SMSE PRECODER DESIGN IN A MULTIUSER MISO SYSTEM WITH LIMITED FEEDBACK

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ABSTRACT

We design an end-to-end linear transceiver in the downlink of a multi-user (MU) multiple input single output (MISO) system with quantized channel state information at the transmitter (CSIT). The design minimizes the sum mean squared error (SMSE) under a sum power constraint. The contribution of this paper is two-fold. First, unlike previous approaches, we quantize the channels using mean squared inner product (MSIP) vector quantization (VQ) and derive an SMSE-based algorithm that considers MSIP quantization error as an integral component of the whole system. This decreases the bit error rate (BER) at high signal-to-noise ratio (SNR) and outperforms previously derived MU MISO linear transceivers that exist in the limited feedback literature. Second, we show analytically why the BER, in the high SNR regime, increases if quantization error is not considered.

Index Terms— MISO precoder/decoder design, MU schemes, limited feedback, MSIP

1. INTRODUCTION

The advantages of spatial diversity and multiplexing has led much research into MU MISO downlink channels [1–4] for cellular networks. In these systems, MU interference and channel fading are the main concerns. These can be mitigated by precoding the signals at the base station (BS), in turn requiring CSIT. In frequency-division duplex and broadband time-division duplex systems [5] channel information needs to be estimated at the receiver and fed back to the BS. Therefore, providing accurate CSIT and reducing feedback overhead are important considerations in precoder design. This paper focuses on CSIT feedback and accounts for the resulting imperfect CSIT in precoder design. We assume perfect channel estimation at the receiver end and zero-delay noiseless feedback.

There are several feedback schemes in the literature, generally based on forming a codebook. The most popular are Grassmanian line packing [6], VQ using mean squared error (MSE) as the optimality criterion [7] and random vector quantization (RVQ) [3, 4, 8]. Most of these works focus on the zero forcing beamformer and show that the throughput gets interference limited at high SNR. [9] and [10] work on SMSE precoder design with a given channel uncertainty but do not consider the feedback policy in their approach. The work in [7] appears to be the state-of-the-art in terms of limited feedback precoder design to minimize MSE. Their *simulations* show that, if quantization error is ignored, the BER of a MU MISO system increases with SNR in the high SNR regime. They use the generalized Lloyd algorithm [11] for quantization and take quantization error into account using rate-distortion theory.

In this paper we investigate quantization based on the MSIP criterion [12] which is computationally efficient compared to [7] and results in a codebook with the same MSIP [12]. In our scheme, each user quantizes its channel using VQ MSIP and feeds the corresponding index to the BS. We use the resulting quantization error in conjunction with the SMSE minimization algorithm of [1, 2]. At the end, receivers implement minimum mean square error (MMSE) decoders while receiving actual data. Our results show that the combination of using MSIP, its quantization error and the SMSE precoder improves on the state-of-the-art. We also show why the BER rises with SNR if quantization error is not accounted for.

2. SYSTEM MODEL

We consider a MU MISO system: a single base station equipped with M transmit antennas and L individual users. The data symbol for user k, x_k, is processed by a unit norm precoding vector $\mathbf{u}_k$. The overall precoder and data vector are $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_L]$ and $\mathbf{x} = [x_1, ..., x_L]^T$. Let $\mathbf{p} = [p_1, p_2, .., p_L]^T$ be the allocated power for different data streams and define the downlink power matrix $\mathbf{P} = diag(\mathbf{p})$. $P_{max} \geq ||\mathbf{p}||_1$ is the total available power. The channel between the BS and the user is assumed to be flat for each block and represented by $1 \times M$ dimensional vector $\mathbf{h}_k^H$ where $(\cdot)^H$ denotes the conjugate transpose. The global channel matrix is $\mathbf{H}^H$, with $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_L]$. Based on this model, user k receives (DL denotes the downlink),

$$y_k^{DL} = \mathbf{h}_k^H \mathbf{U} \sqrt{\mathbf{P}} \mathbf{x} + n_k. \tag{1}$$

Here $n_k \sim \mathcal{N}\left(0, \sigma^2\right)$ represents the additive white gaussian noise at the k-th user's receive antenna. To estimate the transmitted symbol user k processes the received symbols with a

v_k. So,

$$\widehat{x}_k^{DL} = v_k^H \mathbf{h}_k^H \mathbf{U}\sqrt{\mathbf{P}}\mathbf{x} + v_k^H n_k \quad (2)$$

