A Pragmatic Approach to Adaptive Antennas

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Abstract

This paper presents a novel approach for efficient computation of adaptive weights in phased-array antennas. The fundamental philosophical differences between adaptive antennas and adaptive signal-processing methodology are also delineated in the introduction. This approach, unlike the conventional statistical techniques, eliminates the requirement for an interference covariance matrix, and represents a rethinking of the entire conventional approach to adaptive processing. This approach provides greater flexibility in solving a wider class of problems, at the expense of a slightly reduced number of degrees of freedom. It is important to note that the application of a deterministic approach to address stochastic problems with an ergodic structure can be seen in the works of Norbert Wiener and A. N. Kolmogorov, as outlined in the introduction. This paper presents examples to illustrate the effectiveness and uniqueness of this new pragmatic approach.

Keywords: Adaptive arrays; adaptive signal processing; shaped beam antennas; array signal processing; sidelobe cancellation; multipath channels

1. Introduction

This paper deals with the application of a direct data-domain least-squares algorithm to adaptive antennas. The basic difference between adaptive antennas and adaptive signal processing is that an antenna is a spatial filter, and therefore processing occurs in the angular domain, whereas a signal-processing algorithm is applied in the temporal domain. To identify whether one is dealing with adaptive antennas or adaptive signal processing is to ask the question, "Can the adaptive system separate a desired signal from a mixture of itself along with its coherent multipath component?" In this case, there is not only signal, but also a multipath component that is correlated with the signal and interacts (in either an additive or destructive fashion) with the signal. Only an adaptive antenna can isolate the signal from its coherent multipath, as the information on how to separate them is available spatially. In a conventional signal-processing algorithm, this type of coherent multipath separation is not a trivial problem, and secondary processing that utilizes the electromagnetic spatial concepts is necessary. The point here is that purely temporal processing cannot separate signals spatially, as the information exists in different domains. The direct-domain, least-squares approach is unlike the conventional method-

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ology, which needs to evaluate second-order statistics (i.e., the covariance matrix) of the data.

This paper has eight sections, followed by a list of references. The goal of the references is to provide information as to where supplementary materials may be available, which will further illustrate the points made in this paper in a more elaborate fashion. The second section describes the anatomy of an adaptive process. It is seen that to formulate an adaptive methodology, it is absolutely essential to have some knowledge about the desired signal to be estimated from a noisy environment. Historically, statistical methods have been used extensively in the adaptive methodology, as described in Section 3. In Section 4, it is shown how the analog adaptive process was modified with the advent of digital techniques. However, the shortcomings of a statistical process can be overcome by using a deterministic methodology, based on a single snapshot. This approach is presented in Section 5, along with some numerical examples. It is seen that at least four different deterministic approaches may be applied simultaneously to solve the same adaptive problem, without making any assumptions about the underlying process. Hence, the level of confidence in the final computed solution can be increased to an extremely high level, by comparing the solution obtained by the different direct data-domain least-squares procedures. Thus, with this approach, one can not only obtain a useful solution, but can also have a high degree of confidence in the result. For example, if all the three techniques provide similar estimates for the unknown signal, then one can say with a high level of confidence that the solution is probably correct, although the true solution remains unknown. If properly implemented, this procedure can significantly minimize the false-alarm probability.

It is quite easy to demonstrate that in a more scientific approach, we apply a deterministic least-squares method to solve the problem when the underlying probability density functions are not known a-priori. As William A. Gardner points out in his book [24], in most cases, a deterministic approach can be applied without taking recourse to a statistical methodology.

Gardner points out, in the preface of his other book [22],

> The book grew out of an enlightening discovery I made a few years ago, as a result of a long term attempt to strengthen the tenuous conceptual link between the abstract probabilistic theory of cyclostationary stochastic processes and empirical methods of signal processing that accommodate or exploit periodicity in random data. After a period of unsatisfactory progress toward using the concept of ergodicity to strengthen this link, it occurred to me (perhaps wishfully) that the abstraction of the probabilistic framework of the theory might not be necessary. As a first step in pursuing this idea, I set out to clarify for myself the extent to which the probabilistic framework is needed to explain various well-known concepts and methods in the theory of stationary stochastic processes. To my surprise, I discovered that all the concepts and methods of empirical spectral analysis can be explained in a more straight forward fashion in terms of a deterministic theory, that is, a theory based on time averages of a single time series rather than ensemble-averages of hypothetical random samples from an abstract probabilistic model. To be more specific, I found that the fundamental concepts and methods of empirical spectral analysis can be explained without use of probability calculus or the concept of probability and that probability calculus, which is indeed useful for quantification of the notion of degree of randomness or variability, can be based on time averages of a single time-series without any use of the concept or theory of a stochastic process defined on an abstract probability space.

Norbert Weiner's generalized harmonic analysis, written in 1930, was entirely devoid of probability theory; and yet there has been only one book written since then for engineers and scientists that provides more than a brief mention of Wiener's deterministic theory [22]. All other such books emphasize the probabilistic theory of A. N. Kolmogorov, usually to the complete exclusion of Wiener's deterministic theory.

Even Kolmogorov, himself, suggested "...way toward the future. Side by side with the vigorous pursuit of the theory of stochastic processes, must coexist a more direct process-free (deterministic) inquiry of randomness of different classes of functions" [22]. T. L. Fine, in the concluding section of his book, *Theories of Probability*, states, "Judging from the present confused status of probability theory, the time is at hand for those concerned about the characterization of chance and uncertainty in the design of incidence and decision making systems to reconsider their long-standing dependence on the traditional statistical and probabilistic methodology....why not ignore the complicated and hard to justify probability statistics structure and proceed 'directly' to those perhaps qualitative assumptions that characterize our source of random phenomena, the means at our disposal, and our task?" [22].

These points have further been enhanced by Ronald N. Bracewell. As he points out, in the preface of [22],

> "The theory of signal processing, as it has developed in electrical and electronics engineering, leans heavily toward the random process, defined in terms of probability distributions applicable to ensembles of sample signal waveforms. But many students who are adapt at the useful mathematical techniques of the probabilistic approach and are quite at home with joint probability distributions are unable to make even a rough drawing of the underlying sample waveforms. The idea that the sample waveforms are the deterministic quantities being modeled somehow seems to get lost...."
>
> The assumption of randomness is an expression of ignorance. Progress means the identification of systematic effects which, taken as a whole, may initially give the appearance of randomness and unpredictability....
>
> Many authors have been troubled by the standard information theory approach via the random process or the probability distribution because it seems to put the cart before the horse. Some sample parameters such as mean amplitudes or powers may be known to precision of measurement but if we are to go beyond pure mathematical deduction and make advances in the realm of phenomena, theory should start from the data. To do otherwise risks failure to discover that which is now built into the model....Problems on the forefront of development are often ones where the probability distributions of neither signal nor noise is known; and such distributions may be essentially unknowable because

repetition is impossible. Thus, any account of measurement, data processing, and interpretation of data that is restricted to probabilistic models leaves something to be desired. A nonprobabilistic model demonstrates a consistent approach from data, those things which in fact are given, and shows that analysis need not proceed from assumed probability distributions of random process. This is a healthy approach and one that can be recommended to any reader."

In addition, Haykin [9] points out that a stochastic methodology leads to the design of an adaptive filter that will operate in a probabilistic sense on average for all the operational environments, assumed to be wide-sense stationary. On the other hand, a deterministic approach provides the solution for the given data at hand, and without invoking any of the stochastic methodology and without assuming the nature of the probability density functions. For example, if one takes a normal coin, when tossed up, on the average it will fall with the "head" facing up 50% of the time, and the "tail" up the other 50%. However, it is not known a priori what is going to happen on a single toss. A deterministic approach provides the solution for that single realization, which operates on the given data for one snapshot only. This philosophy has been further amplified by Hofstetter and Gardner [23].

Section 6 describes the prevention of signal cancellation in an adaptive process by performing adaptive processing with constraints across the beamwidth (3 dB points). In Section 7, a novel method is presented to estimate the accuracy of the assumed direction of arrival of the desired signal, based on the norm of the adaptive weights. This is followed by the conclusion in Section 8, and references.

2. Anatomy of an Adaptive Algorithm

Basically, in an adaptive methodology, the goal is to estimate the desired response in an adaptive fashion, using a model transfer function. Historically, the first method to be developed was the Wiener filter. Below, it is shown how this methodology has progressed over the years, and its relationship to new spatially-based adaptive techniques, as opposed to the time-based methodology.

The anatomy of an adaptive technique is shown in Figure 1. The input signal, $x(t)$, is used to track/approximate a desired signal, $d(t)$, through a linear filter, $h(t)$. The characteristics of the linear filter are changed by a controller. The controller, in turn, is affected by the error signal that is generated by taking the instantaneous difference between the desired signal, $d(t)$, and the output, $y(t)$, from the linear filter. Historically, this problem of finding the linear filter $h(t)$ was solved by Kolmogorov [1, 2], in the analysis of stationary time series, and, simultaneously, by Wiener [3], in the control of antiaircraft guns.

