A Theoretical Analysis of Cooperative Diversity in Wireless Sensor Networks

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***Abstract*— We propose here an analytical framework to quantify the impact of cooperative diversity on the energy consumption and lifetime of sensor networks. It is well accepted that cooperative diversity increases energy efficiency in fading environments. However, previous works have not analyzed, from a theoretical perspective, these benefits in a network setting. This paper presents a theoretical framework to model routing behavior and cooperative relay selection, using this information to predict the lifetime and energy consumption of the network.**

I. Introduction

Wireless sensor networks (WSNs) refer to a broad class of wireless networks consisting of small, inexpensive and energy limited devices [1]. In these networks, sensors have the responsibility of collecting data and communicating this information to one or more processing centers. Our focus is on large-scale WSNs with medium-to-low node spatial densities. These types of WSNs could be used in environmental monitoring applications, e.g., in detecting forest fires.

Due to the fact that nodes are battery powered, energy efficiency is the main challenge in designing WSNs. Researchers have generally developed schemes for energy savings in specific layers of the protocol stack. For example, multi-hop routing and clustering have been shown to improve the energy efficiency of large scale WSNs [2]–[5]. Multi-hop routing is necessary because nodes have a limited transmission range and can communicate directly over small distances only [2], [3]. Theoretical analysis of multi-hop routing is restricted to networks of extremely high densities [2]. The idea of clustering refers to partitioning the network into local clusters, with one node in each cluster a cluster-head (CH). Clustering saves energy by allowing each CH to exploit correlation through data aggregation [4], [5]. The CH may also act as a local server for individual nodes.

Energy saving protocols have also been developed in the physical layer. WSNs, like all other wireless networks, suffer from the effects of fading. *Cooperative diversity* has been shown to mitigate the impact of fading through distributed antenna sharing [6]–[9]. This form of diversity is especially suited towards WSNs since size and power constraints restrict nodes from possessing more than one antenna. Generally analysis of the resulting energy savings have been limited to 3- or 4-node networks [6], [7] or to information theoretic issues such as outage probability [9].

This paper considers the issue of cooperative diversity in large scale, multihop, WSNs from a theoretical perspective. The goal is to develop the theory needed to analyze large-scale WSNs and predict network performance. We present theory to determine the expected number of packets forwarded by a node due to routing and cooperative partner selection. Cooperation is achieved using the simple amplify-and-forward scheme [8]. We then use these results to predict the impact of cooperative diversity on the lifetime of sensor networks. To our knowledge such a theoretical framework for cooperative diversity in network settings has not been developed before.

The outline of this paper is as follows. Section II provides an overview of the system model. Section III and Section IV analyze the behavior of routing and cooperative relay selection respectively. Section V describes the energy analysis used to quantify the impact of cooperative diversity on the lifetime of WSNs. The paper ends with conclusions Section VI.

II. System Model

A. General Network Properties

Our network consists of N sensors communicating with a single data sink. The sensors are uniformly and randomly distributed over a circular area of radius a and the data sink is located at the center of this area. We assume the sensors are energy limited and the data sink has unlimited energy. Since sensors are simple and inexpensive devices, we assume they have fixed transmission power levels. Specifically, these power levels correspond to transmission radii R_1 and R_2 used for multi-hop and cooperative relay selection respectively. These transmission radii are chosen to satisfy the lower bound transmission radius required to provide 99% probability of network connectivity [10]. The sensors and data sink are stationary.

In Sections III and IV we make the implicit assumption that each node knows its own and its neighbors distances from the data sink. This assumption is reasonable since our network is stationary and requires only local information sharing.