Stacking all the individual $\widehat{x}_k$ and defining $\widehat{\mathbf{x}} = [\widehat{x}_1, .., \widehat{x}_L]^T$, $\mathbf{V} = diag\,[v_1, ..., v_k]$ and $\mathbf{n} = [n_1, ..., n_k]^T$ the global downlink system is given by

$$\widehat{\mathbf{x}} = \mathbf{V}^H \mathbf{H}^H \mathbf{U}\sqrt{\mathbf{P}}\mathbf{x} + \mathbf{V}^H \mathbf{n}. \quad (3)$$

We now construct a virtual uplink system which will be useful to prove the uplink downlink duality. Let us assume that the transmit powers are $\mathbf{q} = [q_1, .., q_L]^T$. The global virtual uplink power allocation matrix is $\mathbf{Q} = diag(\mathbf{q})$.

$$\mathbf{y} = \sum_{i=1}^{L} \mathbf{h}_i v_i \sqrt{q_i} x_i + \mathbf{n}; \qquad \widehat{x}_k^{UL} = \mathbf{u}_k^H \mathbf{y} \quad (4)$$

The data, x_k, are mutually independent with unit energy.

Channel Quantization & Model with Feedback Error: Each receiver possesses a single quantization codebook comprising 2^B unit norm quantization vectors $\{\widehat{\mathbf{w}}_1, ..., \widehat{\mathbf{w}}_{2^B}\}$ and feeds back B bits to the BS. The quantization codebook is generated as a VQ problem based on chordal distance i.e. the MSIP optimality criterion [12].The receivers individually quantize their channel direction (we ignore magnitude) based on minimum chordal distance [13],

$$\widehat{\mathbf{h}}_k = \arg \max_{\widehat{\mathbf{w}} \in \{\widehat{\mathbf{w}}_1, .., \widehat{\mathbf{w}}_{2^B}\}} |\overline{\mathbf{h}}_k^H \widehat{\mathbf{w}}| \quad (5)$$

Here, $\overline{\mathbf{h}}_k = \frac{\mathbf{h}_k}{||\mathbf{h}_k||}$. $||\mathbf{h}_k||$ is the euclidean norm of $\mathbf{h}_k$. The quantization error $\widetilde{\mathbf{h}}_k$ can be defined as,

$$\widetilde{\mathbf{h}}_k = \overline{\mathbf{h}}_k - \left(\widehat{\mathbf{h}}_k^H \overline{\mathbf{h}}_k\right) \widehat{\mathbf{h}}_k$$

Therefore, the channel model at the receiver end takes the following form,

$$\begin{aligned} \mathbf{h}_k &= ||\mathbf{h}_k|| \overline{\mathbf{h}}_k \\ &= ||\mathbf{h}_k|| \left(\left(\widehat{\mathbf{h}}_k^H \overline{\mathbf{h}}_k\right) \widehat{\mathbf{h}}_k + \widetilde{\mathbf{h}}_k \right) \end{aligned} \quad (6)$$

Since we only send back the index of the unit norm quantized channel $\widehat{\mathbf{h}}_k$ to the BS, we consider the following channel model at the BS,

$$\mathbf{h}_k = \widehat{\mathbf{h}}_k + \widetilde{\mathbf{h}}_k \quad \text{or} \quad \mathbf{H} = \widehat{\mathbf{H}} + \widetilde{\mathbf{H}} \quad (7)$$

Here, $\mathbf{H}$ comprises L unit norm channel vectors with the original channel direction. $\widehat{\mathbf{H}}$ denotes the L quantized feedback unit norm vectors. $\widetilde{\mathbf{H}}$ denotes the error in the quantized feedback. Comparing (6) and (7), we can see that there is a phase shift of $\left(\widehat{\mathbf{h}}_k^H \overline{\mathbf{h}}_k\right)$ between the original $\mathbf{h}_k$ at the receiver and the assumed $\mathbf{h}_k$ at the BS. However, since we propose MMSE decoder while receiving data, the system performance will be invariant to this phase shift.

We approximate that the quantization error matrix $\widetilde{\mathbf{H}}$ has $L \times M$ independent identically distributed (i.i.d.) elements with zero mean and a variance of σ_E^2/M. The analysis section will give more insight into the form of σ_E^2. We also assume that $\widetilde{\mathbf{H}}$ is independent of $\mathbf{x}$, $\mathbf{n}$ and $\widehat{\mathbf{H}}$.