The approaches by Kolmogorov and Wiener are very similar. The methodology starts by defining the error signal, $e(t)$, as

$$e(t) = d(t) - y(t) = d(t) - x(t) \otimes h(t), \quad (1)$$

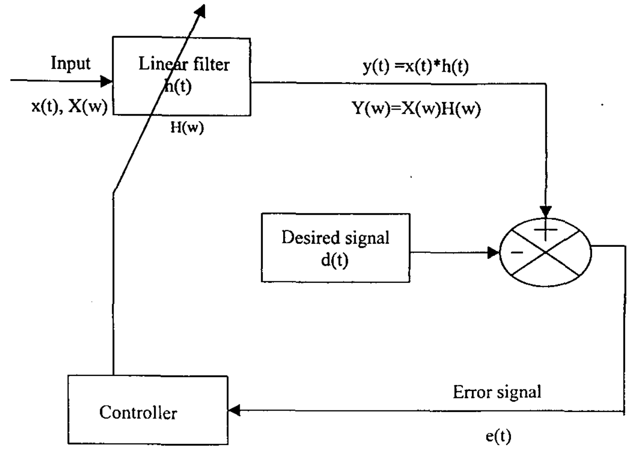

Figure 1. The anatomy of an adaptive algorithm.

where $\otimes$ represents a convolution, and, therefore,

$$e(t) = d(t) - \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau .$$

Next, the expected value of the squared error is taken, to yield

$$E = \mathcal{E}[\ e(t)\ e^*(t)\ ] = \mathcal{E}[\ |e(t)|^2\ ], \quad (2)$$

where $\mathcal{E}[\bullet]$ is the expectation operator, and * represents the complex conjugate.

To find the optimum filter $h(t)$, the error, E, needs to be differentiated with respect to h, and this leads to the orthogonality of the error with the input $x(t)$. In other words,

$$\mathcal{E}\left[x(t) e^*(t)\right] = 0. \quad (3)$$

If we define

$$R_{xd}(t) = \mathcal{E}\left[x(t) d^*(t)\right], \quad (4)$$

$$R_{xx}(t) = \mathcal{E}\left[x(t) x^*(t)\right]. \quad (5)$$

Then, one obtains

$$R_{xd}(t) = \int_{-\infty}^{\infty} R_{xx}(t-\tau) h(\tau) d\tau . \quad (6)$$

This integral equation provides the filter $h(t)$ that is going to match $d(t)$ for a given input $x(t)$ in an optimum fashion [4].

The basic principles of an adaptive technique thus illustrate that there must be something known about the desired signal in order to define the adaptive process. This knowledge could be about the constant amplitude (as in a digital signal), or some hidden structure in the spectral characteristics (like cyclostationarity and so on). An adaptive procedure cannot be completely defined without some knowledge of the desired signal. What that information is may change from problem to problem, but it has to exist.

In adaptive-antenna problems, it is not enough to assume there exists a signal $d(t)$, or to match the output to a desired signal. We need to know more about the signal. Such information may be related to the angle of arrival of the desired signal. Or, it may be associated with the modulation technique used. In many digital-communications applications, the binary signal may be of constant magnitude, and what needs to be solved for is the sign of the signal. Alternately, for other types of signals used in mobile communications, the spectrum of the signal may have conjugate-transpose symmetry–namely, the spectrum may be cyclostationary. This may be equivalent, in some cases, to saying that the autocorrelation function of the desired signal may have periodic properties.

To further explain the situation, for most adaptive-antenna problems we know the Doppler and direction of arrival of the signal, and the goal is to estimate its strength in the presence of jammers, clutter, and noise. An important class of jammers are multipath signals, which may be coherent with the signal of interest. Sometimes, we will also deal with jammers that may be in the main lobe of the antenna, and could be intermittent (i.e., blinking jammers). In mobile communications, the direction of arrival is not known, a priori. There, we know that for digitally transmitted signals, in many cases, the signal has a constant magnitude only, and its sign needs to be estimated. This leads to the class of constant-modulus algorithms.

In some other applications–like, for example, BPSK or QPSK transmission–we know that the spectrum has some conjugate symmetric property, or, equivalently, that the autocorrelation function of the signal of interest may be periodic or cyclostationary. We exploit this information to extract the signal in the presence of jammers, clutter, and noise. In the literature, these algorithms are called blind-equalization techniques. However, in this paper we will focus on the situation where the direction of arrival of the signal is known a priori.

In summary, all adaptive techniques require some knowledge about the signal of interest, in order to estimate it in the presence of interference and thermal noise. Without such information, an adaptive procedure is not defined.

3. Historical Background

We now illustrate how the Wiener filter was modified to deal with digital data. In antenna theory, adaptive techniques were first developed by Applebaum [5], at Syracuse, for application in a sidelobe canceler. Simultaneously, Widrow [6] developed the LMS (least-mean-square) algorithm for adaptively canceling interferers in the presence of signals of interest. Both of these techniques were applied to analog signals for continuous operation and iteratively canceling interferers in the presence of signals of interest. They were based on statistical methodologies because, in those years, it was not easy to quantify the analog signals of interest.

With the advent of digital technology, these techniques were re-employed, this time dealing with digitally sampled data. However, with the design of faster processors, the Wiener-filter theory, developed in the previous section, also became available for the enhancement of signals in a noisy environment. With the availability of high-speed signal processors and analog-to-digital converters, these techniques were essentially employed in the digital domain. It can be seen [7, 20] that the speed of the adaptive processes were greatly enhanced by replacing the LMS algorithm by a conjugate-gradient method, saving several orders-of-magnitude of CPU time. Also, the method can be used to improve the reliability of the estimate [21] while performing adaptive processing. However, these methods were basically applications of the same procedure for calculating the instantaneous error signal and then applying a "forgetting factor" to decimate the old data as new data arrived.

Next, the class of algorithms based on the method of least squares is discussed. We efficiently use digital signal processors to solve adaptive problems. In these procedures, a model-dependent procedure, using the method of least squares (without invoking any assumptions about the statistics of the signals that are to be tracked), is utilized. This gives rise to the minimum-variance distortionless response (MVDR), based on a statistical methodology, but using the data in the test cell, only. As pointed out by Gardner [22], the stochastic approach has become prevalent because one is dealing with analog signals and, secondly, because communications engineers want to design systems that will perform well, on average, over the ensemble. However, since it is not feasible to make measurements over many realizations (systems), the communications engineers have settled on characterizing system performance, in practice, by averaging over time for a single system. In order to replace the ensemble averages by time averages, one needs to assume wide-sense stationarity. Furthermore, since the measurement is limited to one system, one has to invoke the concept of ergodicity. This is equivalent to using stationary stochastic-process models that are ergodic, so that the mathematically calculated expected values (ensemble averages) will equal the measured time averages.

Hofstetter [23] states that "...unfortunately, however the logic seems to have stopped at this point. It apparently was not recognized (except by too few to make a difference) that once consideration was restricted to ergodic stationary models, the stochastic process and its associated ensemble could be dispensed with because a completely equivalent theory of statistical interference and decision that is based entirely on time averages over a single record of data could be used." Gardner [22] further points out that

> "Any calculations made using a model based on the time average theory could be applied to any one member of an ensemble if one so desired because the arguments that justify the ergodic stochastic model also guarantee that the time-average for one ensemble member will be same (with probability one) as the time average for any other ensemble member. Whenever transient behavior is of interest ergodic models are ruled out, because all transient behavior is lost in an infinitely long time-average. Thus to counter the conceptual simplicity and realism offered by the time-average approach, the stochastic-process approach offers the advantage of more general applicability."

These considerations lead us to apply the direct data-domain approach to adaptive processing. Furthermore, we do away with

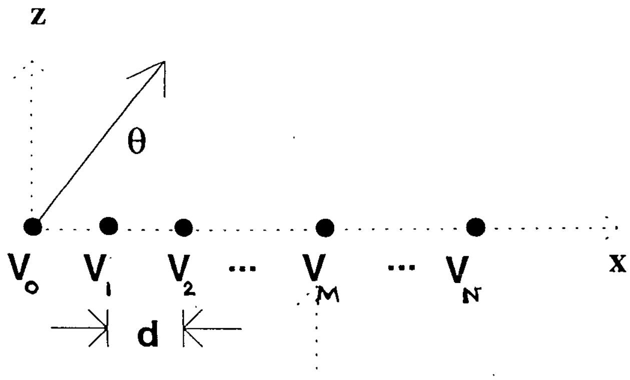

Figure 2. A linear array containing $N+1$ elements.

the time averages and focus on spatial averaging. In addition, we solve an estimation problem, rather than a detection problem.

Consider a linear array of $(N+1)$ uniformly spaced (isotropic) receiving elements, separated by a distance d, as shown in Figure 2. We have $(N+1)$ sensors in the array. We further assume that narrowband signals, consisting of a desired signal and interference with center frequency f_0, are impinging on the array from various angles θ, measured from the end-fire direction of the array, with the constraint $0 \le \theta \le 180°$. The signal we want to estimate is arriving from an a priori known angle, θ_s, and the various jamming signals are arriving from various angles θ_J, including coherent/non-coherent multipaths. The jamming signals may be located in the main beam of the array, i.e., $\theta_s - \theta_J \le 50.8° / (L/\lambda)$, where L is the length of the array, λ is the wavelength corresponding to the frequency f (i.e., $\lambda = 2\pi c / f_0$, where c is the velocity of light), and, typically, d (the spacing between the elements) is chosen to be $\lambda/2$.