B. Clustering Protocol

We assume out network is clustered using a distributed algorithm where CHs are selected randomly [3], [4]. These

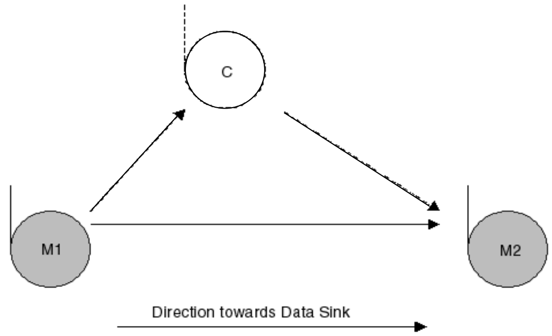

Fig. 1. Example of a Section Belonging to a Multi-Hop Path

class of algorithms are practical to implement in WSNs since WSNs are organized in a distributed fashion. We also assume that the CH role is evenly distributed over the network and each CH performs ideal aggregation, i.e., all cluster data is aggregated into a single packet.

C. Routing Protocol

We assume that min-hop routing (MHR) is used to establish the multi-hop path from each CH to the data sink. MHR is known to perform well in stationary networks comprised of nodes with fixed transmission power levels. We use a simple iterative algorithm that begins with nodes neighboring the data sink broadcasting their hop number. In turn, neighboring nodes update and broadcast their hop number if necessary and the process continues until each node in the network has determined its min-hop path to the data sink. Note that when traversing any min-hop path in decreasing hop number we have implicitly assumed that the distance between the node and data sink is strictly decreasing. This assumption is suspect for networks with low densities, but becomes valid with increasing network density. This assumption significantly simplifies the theoretical analysis developed below.

D. Cooperative Diversity

If we assume each sensor in the multi-hop path has a cooperative partner, then each hop is no different than the three node network studied in [8]. Figure 1 illustrates one section of a multi-hop path where nodes M_1 and node M_2 belong to the multi-hop path and node C represents a potential cooperative relay. All channels are modelled as slow and flat. The receiver to any transmission is assumed to know the channel perfectly. The cooperating node, C in Fig. 1, helps in the communication between nodes M_1 and M_2 using the amplify-and-forward (AF) protocol. Thus node C receives a noisy version of M_1's transmitted signal and transmits an amplified version of this signal to M_2. This protocol creates spatial diversity since node M_2 receives two independently faded signals.

The performance of the AF protocol depends on the quality of the channel between the source and relay and between the relay and destination. Since generally channel quality decreases with distance, we restrict the selection of relays to a node's forward transmission region. Section IV describes the process of cooperative relay selection in more detail.

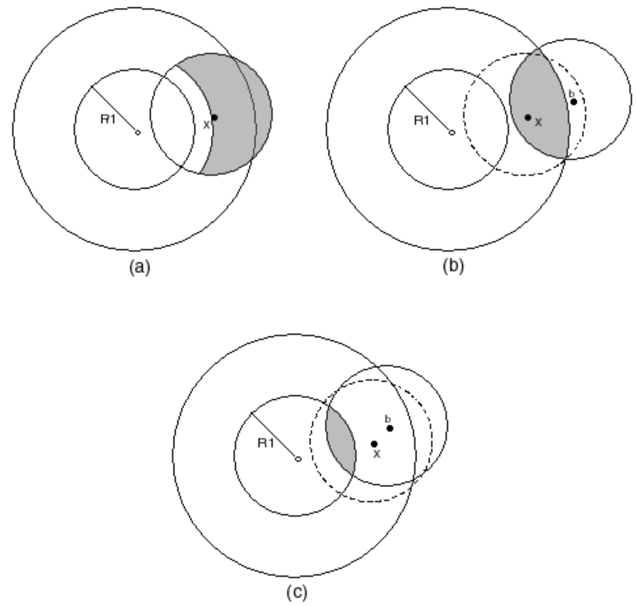

Fig. 2. Example Topology for Multi-Hop Routing

III. Analysis of Multi-Hop Routing

In this section, we analyze the behavior of packet forwarding without cooperation. We assume the transmission of packets dominates the energy consumption of sensors. Thus this analysis represents the first step towards predicting the energy consumption of the network. The focus here is on low-to-medium density networks, as opposed to the high-density networks considered in the available literature. The analysis uses a layered structure, similar to [2], where each layer has a width R_1 (corresponding to the communication radius used by nodes for multi-hop transmission). We let r denote the distance of a node from the data sink. When MHR is used, we observe that at even low densities we can roughly approximate the number of hops between a node and the data sink by $\lceil \frac{r}{R_1} \rceil$.