3. LINEAR TRANSCEIVER DESIGN

Let e_k^{DL} be the MSE of user k in the downlink where

$$e_k^{DL} = E\left[\left(\widehat{x}_k - x_k\right)\left(\widehat{x}_k - x_k\right)^H\right]. \quad (8)$$

The SMSE minimization problem can be formulated as,

$$\min_{\mathbf{p}, \mathbf{U}, \mathbf{V}} \sum_{k=1}^{L} e_k^{DL}; \quad \text{subject to} \quad ||\mathrm{p}||_1 \leq P_{max}. \quad (9)$$

3.1. Precoder Design in the virtual uplink

We will at first solve the problem in the virtual uplink and then transfer the solution via duality. Using (4) for $\widehat{x}_k$ and expanding (8) in the virtual uplink,

$$e_k^{UL} = \mathbf{u}_k^H \left(\mathbf{H}\mathbf{Q}\mathbf{H}^H + \sigma^2 \mathbf{I}\right) \mathbf{u}_k + 1 - \left(\mathbf{u}_k^H \mathbf{h}_k + \mathbf{h}_k^H \mathbf{u}_k\right) \sqrt{q_k} \quad (10)$$

Since BS has knowledge of only $\widehat{\mathbf{H}}$, not $\mathbf{V}$, $\mathbf{V} = \mathbf{I}$ was enforced to simplify the analysis in (10). Minimization of (10) w.r.t $\mathbf{u}_k$, based on the channel model of (7), was solved in [9]. For brevity, we just present the solutions here,

$$\mathbf{u}_k^{SMSE} = \mathbf{J}^{-1} \widehat{\mathbf{h}}_k \sqrt{q_k} \quad (11)$$

$$\mathbf{J} = \widehat{\mathbf{H}}\mathbf{Q}\widehat{\mathbf{H}}^H + \sigma^2 \mathbf{I}_M + \frac{\sigma_E^2}{M}\left(q_1 + .. + q_L\right) \mathbf{I}_M \quad (12)$$

$$e_k^{UL,SMSE} = 1 - \sqrt{q_k}\widehat{\mathbf{h}}_k^H \mathbf{J}^{-1} \widehat{\mathbf{h}}_k \sqrt{q_k} \quad (13)$$

Using (13) the SMSE is,

$$\begin{aligned} SMSE^{UL} &= \sum_{k=1}^{L} e_k^{UL,MSE} \\ &= \sum_{k=1}^{L} 1 - \sum_{k=1}^{L} \sqrt{q_k}\widehat{\mathbf{h}}_k^H \mathbf{J}^{-1} \widehat{\mathbf{h}}_k \sqrt{q_k} \\ &= L - tr\left[\widehat{\mathbf{H}}\mathbf{Q}\widehat{\mathbf{H}}^H \left(\widehat{\mathbf{H}}\mathbf{Q}\widehat{\mathbf{H}}^H + \left(\sigma^2 + \frac{\sigma_E^2 \sum_{k=1}^{L} q_k}{M}\right)\mathbf{I}_M\right)^{-1}\right] \\ &= L - M + (\sigma^2 + \frac{\sigma_E^2 \sum_{k=1}^{L} q_k}{M}) tr[\mathbf{J}^{-1}] \end{aligned} \quad (14)$$

where, tr[$\cdot$] denotes the trace operator and $\mathbf{J}$ takes the form of (12). As $\widehat{\mathbf{H}}$ is fixed, the SMSE expression is a function of uplink power allocation $\mathbf{Q}$.