4. Minimum-Variance Distortionless-Response (MVDR) Technique

The MVDR response is based on the statistical methodology of Capon [8], but without invoking the statistics of the underlying signal. We assume that the direction of the arrival of the signal is known. The goal is to estimate its strength in the presence of jammers, clutter, and thermal noise. We deal with discrete signals, and the linear filter of Figure 1 is now replaced by the weight vectors, $w(k)$, and the received signal is replaced by $x(k)$. The goal is to estimate the weights, and to use them to find the signal of interest, $s(k)$, embedded in the received signals, $x(k)$. The desired signal, $d(k)$, is now a function of the input signals. As pointed out by Haykin [9], "This methodology may be viewed as an alternate to Wiener filter theory. Basically, Wiener filters are derived from ensemble averages (which is achieved by taking the expected value) with the result that one filter (in a probabilistic sense) is obtained for all realizations of the operational environment, assumed to be wide-sense stationary. On the other hand, the method of least squares is deterministic in approach. Specifically, it involves the use of time averages, with the result that the filter depends on the number of samples used in the computation." This has been implemented by Owsley [10, 11].

Consider the case where we have an array of $N+1$ sensors. The signals received in the $N+1$ sensors are $x(i)$, $x(i-1)$,..., $x(i-N+1)$ at the ith time instance. We consider the case where a set of N weights is attached to the last N sensors. For the first sensor, the weight is equal to unity. We then define

$$y(i) + \sum_{k=1}^{N} w^*(k)\, x(i-k) = 0. \qquad (7)$$

It is further assumed that we can predict the first sample from the next N samples. Hence,

$$d(i) = x(i). \qquad (8)$$

Therefore, in this system we are using the last N samples of $x(i-1),...,x(i-N+1)$ to predict the ith sample of $x(i)$. Then, the error is defined (with $w(0)=1$) to be

$$e(i) = d(i) - y(i) = \sum_{k=0}^{N} w^*(k)\, x(i-k) = [w]^H [x], \qquad (9)$$

where H denotes the conjugate transpose and $[\bullet]$ denotes a matrix. Without any loss of generality, we no longer assume that $w(0)=1$, but that it has some value. The goal is to extract the signal of interest, $s(k)$, which is embedded in the received signal, $x(k)$, in the presence of interferers, jammers, and the like. Next, the objective is to find the $w(k)$s by minimizing the sum of the errors $e(i)$.

$$E = \sum_{i=i_1}^{i_2} |e(i)|^2, \qquad (10)$$

where the summation runs over indices from i_1 to i_2. The exact values of i_1 and i_2 are determined from the given data.

However, the information by itself is not sufficient to solve the adaptive problem. Additionally, one needs to specify the direction of arrival of the signal of interest, $s(k)$. This is achieved by maintaining the main-beam gain of the array,

$$\sum_{k=0}^{N} w^*(k)\, e^{-j\frac{k2\pi d}{\lambda}\cos\theta_s} = 1, \qquad (11)$$

where $\frac{d}{\lambda}$ is the inter-element spacing of the sensors in terms of the wavelength of the operating frequency, and θ_s is the direction of arrival of the signal from the end-fire direction of the array (here, $\theta = 90°$ is the broadside direction. Also, note that a linear array cannot resolve the ambiguity of $+\theta$ or $-\theta$, i.e., of from which side of the array the signal is coming). Hence, the objective of the weights is to minimize the cost function, E, formed through the Lagrange multiplier, λ (a complex constant):

$$E = \sum_{i=N+1}^{I} |e(i)|^2 + \lambda\left(\sum_{k=0}^{N} w^*(k)\, e^{-j\frac{2\pi dk}{\lambda}\cos\theta_s} - 1\right)^*, \qquad (12)$$

where I is the total number of time samples available. Here, the index i runs from $N+1$ to I because of Equation (9).

Let $\phi_0 = \frac{2\pi d}{\lambda}\cos\theta_s$. Then,

$$E = \sum_{i=N+1}^{I}\left|\sum_{k=0}^{N} w^*(k)x(i-k)\right|^2 + \lambda\left(\sum_{k=0}^{N} w^*(k)e^{-j\phi_0 k} - 1\right). \quad (13)$$

By minimizing the cost function E in terms of the weights, one obtains

$$\begin{aligned}\frac{\partial E}{\partial w^*(k)} &= 2\sum_{i=N+1}^{I} x(i-k)y^*(k) + \lambda e^{-jk\phi_0} \\ &= 2\sum_{t=o}^{N} w(k)\sum_{i=N+1}^{I} x(i-k)x(i-v) + \lambda e^{-jk\phi_0}.\end{aligned} \quad (14)$$

Since at the minimum, the first derivative is zero, this yields

$$\sum_{i=0}^{M} w(k)R(v,k) = \frac{1}{2}\lambda e^{-jk\phi_0} \quad \text{for } k = 0,1,\ldots,N, \quad (15)$$

where

$$R(v,k) = \sum_{i=N+1}^{I} x(i-k)x(i-v). \quad (16)$$

Or, utilizing a matrix notation,

$$[R]_{(N+1)\times(N+1)}[w]_{(N+1)\times 1} = \frac{1}{2}\lambda\left[s(\phi_0)\right]_{(N+1)\times 1},$$

where

$$\left[s(\phi_0)\right]_{(N+1)\times 1} = \left[1, e^{-j\phi_0}, \ldots, e^{-jN\phi_0}\right]^T, \quad (17)$$

and T denotes the transpose of a matrix. The optimum weight vectors are given by

$$[w]_{opt} = -\frac{1}{2}\lambda[R]^{-1}\left[s(\phi_0)\right]. \quad (18)$$

If H represents the conjugate transpose of a matrix, then from Equation (11),

$$[w]^H\left[s(\phi_0)\right] = 1. \quad (19)$$

Utilization of Equation (19) in Equation (18) results in

$$\lambda = \frac{-2}{\left[s(\phi_0)\right]^H[R]^{-1}\left[s(\phi_0)\right]}. \quad (20)$$

Therefore, the final result for the optimum weights is given by

$$[w]_{opt} = \frac{[R]^{-1}\left[s(\phi_0)\right]}{\left[s(\phi_0)\right]^H[R]^{-1}\left[s(\phi_0)\right]}, \quad (21)$$

and the desired signal, $s(i)$, is estimated from

$$s(i) = e_{\min}(i) = \sum_{k=0}^{N} w_{opt}^*(k)x(i-k) = \left[w_{opt}\right]^H[x]. \quad (22)$$

This optimum solution has several interesting properties, as originally outlined by Capon [8] and implemented by Owsley [10, 11], and summarized by Haykin [9]. Namely, the optimum weights are unbiased if the sequence $x(k)$ contains noise that is zero mean. In addition, this least-squares estimate, w_{opt}, is the best linear unbiased estimate. Finally, when the additive noise in $x(k)$ is white and Gaussian with zero mean, the least-squares estimate achieves the Cramer-Rao lower bound for unbiased estimates. In addition to these advantages, there are some drawbacks, which are as follows:

(i) Computation of the matrix $[R]$, used in the evaluation of the optimum weights in Equation (21), is an $(N+1)^2 \bullet (I-N+1)$ process, which is difficult to carry out in real time.

(ii) Computation of $[R]^{-1}$ can also be expensive, and computationally unstable. For example, evaluation of the inverse requires an $O(N+1)^3$ process, as the dimension of $[R]$ is $N+1$. In addition, in the absence of noise, $[R]$ is singular. The presence of additive noise may make it nonsingular, but this could be numerically unstable.

(iii) In the evaluation of the elements of matrix $[R]$, a time averaging is carried out, as shown in Equation (16). Hence, if there are intermittent (blinking) jammers or a coherent multipath then this method cannot eliminate them. Coherent multipath depicts a signal $s(t)$ in terms of the multipath, $gs(t)$. If g is -1, then (complete) fading occurs and the signal is cancelled and, hence, the adaptive technique cannot reconstitute the signal, as the multipath can only be detected in the spatial domain of the arrays.

(iv) Inherent in this development is the assumption that the signal of interest is arriving from an angle θ_s. However, due to misadjustment or for some other reasons, the signal may be arriving from an angle $\theta_s + \Delta\theta$, and not exactly at θ_s. In this case, the adaptive processor considers the actual signal at $\theta_s + \Delta\theta$ as a jammer, and cancels it. This issue results in a problem of signal cancellation due to mismatch. A possible solution is to have a number of constraints, instead of a single constraint as given by Equation (11). This is equivalent to defining the number of constraints required to characterize the 3 dB beamwidth of the adaptive array. This would require modifying Equation (13) with a number of Lagrange multipliers for a number of points as constraints defining the 3 dB beamwidth.