The analysis below is based on a preferential routing framework. This is an approximation to the MHR protocol, but significantly simplifies the analysis. We differentiate our framework from [2] by allowing nodes to forward packets within their own layer. We can thereby approximate MHR at much lower node densities than considered in [2]. As shown in Fig. 2(a), for a given node x, nodes that may potentially forward packets to x lie in a circle of radius R_1 centered at x. Since nodes are assumed to transmit forward towards the data sink only, x can only receive packets from nodes in the shaded region of Fig. 2(a). The layer structure allows for differentiation of the routing behavior of nodes in this shaded region based on whether they are located in the same layer as x.

If a willing multi-hop partner is available in a higher layer, a node preferentially forwards its data to the higher layer. For example, consider Fig. 2(b). where node x and node b are in different layers. We model b's routing behavior as being indifferent to forwarding packets to x and any other node in the shaded area of Fig. 2(b). Now consider the case where x and b are located in the same layer. We model b's routing behavior to prefer forwarding packets to the shaded region in Fig. 2(c). over forwarding to x. Consequently if no nodes exist in this region, we then model b to be indifferent to forwarding packets to node x and any other node in the area of intersection of b's transmission region and b's layer.

To express this framework rigorously, define the following variables (detailed in the appendix):

- $A_1(x,b)$=The area of intersection of b's transmission region and the layer above,
- $A_2(x,b)$=The area of intersection of b's transmission region and b's layer,
- N_{A_i}=The number of nodes in $A_i(x,b)$, where $i\epsilon(1,2)$,
- λ=The density of nodes in the network, $\lambda = \frac{N}{\pi a^2}$,
- $N(r)$=The expected number of packets forwarded at a distance r from the data sink.

As mentioned above, the probability of node b choosing node x as its next hop, denoted by $p(b,x)$, depends on whether b and x are in the same layer. For the case where b and x are in the different layers,

$$p(b,x) = \sum_{n=1}^{N} \frac{1}{n} \Pr\left(N_{A_1} = n | N_{A_1} \geq 1\right), \quad (1)$$

and for the case where b and x are in the same layer,

$$p(b,x) = \sum_{n=1}^{N} \frac{1}{n} \Pr(N_{A_1} = 0) \Pr(N_{A_2} = n | N_{A_2} \geq 1). \quad (2)$$

where we use the binomial distribution to determine the probability of having n nodes in an area A.

To determine $N(|x|)$, the expected number of packets forwarded by a node at distance $|x|$ from the data sink, we have to recursively integrate over the shaded region in Fig. 2(a) and add one to account for the packet originating at x. Thus $N(|x|)$ is expressed as

$$N(|x|) = 1 + \lambda \int_{|x|}^{|x|+R_1} p(b,x) N(|b|) 2\gamma |b| dr, \quad (3)$$

where λ is the spatial density of nodes and

$$\gamma = \arccos\left(\frac{|x|^2 + |b|^2 - R_1^2}{2|b||x|}\right). \quad (4)$$

Note that we assume all nodes in the first layer are directly connected to the data sink. Thus for the case where $|x| \leq R_1$, we replace $|x|$, the lower limit for the integration in (3), with R_1.

Figure 3 compares the number of packets forwarded versus distance determined theoretically using (3) and (4) with simulations based on MHR. The simulations average over 200 different networks at a density of $30/(\pi R_1^2)$, where R_1=1 and network radius $a = 4R_1$. This density is considerably lower than what has been used in the only available literature [2]. In [2], the authors use a density of $100/(\pi R_1^2)$. Clearly the theoretical analysis compares fairly well to the simulations, especially so in the first layer, the most critical layer in the network.