Proposition 1 : The optimization problem,

$$\mathbf{Q}^{opt} = \min_{\mathbf{Q}} \left(\sigma^2 + \frac{q_1 + ... + q_L}{M}\sigma_E^2\right) tr(\mathbf{J}^{-1}) \quad (15)$$

subject to $tr[\mathbf{Q}] \leq P_{max}, q_k \geq 0$ for all k is convex in $\mathbf{Q}$
Proof: [10] shows that SMSE remains a nonincreasing function of SNR with equal channel uncertainty. Therefore, we assume that all power is used and $tr[\mathbf{Q}] = P_{max}$. The convex optimization problem then takes the following form,

$$\min_{\mathbf{q}} \left(\sigma^2 + \frac{1}{M} P_{max}\sigma_E^2\right) tr(\mathbf{J}^{-1})$$
$$\mathbf{J} = \widehat{\mathbf{H}}\mathbf{Q}\widehat{\mathbf{H}}^H + \left(\sigma^2 + \frac{1}{M} P_{max}\sigma_E^2\right) \mathbf{I}_M \qquad (16)$$

Here $(\sigma^2 + \frac{1}{M} P_{max}\sigma_E^2)$ is a constant and $\mathbf{J}$ is a positive definite matrix. Therefore, the optimization problem is convex in $\mathbf{J}$ [14]. Since $\mathbf{J}$ is linear in $\mathbf{q}$, it can be readily proved that the problem is convex in $\mathbf{q}$. ■
Proposition 2: Given $\mathbf{U}$ and P_{max}, $MSE_k^{UL} = MSE_k^{DL} \forall k$.
Proof: The proof follows the same reasoning as in [15] & [16] and is not reproduced here for lack of space. ■

Since quantization error is present in the system, the downlink power allocation vector takes the following form [15],

$$\mathbf{p} = \sigma^2 \left(\mathbf{D}^{-1} - \mathbf{\Psi}\right)^{-1} \mathbf{1} \qquad (17)$$

$$\mathbf{D} = diag\left(\frac{\gamma_1}{\left|\hat{\mathbf{h}}_1^H \mathbf{u}_1\right|^2 + \frac{\sigma_E^2}{M}}, ..., \frac{\gamma_L}{\left|\hat{\mathbf{h}}_L^H \mathbf{u}_L\right|^2 + \frac{\sigma_E^2}{M}}\right) \qquad (18)$$

$$(\mathbf{\Psi})_{ik} = \left\{ \begin{array}{ll} |\hat{\mathbf{h}}_i^H \mathbf{u}_k|^2 + \frac{\sigma_E^2}{M} & k \neq i \\ 0 & k = i \end{array} \right\} \qquad (19)$$

where $\mathbf{1} = [1, .., 1]^T$ is a length-L vector of ones and $\gamma_k = SINR_k^{UL}$ where $SINR_k^{UL}$ is the signal-to-interference-plus-noise ratio for user k in the virtual uplink:

$$SINR_k^{UL} = q_k \frac{\mathbf{u}_k^H \mathbf{S}_k^{UL} \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{T}_k^{UL} \mathbf{u}_k}. \qquad (20)$$

where $\mathbf{S}_k^{UL} = \widehat{\mathbf{h}}_k \widehat{\mathbf{h}}_k^H + \frac{\sigma_E^2}{M}\mathbf{I}_M$ and

$$\mathbf{T}_k^{UL} = \sum_{j=1, j\neq k}^{L} q_j \left(\widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \frac{\sigma_E^2}{M}\mathbf{I}_M\right) + \sigma^2 \mathbf{I}_M.$$

Given the duality result in Proposition 2, the BS can find the uplink power allocation via convex optimization, solve for the corresponding downlink precoder in the virtual uplink and then obtain the downlink power allocation using (17). Solving the problem in the virtual uplink significantly reduces computational complexity.

3.2. Receiver Design

MMSE receivers are implemented at the user side while they receive data. Therefore,

$$\mathbf{v}_k = \left(\mathbf{h}_k^H \mathbf{U}\mathbf{P}\mathbf{U}^H \mathbf{h}_k + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_k^H \mathbf{u}_k \sqrt{p_k}, \qquad (21)$$

which can be normalized to make $||\mathbf{v}_k|| = 1$. The MMSE receiver can be implemented by sending dedicated symbols from the BS and using training. Note that the MMSE receiver cannot be implemented at the time of channel quantization since the precoder matrix $\mathbf{U}$ was not designed at that time.

3.3. Overall Algorithm

Using the development above, the transceiver design algorithm is:
1. Each user sends its quantized channel $\widehat{\mathbf{h}}_k$ using (5);
2. solve for virtual uplink power allocation $\mathbf{Q}^{opt}$ using (15);
3. solve for the downlink precoder using (11);
4. solve for the downlink power allocation using (17).
5. solve for the receiver decoder using (21).