In the next section, an alternate methodology is presented where many of these problems can be mitigated. This new method is based on spatial analysis of the data, rather than dealing with the time variable. Therefore, we are processing the data on a snapshot-by-snapshot basis. A snapshot is defined as consisting of the voltages induced in the $N+1$ elements of the array at a particular time instance $t = T_0$ (say, for example).

5. Spatial-Domain Least-Squares Approach

In the conventional adaptive-beam-forming methodology, we have assumed that the weights are applied to each of the antenna elements, and the processing information is generated over time, as the correlation matrix, $[R]$, of the data needs to be formed [as represented by Equations (16) and (18)]. We have seen, in the previous section, that this may have some shortcomings, which we would like to counteract. Hence, in the current development we deal with a single frame or a single snapshot. A single snapshot is defined as the complex voltages, V_n, measured at each one of the $(N+1)$ antenna elements at a particular instant of time. These measured voltages, V_n, $n = 0,1,\ldots,N$, contain the desired signal, jammer, clutter, and thermal noise components. Hence, in this development, one can allow for blinking jammers, time-varying clutter, and coherent multipath components. The price one pays for dealing with a snapshot/frame is that the degrees of freedom are limited to $N/2$, as opposed to $N+1$ in the previous covariance-matrix-based approach. However, this serious limitation will be alleviated later on, where we show that we can essentially double the data-set size via processing. The number of weights increase by 50%, thereby achieving close to the same number of degrees of freedom as outlined before, without sacrificing the flexibility of spatial processing on a snapshot-by-snapshot basis. In this new approach, we utilize the idea of Frost [12] by forming a matrix generated by taking the weighted differences between neighboring measured voltages. The weights are related to the direction of arrival of the signal, and are quite separate from the adaptive weights, $w(k)$, used in beam-forming.

5.1 Direct Method Based on Solution of an Eigenvalue Equation [13, 14, 17]

Consider the same linear array of $N+1$ uniformly spaced elements as shown in Figure 2. Let us assume that the signal is coming from θ_s, and our objective is to estimate its amplitude. Let us define by S_n the complex voltage received at the nth element of the linear array due to a signal of unity amplitude coming from a direction θ_s. [For notational convenience and to differentiate the methodology from the MVDR technique, we use the subscript n to represent a voltage at an element, whereas the index within the bracket represents the time instance. Thus, S_n is really $S_n(k)$, where S_n is the voltage at the nth sensor measured at the kth instance of time. Since we are going to deal with a single snapshot, i.e., all the values are measured at the kth instance of time, the term (k) has been dropped from all the variables]. The signal-induced voltages are under the assumed array geometry and narrowband signal, a complex sinusoid. Let X_n be the complex voltages that are measured at the nth element due to the actual signal of complex amplitude α, jammers which may include multipaths of the actual signal, clutter (which is the reflected electromagnetic energy from the surrounding environment), and thermal noise. If we now form the matrix pencil consisting of matrices of dimension $(M+1)$, we have

$$[X] - \alpha[S], \tag{23}$$

where

$$[X] = \begin{bmatrix} X_0 & X_1 & \cdots & X_M \\ X_1 & X_2 & \cdots & X_{M+1} \\ \vdots & & & \\ X_M & X_{M+1} & \cdots & X_N \end{bmatrix}_{(M+1)\times(M+1)} . \tag{24}$$

Then, the difference at each element, $X_n - \alpha S_n$, represents the contribution due to signal multipaths, jammers, and clutter (i.e., all noise components except the signal). It is interesting to observe that in this procedure, $N = 2M$, and the total number of antenna elements, $N+1$, is always odd. This is because if there are P jammers, then we have in total $2P+1$ unknowns to deal with. For each jammer, the direction of arrival and its complex amplitude are unknown, and that accounts for the $2P$ terms. Now, for the signal, we know the direction of arrival, but do not know its strength. Hence, the +1 term takes care of the unknown signal strength. Therefore, the total number of unknowns is always $2P+1$ in this procedure, and so $N+1$ is an odd number.

Note that the elements of the matrices in Equation (23) are of the following form. Let S_n be the voltage induced in the antenna element n due to the incident wave of unit amplitude:

$$S_n = e^{j2\pi\frac{(n-1)d}{\lambda}\cos\theta_s}, \tag{25}$$

and let X_n be the voltage induced in the antenna element n due to the signal, jammers, clutter C_n, and thermal noise, z_n:

$$X_n = s e^{j2\pi\frac{(n-1)d}{\lambda}\cos\theta_s} + \sum_{p=1}^{P} A_p e^{j2\pi\frac{(n-1)d}{\lambda}\cos\theta_s} + C_n + z_n, \tag{26}$$

where A_p and θ_p are the amplitude and direction of arrival of the pth jammer signal. It is assumed that there are P such jammers and $P \le N/2$, and that C_n is the contribution due to clutter, and z_n is the thermal noise at the antenna elements.

Now, in an adaptive processing, the weights, $[W]$, are chosen in such a way that the contribution from the jammers, clutter, and thermal noise are equated to zero. Hence, if we define the following generalized eigenvalue problem,

$$[U]_{(M+1,M+1)}[W]_{(M+1)\times 1} = \{[X] - \alpha[S]\}_{(M+1)\times(M+1)} \cdot [W]_{(M+1)\times 1} = 0, \tag{27}$$

then α, which will be equal to s of Equation (26) (the strength of the signal), is given by the generalized eigenvalue, and the weights $[W]$ are given by the generalized eigenvector. Since we have assumed that there is only one signal arriving from θ_s, the matrix is of rank unity, and hence the generalized eigenvalue equation, given by Equation (27), has only one eigenvalue, and that eigenvalue provides the strength of the signal.

Alternately, one can view the left-hand side of Equation (27) as the total noise signal at the output of the adaptive processor due to jammer, clutter, and thermal noise. Hence, the total noise is

$$N_{\text{out}} = [U][W] = \{[X] - \alpha[S]\}[W]. \tag{28}$$

Therefore, the total noise power is given by

$$N_{power} = [W]^H \{[X] - \alpha[S]\}^H \{[X] - \alpha[S]\}[W]. \tag{29}$$

Our objective is to set the noise power to zero by selecting $[W]$ for a fixed signal strength α. This yields Equation (27).

From a computational point of view, one could alternately look at solving for α by making the determinant of the matrix

$$\det\{[X] - \alpha[S]\} = 0 \tag{30}$$

for a suitable value of α. For lengthy and unstable (as the matrix S is of rank one and not positive definite) computational reasons, we reformulate the problem in terms of the solution of a matrix equation.

In real-time applications, it may be difficult to solve the generalized eigenvalue problem in an efficient way, particularly if the dimension, M–the number of weights–is large. Also, when $[S]$ is of rank one, it may be numerically unstable to solve the generalized eigenvalue problem. For that reason, we convert the solution of a nonlinear eigenvalue problem in Equation (27) to the solution of a linear matrix equation.

5.2 Direct Methods Based on the Solution of the Matrix Equations

5.2.1 Forward Method

Note that the (1,1) and (1,2) elements of the noise matrix, $[U]$, are given by

$$U(1,1) = X_0 - \alpha S_0, \tag{31}$$

$$U(1,2) = X_1 - \alpha S_1, \tag{32}$$

where X_0 and X_1 are the voltages received at antenna elements 0 and 1 due to signal, jammer, clutter, and noise, whereas S_0 and S_1 are the values of the signals only, at those elements, due to a signal of unit strength. Define

$$Z = \exp\left[j2\pi \frac{d}{\lambda} \cos\theta_s\right]. \tag{33}$$

Then, $U(1,1) - Z^{-1}U(1,2)$ contains no components of the signal, as

$$S_0 = \exp\left[j2\pi \frac{id}{\lambda} \cos\theta_s\right] \text{ with } i = 0, \tag{34}$$

and

$$S_1 = \exp\left[j2\pi \frac{d}{\lambda} \cos\theta_s\right] \text{ with } i = 1. \tag{35}$$

Therefore, one can form a reduced-rank matrix $[T]_{(M-1)\times M}$, generated from $[U]$ such that

$$[T] = \begin{bmatrix} X_0 - Z^{-1}X_1 & X_1 - Z^{-1}X_2 & \cdots & X_M - Z^{-1}X_{M+1} \\ \vdots & \vdots & \vdots & \vdots \\ X_{M-1} - Z^{-1}X_M & X_M - Z^{-1}X_{M+1} & \cdots & X_{N-1} - Z^{-1}X_N \end{bmatrix}_{M\times(M+1)} = [0]. \tag{36}$$

In order to restore the signal component in the adaptive processing, we fix the gain of the subarray formed by evaluating a weighted sum of the voltages $\sum_{i=0}^{M} W_i X_i$. Let us say the gain of the subarray is C in the direction of θ_s. This provides an additional equation, resulting in

$$\begin{bmatrix} 1 & Z & Z^2 & \cdots & Z^M \\ X_0 - Z^{-1}X_1 & & & & X_M - Z^{-1}X_{M+1} \\ \vdots & & \vdots & & \vdots \\ X_{M-1} - Z^{-1}X_M & & \cdots & & X_{N-1} - Z^{-1}X_N \end{bmatrix}_{(M+1)\times(M+1)} \bullet \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_M \end{bmatrix}_{(M+1)\times 1} = \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(M+1)\times 1} \tag{37}$$

or, equivalently,

$$[F][W] = [Y]. \tag{38}$$

Once the weights are solved for by using Equation (37), the signal component, α, may be estimated from

$$\alpha = \frac{1}{C} \sum_{i=0}^{M} W_i X_i. \tag{39}$$

The proof of Equation (39) is available in [14].

