfy invertibility requirements of the CIR estimation.

The received time domain response of the channel is given by

$$y(t) = \sum_{m=1}^{M} \alpha_m s(t - \tau_m) + n(t), \tag{3}$$

where $n(t)$ is the AWGN produced by the channel. The frequency response is then obtained using a Fourier transform of this CIR.

III. Matrix Pencil

From Eqn. (2), the frequency domain response of the estimated noisy CIR is modelled as

$$H(j2\pi k\Delta f) = \sum_{m=1}^{M} \alpha_m z_m^k + n_m, \tag{4}$$

where n_m represents the AWGN of the channel; and $z_m = e^{(-j2\pi\Delta f \tau_m)}$, with $\Delta f = 1/N\Delta t$. Note that if we can estimate z_m we can also estimate the component delays, τ_m. This frequency domain expression falls into the same format as the data for direction of arrival (DOA) estimation. Super resolution techniques used for DOA estimation, such as Matrix Pencil and MUSIC, can therefore be applied to estimate τ_m.

A. Theory

The Matrix Pencil algorithm was originally developed in order to estimate the poles of a system. The k-th frequency sample at $k\Delta f$ of the ML estimate is represented as $H_{ML}(k)$, $k = 0, \ldots N-1$. Consider the following $(N-L) \times (L+1)$ matrix formed using data from this single sample in time,

$$\mathbf{X} = \begin{bmatrix} H_{ML}(0) & \cdots & H_{ML}(L) \\ H_{ML}(1) & \cdots & H_{ML}(L+1) \\ \vdots & \ddots & \vdots \\ H_{ML}(N-L-1) & \cdots & H_{ML}(N-1) \end{bmatrix}, \tag{5}$$

where L is called the pencil parameter. Define two $(N-L) \times L$ matrices $\mathbf{X}_0$ and $\mathbf{X}_1$ as the first L and last L columns of $\mathbf{X}$, i.e. in MATLAB notation

$$\mathbf{X_0} = \mathbf{X}(:, 1:L) \tag{6}$$

$$\mathbf{X_1} = \mathbf{X}(:, 2:L+1). \tag{7}$$

These two matrices can be written as

$$\mathbf{X_0} = \mathbf{Z}_1 \mathbf{A} \mathbf{Z}_2, \tag{8}$$

$$\mathbf{X_1} = \mathbf{Z}_1 \mathbf{A} \mathbf{Z}_0 \mathbf{Z}_2, \tag{9}$$

where,

$$\mathbf{Z}_1 = \begin{bmatrix} 1 & \cdots & 1 \\ z_1 & \cdots & z_M \\ \vdots & \ddots & \vdots \\ z_1^{(N-L-1)} & \cdots & z_M^{(N-L-1)} \end{bmatrix}_{(N-L)\times M} \tag{10}$$

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ 1 & z_2 d & \cdots & z_2^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M & \cdots & z_M^{L-1} \end{bmatrix}_{M\times L}, \tag{11}$$

$$\mathbf{Z}_0 = \text{diag}\begin{bmatrix} z_1 & z_2 & \cdots & z_M \end{bmatrix}, \tag{12}$$

$$\mathbf{A} = \text{diag}\begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_M \end{bmatrix}. \tag{13}$$

Based on eqn. (9), our goal is to estimate the entries of the matrix $\mathbf{Z}_0$.

Consider the following matrix pencil,

$$\mathbf{X_1} - \lambda \mathbf{X_0} = \mathbf{Z}_1 \mathbf{A} \left[\mathbf{Z}_0 - \lambda \mathbf{I}\right] \mathbf{Z_2}. \tag{14}$$

Choosing $\lambda = z_m$, for some m, reduces the rank of the pencil by one. The estimates for z_m are, therefore, the generalized eigenvalues of the matrix pair $[\mathbf{X}_1, \mathbf{X}_0]$.

The amplitudes α_m can then be obtained by a least squared estimate [2]. These amplitudes are not required for delay estimation, though are useful to compare the recovered Matrix Pencil channel estimates with the ML channel estimate.

In Section IV we compare the performance of Matrix Pencil to the MUSIC algorithm. The MUSIC algorithm begins with an estimate of the signal covariance matrix. Theoretically, estimating the delays associated with M components requires a minimum of M independent multiple estimates of the same channel. In practice, many more than M estimates are required, i.e. the channel estimation process must be executed repeatedly. This is a time consuming and wasteful process. As the channel estimation is based on a training sequence, repeated channel estimations result in reduced data throughput. Furthermore, the channel must remain constant over the several estimates. In rapidly varying channels, the use of only a single channel estimate is crucial. Finally, to be able to distinguish between correlated channel components, the MUSIC algorithm requires the frequency equivalent of spatial smoothing [7]. This further increases the computation load while reducing accuracy.

TABLE I

Matrix Pencil Data. 1 snapshot delays=[11, 12, 15, 16]

True delay	1000 bits, 14dB	6000 bits, 10dB
16.0	15.9899	15.9350
11.0	10.9384	11.0143
15.0	14.9924	15.0232
12.0	11.9977	11.9992

In this regard, the Matrix Pencil algorithm, as described here, has some significant advantages. As shown in [4], the algorithm requires about half the computation load of covariance matrix based techniques such as MUSIC. Furthermore, only a single channel estimate is required. Since only a single channel estimate is used, Matrix Pencil can distinguish between correlated components. Note that if multiple estimates of the channel are available, the Matrix Pencil algorithm can be applied using an average estimate.