4. QUANTIZATION ERROR AND SMSE ANALYSIS

4.1. Quantization Error Analysis

Due to the formulation of MSIP, its error variance is measured in terms of the angle spread between the original and quantized vectors. In [12], the quantization error of $\widetilde{\mathbf{h}}$ was given the following form,

$$\sigma_E^2 = E\left[\sin^2\left(\angle\left(\mathbf{h}_k, \widehat{\mathbf{h}}_k\right)\right)\right]$$

where, σ_E^2 is *upper bounded* by $2^{\frac{-B}{M-1}}$ [12]. This assumption of σ_E^2 as the variance of $\widetilde{\mathbf{h}}$ in (7) is heuristic and deviates from the previous literature works that use MSE based VQ for quantization and assume,$\sigma_E^2 = E\left|\left|\widetilde{\mathbf{h}}\right|\right|^2 = E\left|\left|\mathbf{h} - \widehat{\mathbf{h}}\right|\right|^2$. However, the performance of MSIP quantization depends on the alignment of the original channel vectors with the quantized feedback vectors and not on the distance between the tip of these vectors in the M-dimensional unit norm sphere. Therefore, our assumption remains valid in this system model.

If B feedback bits are used to quantize a length-M vector using VQ based on MSE, the expected quantization error becomes $2^{\frac{-B}{M}}$ [7]. For the Rayleigh distributed channel model, this expectation turns into a *lower bound* [7]. By changing the exponent of the error, MSIP leads to reduced quantization error for the direction of the channel and thus provides better performance.

4.2. SMSE Analysis

In the absence of quantization error, SMSE of the traditional precoder [2] (where quantization error is not considered) is

$$SMSE = L - M + \sigma^2 tr\left[\left(\mathbf{H}\mathbf{Q}\mathbf{H}^H + \sigma^2 \mathbf{I}_M\right)^{-1}\right]$$
$$= L - M + tr\left[\left(\frac{P_{max}}{L\sigma^2}\mathbf{H}\mathbf{H}^H + \mathbf{I}_M\right)^{-1}\right] \qquad (22)$$

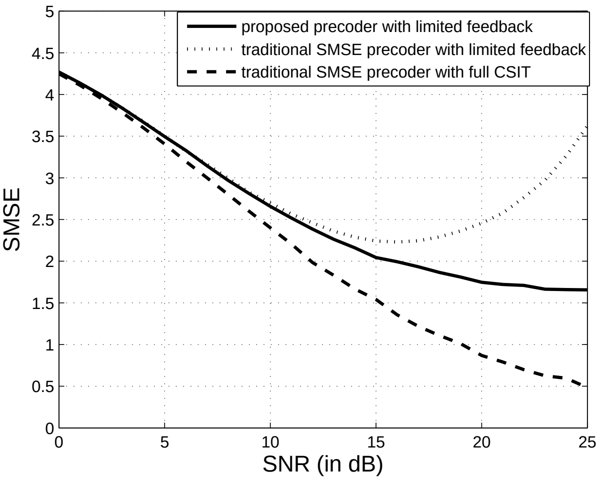

Fig. 1. SMSE analysis for the proposed precoder

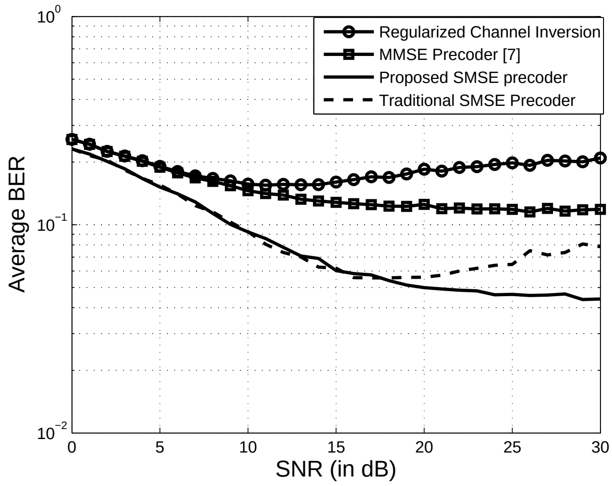

Fig. 2. Performance of the proposed SMSE precoder compared to other existing precoding techniques with M = 4, L = 4, B = 10 bit per user, QPSK