It is also possible to estimate α from any of the following $M+1$ equations:

$$\alpha = \frac{1}{C Z^k} \sum_{i=0}^{M} W_i X_{i+k} \text{ for } k = 0, \ldots, M, \tag{40}$$

or by averaging any one of the equations given by the set of $M+1$ equations in Equation (40). However, it is interesting to note that because of Equation (36), averaging $M+1$ estimates of α obtained from Equation (40) is no better than using Equation (39)!

As noted in [14, 15], Equation (37) can be solved very efficiently by applying the conjugate-gradient method, which may be implemented to operate in real time utilizing a DSP32C signal-processing chip [15, 16].

For the solution of $[F][W]=[Y]$ in Equation (38), the conjugate-gradient method starts with an initial guess, $[W]_0$, for the solution, and lets [16]

$$P_0=-b_{-1}[F]^H[R_0]=-b_{-1}[F]^H\{[F][W_0]-[Y]\}, \quad (41)$$

where H denotes the conjugate transpose of a matrix. At the nth iteration, the conjugate-gradient method develops the following:

$$t_n=\frac{1}{\left\|[F][P]_n\right\|^2}, \quad (42)$$

$$[W]_{n+1}=[W]+t_n[P]_n, \quad (43)$$

$$[R]_{n+1}=[R]_n+t_n[F][P]_n, \quad (44)$$

$$b_n=\frac{1}{\left\|[F]^H[R]_{n+1}\right\|^2}, \quad (45)$$

$$[P]_{n+1}=[P]_n-b_n[F]^{\mathrm{H}}[R]_{n+1}. \quad (46)$$

The norm is defined by

$$\left\|[F][P]_n\right\|^2=[P]_n^H[F]^H[F][P]_n. \quad (47)$$

The above equations are applied in a routine fashion until the desired error criterion for the residuals, $\left\|[R]_n\right\|$, is satisfied. In our case, the error criterion is defined as

$$\frac{\left\|[F][W]_n-[Y]\right\|}{\|\mathrm{Y}\|}\le 10^{-6}. \quad (48)$$

The iteration is stopped when the above criterion is satisfied.

The computational bottleneck in the conjugate-gradient method is the computation of the matrix-vector product in $[F][P]_n$, and in the computation of $[F]^H[R]_{n+1}$. Typically, matrix-vector products in real-time computations can slow down the process. However, in our examples, these bottlenecks can be streamlined through the utilization of the block-Hankel structure in the matrix. The matrix-vector product can be carried out efficiently through the use of the fast Fourier transform (FFT) [16]. This is accomplished as follows.

Consider the following matrix-vector product:

$$\begin{bmatrix} f_1 & f_2 & f_3 \\ f_2 & f_3 & f_4 \\ f_3 & f_4 & f_5 \end{bmatrix}_{(k\times k)} \bullet \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}_{(k\times 1)}. \quad (49)$$

This is usually accomplished in K^2 operations, where K is the dimension of the matrix. However, since the matrix has a Hankel structure, we can write it as the convolution of two sequences, so that $\{f\}*\{w\}=\{f_1 \;\; f_2 \;\; f_3 \;\; f_4 \;\; f_5\}*\{w_3 \;\; w_2 \;\; w_1 \;\; 0 \;\; 0\}$, and considering the last three elements of the convolution, which provides the correct matrix-vector product. Hence, the total operation, in this case, is $\mathrm{FFT}^{-1}\left[\mathrm{FFT}\{f\}\bullet\mathrm{FFT}\{w\}\right]$. This results in a total operation count of $3[2K-1]\log[2K-1]$. For K typically greater than 30, this procedure becomes quite advantageous, as the operation count is of the order of ($K\log K$), as opposed to K^2 for a conventional matrix-vector product. Also, in this new procedure there is no need to store an array, and so time spent in accessing the elements of the array on the disk is virtually nonexistent: everything is now one-dimensional and can be stored in the main memory. This procedure is quite rapid, and easy to implement in hardware [15].

5.2.2 Backward Procedure

It is well-known in the parametric spectral-estimation literature that a sampled sequence can be estimated either by observing it in the forward direction or in the reverse direction. If we now conjugate the data and form the reverse sequence, then one gets an equation similar to Equation (37) for the solution of the weights W_m:

$$\begin{bmatrix} 1 & Z & \cdots & Z^M \\ X_N^*-Z^{-1}X_{N-1}^* & X_{N-1}^*-Z^{-1}X_{N-2}^* & \cdots & X_M^*-Z^{-1}X_{M-1}^* \\ \vdots & \vdots & \cdots & \vdots \\ X_{M+1}^*-Z^{-1}X_M^* & X_M^*-Z^{-1}X_{M-1}^* & \cdots & X_1^*-Z^{-1}X_0^* \end{bmatrix}_{(M+1)\times(M+1)} \bullet \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_M \end{bmatrix}_{(M+1)\times 1} = \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(M+1)\times 1}, \quad (50)$$

or, equivalently,

$$[B][W]=[Y]. \quad (51)$$

The signal strength, α, can again be determined by Equation (39) or (40), once Equation (51) is solved for the weights. C is the assumed gain of the antenna array along the direction of the arrival of the signal. There is no loss of generality by assuming $C=1$. This is because this factor also appears in the evaluation of the signal strength, α, in Equation (39).

Note that in both the two cases, 5.2.1 and 5.2.2, $M=N/2$.

5.2.3 Forward-Backward Method

In the forward-backward model, we double the amount of data by not only considering the data in the forward direction, but by also conjugating it and reversing the direction of increment of the independent variable. This type of processing can be done as long as the series to be approximated can be fit by exponential functions of purely imaginary argument. This is always true for the adaptive-array case. So by considering the data sets $x(k)$ and $x^*(-k)$ we have essentially doubled the amount of data without any penalty, as these two data sets for our problem are linearly independent. So in this case, there can be a free lunch, after all!?

An additional benefit accrues in this case. For both the forward and the backward method, the maximum number of weights we can consider is given by $N/2$, where $N+1$ is the number of antenna elements. Hence, even though all the antenna elements are being utilized in the processing, the number of degrees of freedom available for the new approach is essentially $N/2$. For the forward-backward method, the number of degrees of freedom can be significantly increased without increasing the number of antenna elements. This is accomplished by considering the forward and backward versions of the array data. For this case, the number of degrees of freedom can reach $N/1.5+1$. This is approximately equal to 50% more weights or degrees of freedom than for the two previous cases. The equation that needs to be solved for the weights is given by combining Equations (37) and (50) into

$$\begin{bmatrix} 1 & Z & \cdots & Z^M \\ X_0 - Z^{-1}X_1 & X_1 - Z^{-1}X_2 & \cdots & X_M - Z^{-1}X_{M+1} \\ \vdots & \vdots & \vdots & \vdots \\ X_{M-1} - Z^{-1}X_M & X_M - Z^{-1}X_{M+1} & \cdots & X_{N-1} - Z^{-1}X_N \\ X_N^* - Z^{-1}X_{N-1}^* & X_{N-1}^* - Z^{-1}X_{N-2}^* & \cdots & X_M^* - Z^{-1}X_{M-1}^* \\ \vdots & \vdots & \vdots & \vdots \\ X_{M+1}^* - Z^{-1}X_M^* & X_M^* - Z^{-1}X_{M-1}^* & \cdots & X_1^* - Z^{-1}X_0^* \end{bmatrix}_{(M+1)\times(M+1)} \bullet \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_M \end{bmatrix}_{(M+1)\times 1} = \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(M+1)\times 1} . \quad (52)$$

The value of M in Equation (52) is now much greater than the value of M in Equations (37) and (50), since in Equation (52), the total amount of data is now doubled. This has been achieved by considering both the forward and the reverse form of the data sequence. In summary, in a conventional adaptive technique, where there is a weight attached to each element and the processing is done in time, the number of degrees of freedom is $N+1$, provided the environment is stationary in time. In the proposed spatial processing, based on a snapshot-by-snapshot analysis, the number of degrees of freedom is $N/1.5+1$, and the processing is very flexible since one can pre-fix the beamwidth of the receiving array, as shown later. However, both are least-squares-based approaches. The advantage of doing snapshot-by-snapshot processing is that the stationarity assumption of the data can be relaxed. The disadvantage is that the number of degrees of freedom is slightly less.

5.2.4 Examples

A set of examples has been chosen where the application of a conventional stochastic methodology may not yield satisfactory results.