We use the Total Least Squares (TLS) implementation of Matrix Pencil [3] to estimate the generalized eigenvalues. The TLS implementation helps in dealing with the noise in the ML channel estimate. The algorithm requires an estimate of the number of components (M). Here we assume that the number of components is known.

IV. Examples

The computational accuracy of Matrix Pencil was examined by varying the length of the training sequence, and the SNR.

A. Example 1: A Single Channel Estimate

This example presents two results illustrating the effectiveness of the Matrix Pencil algorithm using a single channel estimate. In the first case, the training sequence comprises 1000 bits with a SNR of 14dB. The second case uses 6000 bits with a SNR of 10dB. Note that these numbers only set the accuracy of the channel estimate. Any improvement in the channel estimate will clearly improve the Matrix Pencil estimates as well. *Only a single channel estimate is used.* The results are summarized in Table I

As is clear from the resulting data, the Matrix Pencil approach provides excellent estimates of the delays. All four signal delays are recovered with less than 1% error.

B. Example 2: Comparison with MUSIC

For a fair comparison with MUSIC we use 16 independent channel estimates to obtain the signal covariance matrix. Matrix Pencil is applied to the average of the 16 runs. Table II compares the results of Matrix Pencil to MUSIC in the recovery of the channel delays from the 16 snapshots. Matrix Pencil successfully identifies all of the delays while MUSIC is only able to identify two of the four multipath signal components correctly. This is because the channel is kept constant over all 16 estimates, making the different channel components correlated. As we have not used spatial smoothing, MUSIC cannot identify these components correctly.

TABLE II

Comparison of Delay Recovery Methods. 16 snapshots, SNR=$6dB$ delays=$[11, 12, 15, 16]$

True delay	Matrix Pencil	MUSIC
16.0	15.9398	0.8828
11.0	10.8942	10.9076
15.0	15.0183	-0.5316
12.0	12.0186	12.0615

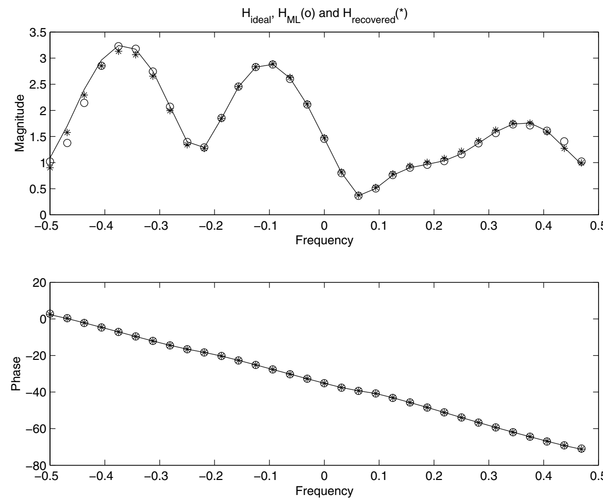

Fig. 1. 16 Snapshots: bits=1000, SNR=6dB, delays=[11,12,15,16]

Figure 1 compares the three frequency plots, H_{ideal}, H_{ML}, and $H_{recovered}$. The recovered channel estimate $H_{recovered}$ is obtained using Eqn. (2) with the parameters α_m and τ_m obtained from Matrix Pencil. The true values of the channel's delay and amplitudes were used to plot H_{ideal}. H_{ML} is the ML channel estimate using the technique of [1]. All three frequency responses were generated with four multipath signals which contribute delays of $[11, 12, 15, 16]\Delta t$. The signal amplitudes were fixed to yield an SNR of 6dB, and the training sequence uses 1000 randomly generated bits. The Fast Fourier Transform size for the transformation to the frequency domain is 32, which was selected within a constraint of being at least twice the highest multipath delay. As is clear, not only does the ML technique estimate the channel accurately, but given the ML estimate, Matrix Pencil is able to recreate the ML estimate exactly.

V. Conclusion

This paper presented a practical approach to the recovery of signal delays from channel estimate data. Matrix Pencil was shown to be a more effective technique than MUSIC. MUSIC's requirement of multiple snapshots involves the time consuming production of repeated channel estimates. MUSIC failed to produce a successful recovery due to coherent multipath. Only one snapshot is required for Matrix Pencil to accurately identify the delays of the multipath components.

The Matrix Pencil algorithm has several important advantages: unlike covariance matrix based techniques, it requires only a single channel estimate, it can be applied in coherent multipath scenarios, it can be applied to relatively rapidly varying channels.

This work was motivated by the necessity of channel delay information for wireless communications applications involving position location of a carrier using time of arrival information. However, the ability to estimate the delays of all multipath components will allow applications such as the RAKE receiver and help in synchronization. The theoretical concerns that arise due to the inherent presence of AWGN in a realistic communication channel were addressed using the TLS Matrix Pencil approach. Recovery using Matrix Pencil was shown to be accurate and feasible.

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