In (22), we assumed $\mathbf{Q} = \frac{P_{max}}{L}\mathbf{I}_L$ i.e. equal power allocation for simplicity of the analysis. At very high SNR, the SMSE approaches zero in (22) as $tr\left(\frac{P_{max}}{L\sigma^2}\mathbf{H}\mathbf{H}^H + \mathbf{I}_M\right)^{-1}$ is a decreasing function of SNR. However, with quantization error, if the original precoder [2] is used,

$$SMSE = \sum_{k=1}^{L}\left(1 - q_k\widehat{\mathbf{h}}_k^H\mathbf{J}^{-1}\widehat{\mathbf{h}}_k + \frac{\sigma_E^2}{M}P_{max}q_k\widehat{\mathbf{h}}_k^H\mathbf{J}^{-2}\widehat{\mathbf{h}}_k\right) \tag{23}$$

where $\mathbf{J} = \widehat{\mathbf{H}}\mathbf{Q}\widehat{\mathbf{H}}^H + \sigma^2\mathbf{I}_M$. Both $q_k\widehat{\mathbf{h}}_k^H\mathbf{J}^{-1}\widehat{\mathbf{h}}_k$ and $\frac{\sigma_E^2}{M}P_{max}q_k\widehat{\mathbf{h}}_k^H\mathbf{J}^{-2}\widehat{\mathbf{h}}_k$ increase with SNR. Since the former term is a linear over affine function and the latter is a quadratic over quadratic function of P_{max}, at high SNR the latter term dominates and SMSE increases with SNR, which explains the results of [7].

In our proposed algorithm,

$$SMSE = L - M + \left(\sigma^2 + \frac{\sigma_E^2}{M}P_{max}\right) \times$$

$$tr\left[\left(\mathbf{H}\mathbf{Q}\mathbf{H}^H + \left(\sigma^2 + \frac{\sigma_E^2}{M}P_{max}\right)\mathbf{I}_M\right)^{-1}\right] \tag{24}$$

$$= L - M + tr\left(\frac{P_{max}}{L\left(\sigma^2 + \frac{\sigma_E^2}{M}P_{max}\right)}\mathbf{H}\mathbf{H}^H + \mathbf{I}_M\right)^{-1} \tag{25}$$

In (25), we again assumed equal power allocation for analysis. $\frac{P_{max}}{L\left(\sigma^2 + \frac{\sigma_E^2}{M}P_{max}\right)}$ is a nonincreasing function of P_{max}. Thus the proposed precoder makes sure that SMSE does not increase with SNR at high SNR region. Figure 1 illustrates all these effects. The simulations were done with independent channel realizations where $M = 5$, $L = 5$ and $B = 10$ bits. The proposed algorithm clearly stabilizes the error rate at high SNR.

Note that, at very high SNR, $\frac{P_{max}}{L\left(\sigma^2 + \frac{\sigma_E^2}{M}P_{max}\right)}$ will become constant and make SMSE saturated. To ensure the decreasing nature of SMSE, the receivers have to decrease the quantization error proportionately to the increase of signal power. This condition can be met by increasing feedback bit with varying power so that $2^{-\frac{B}{M-1}}P_{max}$ remains constant. This relation of feedback bits and varying power was at first noticed in terms of sum-rate in [3].

4.3. Simulation Results

The codebook for the proposed SMSE precoder in Figure 2 was designed using the MSIP criterion [12] with a training set comprising 2×10^5 unit norm independent channel realizations generated with complex gaussian distributions and optimized over 2 independent trials. As the figure shows, the proposed precoder improves over the state-of-the-art algorithm of MMSE precoder [7]. The traditional SMSE precoder that ignores quantization error performs well at lower SNR values, but begins to worsen at SNR = 15dB.

5. CONCLUSION

In this paper, we proposed an SMSE-based precoding technique in limited feedback scenario that uses MSIP VQ for quantizing the channel and takes quantization error into account. The proposed system outperforms the previously existing limited feedback MU-MISO transceivers found in the literature. We have also analytically shown that SMSE increases with increasing SNR if quantization error is not taken into account.

Our proposed system provides shape feedback to the BS. Our simulations show that the proposed system outperforms previous MMSE precoder, employing vector quantization based on euclidead distance, that provide both shape and gain feedback. This will happen if the average magnitude of the channel entries of different users are equal. However, if the users are located at different distances from the BS, channel norm will be needed for optimal power allocation. Our future work lies in the consideration of large scale fading and provide both gain and shape feedback.

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