As a first example, we consider the performance of the various methods due to clutter and thermal noise. For the example, we assume a signal of unity amplitude arriving from $\theta_S = 90°$, impinging on a 19-element array, where the elements are assumed to be a half wavelength apart. So, the antenna beamwidth in this case is approximately 5.5°. We consider clutter arriving at the array from $\theta = 0.1°$ to 85°, and from $\theta = 95°$ to 179°. Here, clutter is modeled by a single plane wave with a complex amplitude that is random. So, the clutter patches contain many specular electromagnetic reflections, which are arriving in azimuth 0.1° apart, with a complex amplitude determined by two random-number generators. The amplitude is determined by a uniformly distributed random-number generator, with values distributed between 0 and 1. The phase is also determined by a uniformly distributed random-number generator, with values between 0 and 2π. In addition, we introduce thermal noise at each of the antenna elements, which is assumed to be uniformly distributed in magnitude between 0 and 1; the phase of the complex signal due to thermal noise is chosen between 0 to 2π. The signal-to-total-thermal-noise power is +23 dB at the array. Figure 3 provides the output signal-to-noise ratio resulting from the various methods, as a function of signal-to-clutter ratio, in dB. Figure 3 illustrates that if the input signal-to-clutter ratio in the array is −10 dB, and if we use the forward or the backward method (namely, if use either Equation (37) or (50)) to do the processing, then the processed output signal-to-noise ratio is about +5 dB. The eigenvalue method, described by Equation (27), also yields a similar value. However, if we utilize the forward-backward method to do the processing (namely, by using Equation (52)), then the processed output signal-to-noise ratio is +8.2 dB. The difference in the processed output signal-to-noise ratio between the forward method (FRW) or the eigenvalue method and the forward/backward method (FB) becomes much larger as the signal-to-clutter ratio increases.

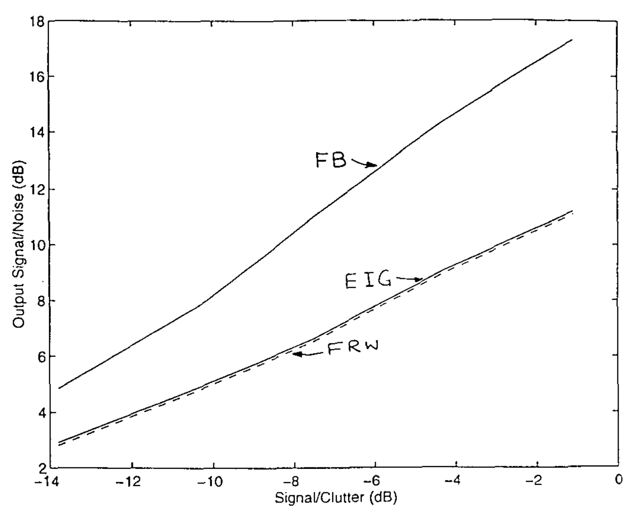

Figure 3. The output signal-to-noise ratio as a function of the input signal in clutter and thermal noise, for the direct-domain least-squares approach.

As a second example, consider the same 19-element array, arranged in such a way so as to receive a signal of 0 dB from 90°. In addition, we have a 69-dB jammer coming from $\theta = 140°$, a 50-dB jammer arriving from $\theta = 95°$, a 60-dB jammer arriving from $\theta = 85°$, and a 56.5-dB jammer arriving from $\theta = 20°$. We also have two clutter patches. The first clutter patch is located from 0.1° to 30°, and is modeled by discrete scatterers located every 0.1°. The second clutter patch extends from 35° to 59°. The complex amplitudes for the point-source clutter returns are generated by two uniformly distributed random-number generators, as outlined before. The total signal-to-clutter ratio is −13.2 dB. In addition, we have thermal noise at each of the antenna elements. The total signal-to-thermal noise at the array is 23 dB. The beamwidth of the antenna is approximately 5.5°. If we utilize the forward-backward (FB) method to do the processing, with the only a priori information being that the signal is arriving from $\theta = 90°$, the processed output signal-to-interference-plus-noise ratio is 26.6 dB. If we use either the forward (FRW) or the backward method, the processed output signal-to-noise ratio is 13.4 dB, whereas for the eigenvalue method (EIG), it is 13.41 dB.

As a third example, consider the same 19-element array receiving a signal of strength 0 dB from $\theta = 95°$. In addition, we have a 50.5-dB jammer coming from $\theta = 50°$, a 60-dB jammer arriving from $\theta = 80°$, a 56.5-dB jammer arriving from $\theta = 70°$, and a 69-dB jammer arriving from $\theta = 20°$. In addition, we have two clutter patches. The first clutter patch is located from $\theta = 15°$ to 50°, and is modeled by discrete scatterers separated in azimuth by 0.1°, with complex amplitudes that are considered random and generated by two uniformly distributed random-number generators. In addition, we have a clutter patch from $\theta = 100°$ to 130°, modeled by discrete scatterers every 0.1° apart. The total signal-to-clutter ratio at the array is −13.2 dB. In addition, we have thermal noise at each of the antenna elements, and the total signal-to-thermal noise at the antenna array is 23 dB. If we utilize the forward-backward method (FB) to do the processing, then the output signal-to-noise ratio at the output is given by 7.4 dB. In contrast, if the processing is done by the eigenvalue method (EIG), the processed output is 1.01 dB, whereas for the forward method (FRW) it is 1.01 dB.

For all the examples, it is seen that the forward-backward equations given by Equation (52) yield a much higher output signal-to-noise ratio than the result given by any of the other methods. This is to be expected. Now, the problem occurs if we increase the number of antenna elements and if we further assume that the direction of arrival of the signal is not exactly $\theta = 95°$, but instead is from $\theta_s \pm \Delta\theta$, where $\Delta\theta$ is not known a priori. The slight deviation in the direction of arrival can also be due to atmospheric refraction. The processed result will not be very good, as all the methods will not find any signal exactly at θ_s. There will, in fact, be signal cancellation. To alleviate such problems of signal cancellation when there is uncertainty in knowing, a priori, the direction of arrival, θ_s, of the signal, we utilize the main-beam constraints as described in the next section.

6. Main-Beam Constraints for Prevention of Signal Cancellation

So far, we have addressed the problem of eliminating unwanted jammers to extract the signal from an arbitrary look direction. However, in practice, the expected signals (target returns) can occur over a finite angular extent. For example, in the radar case, the angular extent is established by the main beam of the transmitted wave (usually between the 3 dB points of the transmitted field pattern). Target returns within the angular extent must be coherently processed for detection, and estimates must be made of target Doppler and angle. Adaptive processing that impacts these processes will lead to unacceptable performance. Correction for this effect is accomplished in the least-squares procedures by establishing look-direction constraints at multiple angles within the transmitter main-beam extent. The multiple constraints are established by using a uniformly weighted array pattern for the same size array as the adaptive array under consideration. Multiple points are chosen on the non-adapted array pattern, and a row is implemented in the matrix equations of Equations (37), (50), and (52) at each of the desired angles; the corresponding uniform complex antenna gains are placed in the Y vector of Equations (37), (50), and (52). Hence, for this problem, the size of the matrix U, for example, is established by the following. Let

L = the number of look-direction constraints
$M+1$ = the number of weights to be calculated.

Therefore, $M - L + 1$ = the number of jammers that can be nulled. The first canceling equation uses data from the $M+1$ elements, and each successive canceling equation is shifted by one element. Therefore, $N - M$ equations are required to effectively use the data from $N+1$ elements. Thus, there are L constraint equations and $N - M$ canceling equations for the case of the forward method described by Equations (37) and (38). The number of equations must equal the number of weights; therefore,

$$M = L + N - M . \tag{53}$$

This leads to the relationship among the number of weights, the number of constraints, and the number of elements:

$$N = 2M - L . \tag{54}$$

Similar constraints can be applied to the backward method and to the forward/backward method.

6.1 Examples

To illustrate the effectiveness of the least-squares approach to the adaptive-array problem, we consider an array of $N+1 = 21$ antenna elements, and we employ the forward method. For all the examples, the value of N will be fixed. The performance across the main beam will be compared for the cases of one, three, and five look-direction constraints. This leads to the following relationships:

- $N+1 = 21$, $L = 1$, and so $M+1 = 11$, and ten jammers can be cancelled;
- $N+1 = 21$, $L = 3$, and so $M+1 = 12$, and nine jammers can be cancelled;
- $N+1 = 21$, $L = 5$, and so $M+1 = 13$, and eight jammers can be nulled.

As an example, consider a target at 94°, and let the main-beam look-direction constraint be placed at 90°. It is seen from the main-beam array pattern, depicted in Figure 4, that the target at 94° has been nulled out. In Figures 5, 6, and 7, the complex array gain is shown for one, three, and five main-beam constraints in the same sets of random noise generated at the 21 elements. For the three cases, the array gain in the target direction (denoted by x in the figures) is reduced more in the one-constraint case (Figure 5) than for the two-constraint case (Figure 6) or the five-constraint case (Figure 7). Also, the 10 vectors for the different simulations of noise are less randomly distributed for the five-constraint case and, hence, some coherent integration gain is possible. For the three-constraint case, the constraints are placed at 85°, 90°, and 95°. For the five-constraint case, the main-beam constraints are placed at 85°, 87.5°, 90°, 92.5°, and 95°.