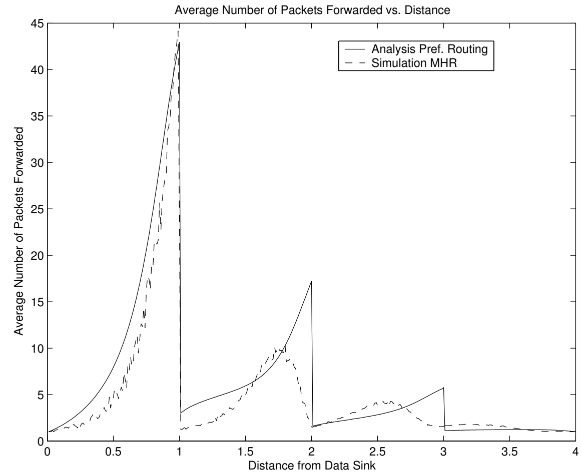

Fig. 3. Expected Number of Packets Forwarded vs. Distance

IV. Analysis of Cooperative Relay Selection

This section analyzes the behavior of cooperative relay selection to determine the number of cooperative packets forwarded as a function of distance from the data sink. To our knowledge this is the first paper to consider such an analysis.

The performance of the AF protocol depends on the position of the relay relative to the source and destination. We assume a node requests cooperation only from other nodes in its forward transmission region with corresponding radius R_2. This ensures that the relay is relatively close to both the source and destination. A given node x can only receive cooperation requests from nodes in the shaded region of Fig. 4(a). Consider node b in this shaded region. We model b's cooperation request behavior to be indifferent to choosing x as its cooperative partner and any other node in the shaded region of Fig. 4(b).

Denote $A(x,b)$ as the area of b's forward transmission region, and N_A as the number of nodes in $A(x,b)$. The probability that b chooses x as its cooperative partner is

$$p_c(b,x) = (1 - \Pr(\text{x is dest.})) \sum_{n=1}^{N} \frac{1}{n} \Pr(N_A = n | N_A \geq 1), \quad (5)$$

where Pr(x is dest.) refers to the probability that x may be b's next hop node, i.e., it is the destination for the current transmission (and therefore cannot act as the cooperating node). Depending on b's layer relative to x, Pr(x is dest.) is expressed as (1) if x and b are in different layers, (2) if x and b are in the same layer, and zero if b is in layer 1.

To determine $C(|x|)$, the expected number of packets received exclusively due to cooperation by a node at distance

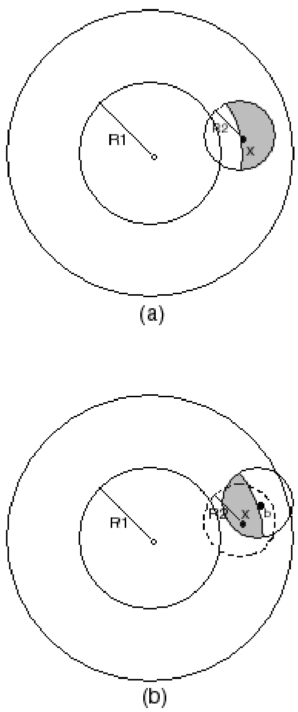

Fig. 4. Example Topology for Cooperative Relay Selection

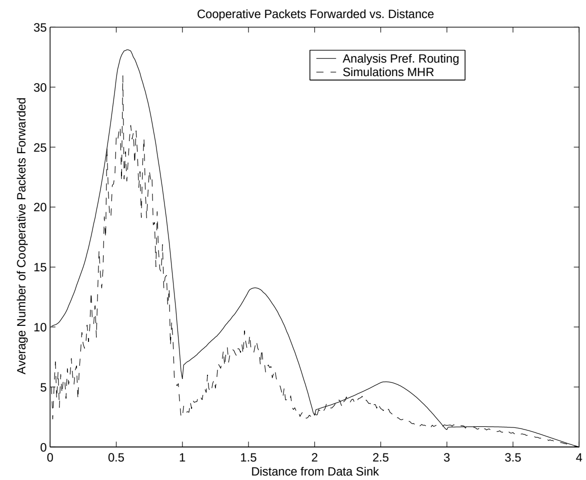

Fig. 5. Expected Number of Cooperative Packets Forwarded vs. Distance

$|x|$ from the data sink, we recursively integrate over the shaded region in Fig. 4(a). Thus $C(|x|)$ is expressed as