Thus, either the first three or the first five rows of the matrix $[F]$ of Equation (37) or the matrix $[B]$ of Equation (51) are of the same form as the first row of the matrices defined above, but with the appropriate steering vector. The excitation function, $[Y]$, would have 1, 3, or 5 nonzero elements, respectively, depending on the number of constraints used for the main beam. For the five-constraint case, $[Y]$ would be of the form

$$[Y]^{\mathrm{T}} = [13,\ 7.72 + j8.32,\ 7.72 - j8.32,\ -0.816 + j7.149,\ -0.816 - j07.149,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0].$$

It is seen that for the five-constraint case, there is no loss in array gain, and the vectors from the 10 different runs are very nearly aligned. The five-constraint approach would permit effective radar processing across the main-beam extent with little loss in performance. For example, Figure 7 shows the main-beam gain in the presence of three jammers, with five constraints in the main beam.

As the signal strength is increased, the distortion of the main beam increases. The above results have been generated, utilizing a 20 dB signal-to-noise ratio per channel per pulse. This would be a strong radar return under most circumstances. Simulation results

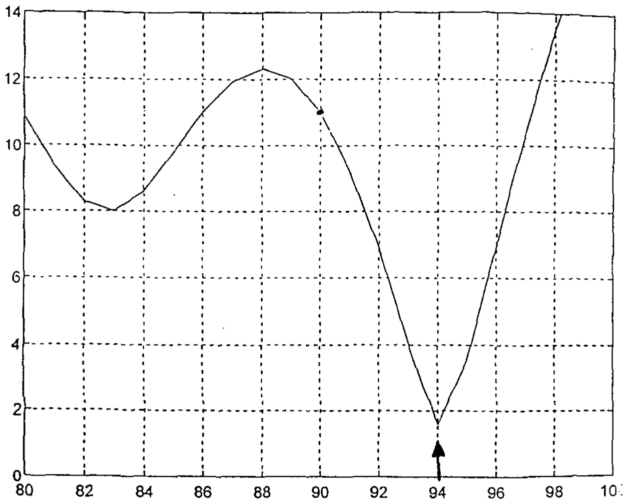

Figure 4. The main-beam gain of the array.

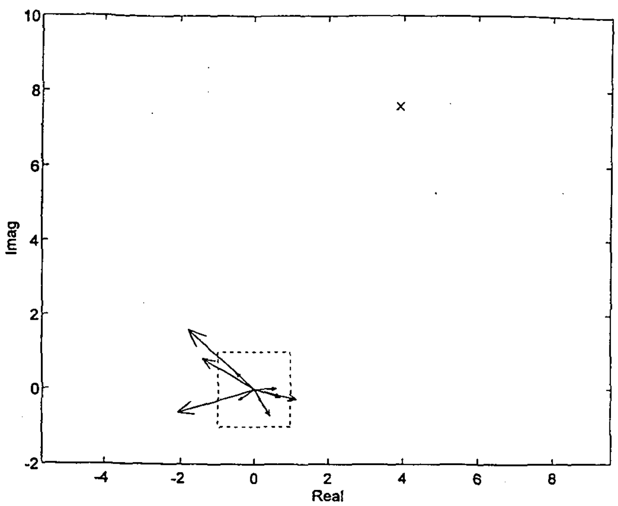

Figure 5. The complex array gain: one-constraint case.

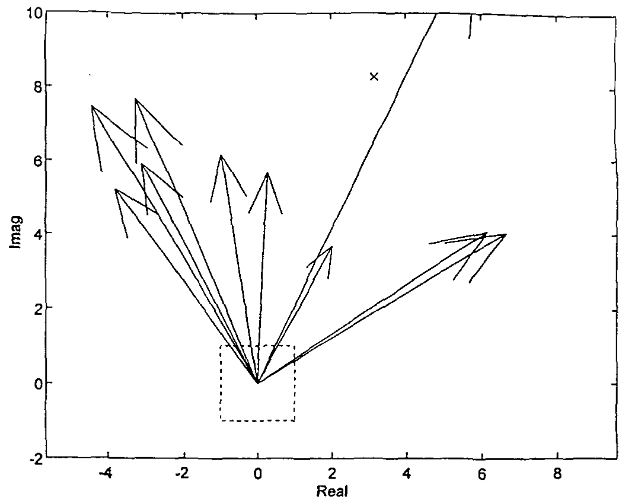

Figure 6. The complex array gain: three-constraint case.

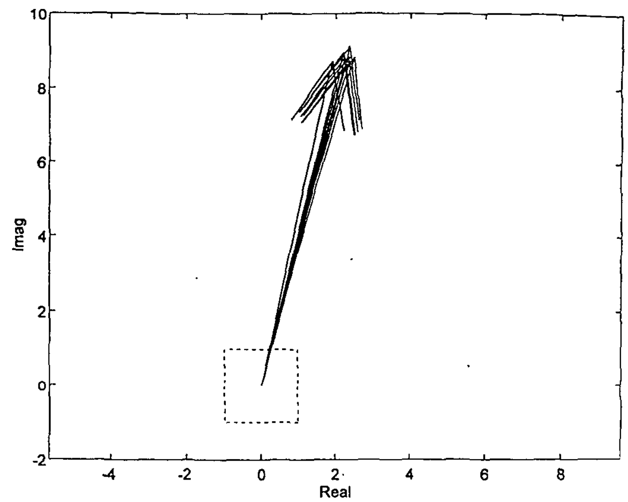

Figure 7. The complex array gain: five-constraint case.

indicate that the five-constraint approach is still effective at a 40 dB signal-to-noise ratio, but that it breaks down at a 60 dB signal-to-noise ratio.

In summary, the main-beam constraint allows a look-direction constraint to be established over a finite beamwidth, while maintaining the ability to adaptively null jammers in the sidelobe region. Although the main-beam gain can become degraded if the signal becomes very strong, this does not appear to be a serious limitation for practical radar-processing cases.

7. Minimum Norm Property of the Optimum Weights

The optimum weights are obtained as a minimum-norm solution of Equation (27) when the assumed direction of arrival coincides exactly with the actual direction of arrival. One of the open problems is how to exactly estimate the direction of arrival of the signal when there is uncertainty associated with the assumed direction of arrival. It has been our experience that the norm of the weights can be used to accurately estimate the direction of arrival of the signal, if that is necessary. Hence, this method can be used as a multiple-step approach that can arrive at a good approximation to the optimum weights, i.e., the weights that would be obtained if the arrival angle of the target return were known exactly. This approach will evaluate the weights at multiple values of the angle assumed to be the correct value in the canceling equations, but will make decisions only on the values of the weights, and will accomplish the detection process only once.

As a step to further developing this approach, consider the data in Figures 8 through 10. Again, the simulation is implemented for a 21-element array, and weights are generated for 13 elements using the five-constraint algorithm. In each, the simulation is repeated for five samples of noise at each element. With the five

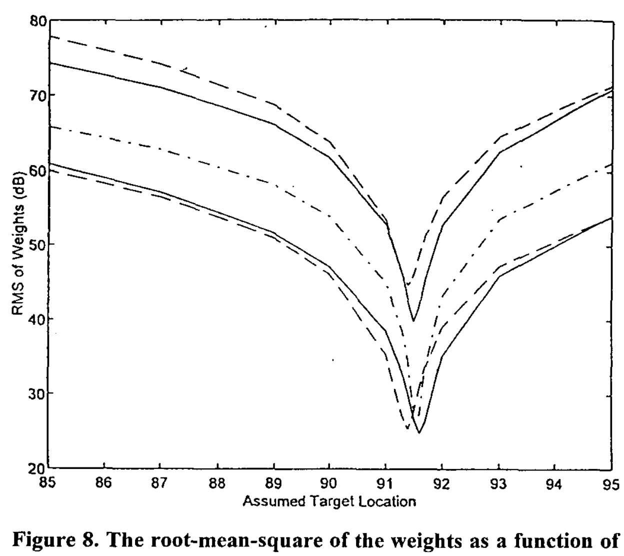

Figure 8. The root-mean-square of the weights as a function of the assumed target location, for a strong target at 91.5°, in the presence of jammers and noise. The five curves represent five different simulations of the problem (different random numbers with the same statistics).

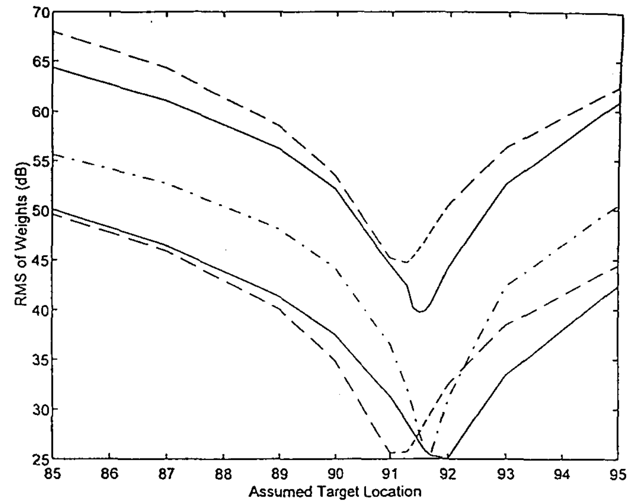

Figure 9. The root-mean-square of the weights as a function of the assumed target location, for a moderate-strength target at 91.5°, in the presence of jammers and noise. The five curves represent five different simulations of the problem (different random numbers with the same statistics).

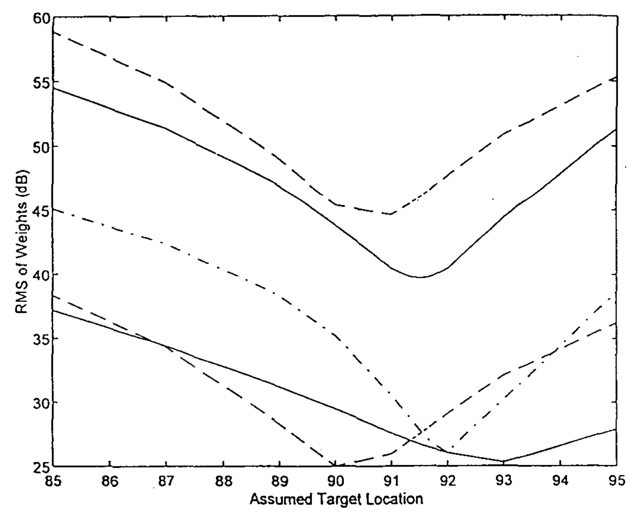

Figure 10. The root-mean-square of weights as a function of the assumed target location, for a weak target at 91.5°, in the presence of jammers and noise. The five curves represent five different simulations of the problem (different random numbers with the same statistics).

constraints in place (at 85°, 87.5°, 90°, 92.5°, and 95°), the assumed location of the target return is varied across the main beam of the array, the adaptive process is implemented, and the sum of the absolute values of the weights is calculated. The actual location of the target is 91.5° in all cases. In these cases (Figures 8 through 10), in addition to target return and noise, there are three jammers present. Figure 8 shows the results for a very strong target return (20 dB S/N at each element). The minimum of the sum of the weights is obtained very close to the true target direction in all five samples of receiver noise. The five different curves in the figures represent five different simulations of the problem. The jammers are effectively nulled in all cases, and the only effect of receiver noise is that the target-return angle is estimated to be between 91.4° and 91.6°, instead of the true 91.5°. The location of the minimum of the weights is easy to identify with a ratio of the largest sum at any location across the main beam to the minimum of about 50. The multiple-constraint approach is thus successful in identifying the true target direction, and the detection process could be implemented optimally at this angle.

Figure 9 repeats the process for a moderate target return (10 dB S/N at each element). The effects of receiver noise are more significant in this case. The estimates of the target location have greater spread (the estimate varies between 91° and 92°). The location of the minimum is still easy to identify, but the ratio between the maximum and the minimum is now only about 20. Figure 10 shows the data for a weak target return (0 dB S/N). The estimated target locations have even greater spread (between 90° and 93°). The plots of the sum of weights now has a relatively broad null, and the ratio between the maximum and minimum is now about 5.

The existence of a minimum in the sum of the weights can be used to estimate the target-return angle. It could also be used to perform the detection process, as well. If there is a large ratio between the minimum value and the maximum value of the sum of the weights across the main beam, then that is an indication that a target is present. The strongest linear progression of the random-noise samples sets a lower limit on that detection process. That component of the random-noise samples that has a linear progression across the array appropriate to a far-field source in a main-beam direction will detect a weak target.

Therefore, the sum of the adaptive weights varies as the value of the assumed target direction is varied across the main beam. When a strong target is present, the ratio between the largest sum of weights and that at the target direction is large. Then, when there is no target present, the ratio between the largest sum and the smallest sum is small. This could lead to a more-accurate estimation of the direction of arrival of the signal or to a more-accurate detection process when this information is not available a priori.

8. Conclusions

A direct data-domain approach, based on the spatial samples of the data, has been presented. In this approach, the adaptive analysis is done on a snap-shot-by-snap-shot basis, and therefore non-stationary environments can be handled quite easily, including coherent multipath environments. This is in contrast to conventional adaptive techniques, where processing is done by taking the time averages, as opposed to the spatial averages. Associated with adaptive processing is the same a priori knowledge about the nature of the signal, which, in this case, is the direction of arrival. The assumption that the target signal is coming from an exactly known direction will probably never be met in any real array. In communications systems, the location of the transmitter may be known only approximately, or the propagation of the signal through the atmosphere may distort the wavefront such that it appears to be coming from a slightly different direction. For example, diffraction could cause enough of an elevation-angle error to be important for some systems. Or, the adaptive receive array may be surveyed into location with small errors, and thus the angle to the transmitter from the broadside of the array will be in error. Other applications of adaptive arrays will also have at least small errors in the direction of arrival of the desired signal. In this paper, two methodologies are presented to treat this signal-cancellation problem. One is through the main-beam constraint, and the other is through the norm of the weight vectors. It has been illustrated that the norm of the adapted weights is a monotonically increasing function of the separation between the assumed direction of the target and the actual direction of the target. It is thus possible to find the true angle of arrival of the desired signal by minimizing the sum of the absolute values of the weights. Once this true angle is known, an optimum set of weight vectors could be formed using the existing algorithm.

A number of issues associated with the direct least-squares algorithm need further investigation. The assumption of a linear wavefront means that effects of mutual coupling between subarrays, and near-field scattering from the structure upon which the array is mounted, have not been taken into account in this presentation. Initial results, however, may be available in [25]. The detection process for this algorithm has only been introduced, and much more work is required to develop an optimum approach.

However, the advantage of this direct data-domain least-squares approach, based on spatial processing of the array data, may provide beneficial over the conventional adaptive techniques, utilizing time averaging of the data. This will be quite relevant in a non-stationary environment.

9. Acknowledgement

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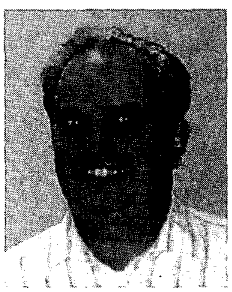

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She has authored a total of 10 contributions for chapters and articles in books, 15 papers in international journals, and 75 papers in international symposiums and workshops, plus a number of national publications and reports. She has lectured in several short courses, some of them in the framework of European Community Programs. She has participated in 19 projects and contracts, financed by international, European, and national institutions and companies. She has been a member of the Technical Program Committee of several international symposiums, and has acted as reviewer for international scientific journals. She has assisted the Comisión Interministerial de Ciencia y Tecnología (National Board of Research) in the evaluation of projects. She has also served in several evaluation panels of the Commission of the European Communities. She has acted in the past as topical Editor for the disk of references of the *Review of Radio Science*. She is a member of the editorial board of two scientific journals. She has served as Vice Chair and Chair of the IEEE MTT/AP-S Spanish Joint Chapter, and is currently serving as Chair of the Spain Section of the IEEE. She has received two individual research awards from national institutions.

IEEE Pre-College Engineer Training Web Site

The IEEE has announced the opening of the IEEE/EAB (Educational Activities Board) Pre-College Engineer Training (PET) Web site. PET is dedicated to an alliance of engineers and educators for the promotion of science and technology education. PET offers specific plans and step-by-step procedures to prepare engineers to serve as resources to pre-college educators, both in and out of the classroom. This site can be accessed through the IEEE Educational Activities Home Page, or at http://www.ieee.org/organizations/eab/pet.

Although open, this site remains under construction. For the next six, the EAB will be providing additional information and periodic updates. The IEEE Region 1 Worcester County, Massachusetts, Section has volunteered to field-test PET in September, 2000. Members will be following the modular step-by-step plan and contacting educators in their communities.

A sample of the material on the site includes the following:

- "The Process?" details various ways of contributing, and how to go about it.
- "Success Stories?" are first-person stories by engineers who have helped in schools.
- "Initial contact?" tells where to start in the school and preparation tips for your first day.
- "Other ways to help?" gives ways to contribute other than in the classroom.
- "Basic tools?" provides helpful advice, and a way to learn about teachers and what they need.
- "Links?" leads to teaching kits and other valuable sites.

PET is intended to enable IEEE members to have a positive impact on and a mutually valuable experience with students, teachers, and school boards. 2000 Educational Activities Board Vice President Lyle Feisel has said, "PET addresses the growing worldwide problem of technological illiteracy, and enables engineers to contribute to its solution."

Initial funding for PET came from the IEEE President's Project Fund and the Life Member Committee. Comments and suggestions for the site can be sent by e-mail to pet@ieee.org, or individuals may e-mail Lynn Murison at l.murison@ieee.org.

[Information for the above item came from an IEEE EAB press release.]