$$C(|x|) = \lambda \int_{|x|}^{|x|+R_2} p_c(b,x) N(|b|) 2\gamma |b| dr, \tag{6}$$

where

$$\gamma = \arccos\left(\frac{|x|^2 + |b|^2 - R_2^2}{2|b||x|}\right), \tag{7}$$

and where $N(|b|)$ refers to the number of packets forwarded determined using (3) in Section III.

Figure 5 is the counterpart of Fig. 3 for networks with cooperation. Again the number of packets forwarded is averaged over 200 different networks at a density of $30/(\pi R_1^2)$, where R_1=1 and network radius $a = 4R_1$. We observe from Fig. 5 that (6) slightly overestimates the number of cooperative packets forwarded. This is due to the dependence on preferential routing to determine $N(|b|)$.

The analysis in (3) and (6) illustrates the need to assume preferential routing and that packets are transmitted forward. Without these assumptions the simple recursion in these equations is invalid, requiring a complicated iterative scheme, thereby making the theory largely useless.

V. Energy Analysis

The essential parameter of interest in a sensor network is energy consumption. The ability to analyze the energy consumption theoretically is the prime motivation for the analysis undertaken in Sections III and IV. In this section, we use the results developed there to analyze the energy consumption, thereby predicting network lifetime. Under the assumption that transmission dominates the energy consumption of nodes we determine $E(r)$, the energy consumption as a function of distance r from the data sink. Note that, due to clustering, the number of packets $N(r)$ and $C(r)$ in (3) and (6) are scaled by a factor of p, the probability the node is a cluster-head. Denote as E_1 and E_2 the required energy to transmit a packet with and without cooperation respectively ($E_1 << E_2$).

Without cooperative diversity the total energy consumed by a node at a distance r from the data sink, $E(r)$, is

$$E(r) = pE_2 N(r). \tag{8}$$

With cooperative multihop transmissions, the energy consumed in transmitting $pC(r)$, the effective number of packets due to cooperative relaying, is $pE_1C(r)$. The energy consumed in transmitting $pN(r)$, the effective number of packets due to MHR, is as $pP_zE_2N(r)$+$(1-P_z)E_1C(r)$ where P_z refers to the probability that there are no cooperating nodes to help in transmission. Combining these two, the total energy consumed by a node at a distance r from the data sink, $E_c(r)$, is

$$E_c(r) = pP_zE_2N(r) + (1-P_z)E_1C(r) + pE_1C(r). \tag{9}$$

To determine E_1 and E_2, we use the error rate of the amplify-and-protocol, given in [8]:

$$P_e = Q(\sqrt{(1-\rho)[\gamma_{s,d} + \gamma_{eq}]}), \tag{10}$$

where

$$\gamma_{eq} = \gamma_{s,r}^{-1} + \gamma_{r,d}^{-1} + \gamma_{s,r}^{-1}\gamma_{r,d}^{-1}, \tag{11}$$

and where $Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^\infty e^{-\frac{x^2}{2}} dx$. Using the above equations and a target error rate, one can determine the required SNR at the receiver and thereby the required transmission energy. For example, under Rayleigh fading, for an error rate of 10^{-3}, $E_1 \simeq E_2/12$.

We use the results of energy consumption to determine network lifetime, defined as the time until the first node in the

TABLE I

NETWORK LIFETIMES WITH AND WITHOUT COOPERATION: THEORETICAL ANALYSIS

Network Density	With Cooperation	Without Cooperation
10	519	231
20	474	145
30	808	116

network dies. Starting with unit energy, the network lifetime is proportional to the inverse of the maximum of $E_c(r)$ or $E(r)$ depending on whether the network utilizes cooperation or not.

Table I lists network lifetimes determined using (8), (9) and system parameters $p = 0.2$ (CH probability) with a target BER of 10^{-3}. The network density is measured in terms of number of nodes per πR_1^2, where R_1 is the transmission radius of each node, i.e., on average each node is able to "see" these many other nodes. Note that the analysis is undertaken for relatively low network densities - an essential distinguishing feature of the analysis presented here is that it is valid for all network densities, not just the extremely high densities assumed in earlier works.

As is expected, and clear from the results in Table I, cooperation significantly increases network lifetime. The contribution in this paper is an approach to quantify this increase *theoretically*. An interesting feature from the results in Table I is that the network lifetime in not a monotonic function. This is because at higher densities all nodes find a partner to cooperate with, saving energy, but at lower densities the nodes nearest the sink have to forward fewer packets (these are first nodes to die).

VI. CONCLUSIONS

This paper has presented a theoretical analysis of cooperation in a *network setting*. The goal is to quantify, theoretically, the gains in key performance measures in using cooperation. Previous works have analyzed cooperation from an information theoretic perspective or focused exclusively on the resulting diversity order. The analysis is based on an approximation to min-hop routing in a multi-hop network and uses clustering for further energy savings.

The analysis here uses knowledge of the spatial distribution of nodes to determine the number of packets to be transmitted as a function of distance from a sink. This number is a sum of packets due to MHR and due to cooperation. These numbers are then used in an energy analysis to determine the average energy used as a function of distance, thereby predicting network lifetime. An essential feature of the analysis here is that it does not assume a high node density. The theory presented quantifies the significant gains in network performance due to node cooperation.

VII. APPENDIX

The area $A_1(x, b)$ in (1)

$$A_1(x,b) = \beta\left(\lfloor\frac{|b|}{R_1}\rfloor\right)^2 - \left(\lfloor\frac{|b|}{R_1}\rfloor R_1\right)^2\frac{\sin(2\beta)}{2} + \alpha R_1^2 - R_1^2\frac{\sin(2\alpha)}{2}, \quad (12)$$

where angles β and α are given by

$$\beta = \arccos\left(\frac{(\lfloor\frac{|b|}{R_1}\rfloor)^2 + |b|^2 - R_1^2}{2|b|(\lfloor\frac{|b|}{R_1}\rfloor)}\right), \quad (13)$$

$$\alpha = \arccos\left(\frac{|b|^2 + R_1^2 - (\lfloor\frac{|b|}{R_1}\rfloor)^2}{2|b|R_1}\right). \quad (14)$$

The area $A_2(x, b)$ in (2)

$$A_2(x,b) = \beta|b|^2 - |b|^2\frac{\sin(2\beta)}{2} + \alpha R_1^2 - R_1^2\frac{\sin(2\alpha)}{2}, \quad (15)$$

where angle β and α are given by

$$\beta = \arccos\left(\frac{|b|^2 + |b|^2 - R_1^2}{2|b||b|}\right), \quad (16)$$

$$\alpha = \arccos\left(\frac{R_1^2 + |b|^2 - |b|^2}{2R_1|b|}\right). \quad (17)$$

The area $A(x, b)$ in (5)

$$A(x,b) = \beta|b|^2 - |b|^2\frac{\sin(2\beta)}{2} + \alpha R_2^2 - R_2^2\frac{\sin(2\alpha)}{2}, \quad (18)$$

where angle β and α are given by

$$\beta = \arccos\left(\frac{|b|^2 + |b|^2 - R_2^2}{2|b||b|}\right), \quad (19)$$

$$\alpha = \arccos\left(\frac{R_2^2 + |b|^2 - |b|^2}{2R_2|b|}\right). \quad (20)$$

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