A Notion of Diversity Order in Distributed Radar Networks

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***Abstract*— We introduce the notion of diversity order in distributed radar networks. Our long-term goal is to analyze the trade-off between distributed detection and centralized detection using co-located antennas. In contrast with the asymptotically high Signal-to-Noise Ratio (SNR) definition in wireless communications, we define the *diversity order* of a distributed radar network as the slope of the probability of detection (P_D) versus SNR curve at $P_D = 0.5$. In this paper we restrict our analysis to noise-limited systems. We evaluate the diversity order of fully distributed systems, and prove that for the OR rule, the gain in diversity is only logarithmic in the number of distributed sensors, denoted K. The AND rule does not lead to any gain in diversity order. We finally present some recent results, and provide preliminary analysis regarding the characterization of a Diversity-Multiplexing tradeoff in distributed radar detection.**

I. Introduction

Distributed detection is receiving renewed attention with the advent of waveform diversity and MIMO radars. This had been a rich area of research after its introduction by Tenney *et al.* in [1]. The main motivation behind this work was to alleviate the excessive bandwidth requirement of joint detection, which requires all the sensors to transmit exact information (likelihood ratios for example) to a fusion center which makes the final decision regarding the presence or absence of a target. In [1], the sensors are required to transmit binary decisions to the fusion center which 'optimally' combines local information into a final system-wide decision.

We denote a system as fully distributed when all the sensors transmit *binary* decisions to the fusion center. The fusion center then adopts an "n out of K" fusion rule. This allows the system three possible data fusion schemes: the OR rule ($n = 1$) wherein a target is declared present if *at least* one sensor declares it present; the AND rule ($n = K$) wherein a target is declared present only if detected by all sensors; and the MAJ (or majority) rules where a target is declared present if *some* majority of the sensors declare it present. A fixed fusion rule is often assumed due to its simplicity; however, varying the fusion rule depending on channel characteristics provides significant enhancements in performance. Finally, Multi-bit detection, where each sensor transmits an M-bit quantization of its information, lies in between joint and fully distributed detection.

The literature relating to distributed detection lacks any formal measure of performance as a function of various system parameters such as the number of sensors, the antenna array size and the transmission bandwidth requirement. We therefore introduce the notion of diversity order as a simple measure to study the behavior of large detection networks.

Diversity was first introduced in Multiple-Input Multiple-Output (MIMO) wireless communications where the diversity order is defined as the slope, in a log-log plot, of either the bit error rate or outage probability versus SNR curve in Rayleigh fading at asymptotically high SNR [2]. This notion paved the path for developing a Diversity-Multiplexing tradeoff between rate and reliability, and it played a crucial role in the development and design of MIMO wireless systems. In addition, even though this is a high-SNR concept, wireless systems usually achieve this asymptotic behavior at practical SNR levels, while radar systems invariably deal with low levels of SNR and an asymptotic definition is not useful.

We define the diversity order of a distributed system as the slope of the probability of detection (P_D) versus SNR curve at $P_D = 0.5$. This provides a convenient, consistent and useful definition for system evaluation and design. We also perform a diversity analysis on fully distributed systems, and we prove that the OR rule, however best fitted for our analysis, only achieves a logarithmic growth in diversity order.

This paper is organized as follows: Section II introduces the system model and presents some background on distributed detection. We then define diversity for distributed detection in Section III, before we analyze the diversity of fully distributed systems in Section IV. Due to space limitations, we have omitted the analysis pertaining to joint detection and 'optimal' binary detection, and the corresponding results are included in Section V. We conclude the paper by some discussions and projections for future work in Section VI

II. System Model and Background

In this section we introduce the system model adopted along this paper, in addition to the various assumptions regarding target model and channel characteristics. We also present an overview of distributed detection under the Constant False Alarm Rate (CFAR) criterion.

A. System Model

The system comprises K distributed sensors attempting to locate a target within a certain region of space. Each sensor k is equipped with an antenna array of N elements, and its corresponding received vector is of the form:

$$\mathbf{z}_k = \begin{cases} \alpha_k \mathbf{s}_k + \mathbf{n}_k, & \text{if target is present} \\ \mathbf{n}_k, & \text{if target is absent} \end{cases}, \tag{1}$$

where $\mathbf{s}_k$ is the space-time target steering vector corresponding to the target look direction and velocity, α_k is the complex amplitude, and $\mathbf{n}_k$ is the additive interference and noise vector.

The target is modeled as a Swerling type-II and consequently, $\{\alpha_k\}_{k=1}^K$ are independent and identically distributed (i.i.d.) drawn from a zero mean complex Gaussian random variable. The corresponding average received signal power is $A^2 = E\{|\alpha_k|^2\}$ where $E\{\cdot\}$ represents the statistical expectation operator. We will restrict our analysis to a noise limited scenario where the noise is assumed to follow a complex Gaussian distribution whose covariance matrix is of the form:

$$\mathbf{R}_n = \sigma^2 \mathbf{I}_N \tag{2}$$

$\mathbf{I}_N$ being the $N \times N$ identity matrix. Interference and adaptive suppression of interference is briefly discussed in section VI.

We assume in this paper that the observations at the sensors are statistically independent given the hypothesis. We also assume that the noise statistics are known, and the corresponding Receiver Operating Characteristics (ROC) can be derived accordingly.

Finally we describe the decision-making procedure. Each sensor k transmits a decision u_k to a fusion center, which makes the final decision u_0 indicating the presence (hypothesis H_1) or absence (hypothesis H_0) of a target in the region of space monitored by the sensors. We will adopt the convention that 1 symbolizes H_1 and 0 symbolizes H_0. $\mathbf{u}$ is the length-K vector of the decisions at the sensors. We assume that the fusion center receives the data from the local sensors without error. The reader is referred to [3] and the references therein for a summary on channel-aware distributed detection.

B. Distributed Detection

In [1], Tenney *et al.* analyzed the problem of distributed detection under the Bayesian criterion. In radar applications, we are particularly interested in preserving CFAR. In [4] the authors prove that the optimal detection rule under CFAR is a Neyman-Pearson test at both the fusion center and the local sensors. At the fusion center, the NP test is of the form:

$$u_0 = \begin{cases} 1, & \text{if} \quad \Pr(\mathbf{u}|H_1) \geq t_0 \Pr(\mathbf{u}|H_0) \\ 0, & \text{if} \quad \Pr(\mathbf{u}|H_1) < t_0 \Pr(\mathbf{u}|H_0) \end{cases}, \tag{3}$$

where t_0 is a global threshold to be determined according to the required false alarm probability (P_F), u_0 and $\mathbf{u}$ were previously defined. By the monotonicity of the optimum fusion rule established in [5], and knowing that the NP test is the most powerful test [6], the local tests at the sensors are also NP tests of the form:

$$u_k = \begin{cases} 1, & \text{if} \quad \Pr(y_k|H_1) \geq t_k \Pr(y_k|H_0) \\ 0, & \text{if} \quad \Pr(y_k|H_1) < t_k \Pr(y_k|H_0) \end{cases}, \tag{4}$$

where y_k and u_k are respectively the output of the processor and the corresponding local decision at sensor k, and $\{t_k\}_{k=1}^K$ are the local thresholds to be determined in order to maintain a desired probability of false alarm at this particular sensor.

The problem is reduced to maximizing the global probability of detection $P_D = \Pr(u_0 = 1|H_1)$ under the constraint that the global probability of false alarm $P_F = \Pr(u_0 = 1|H_0)$ is held constant. This problem in non-convex in general and no global optima are guaranteed by the optimization process. We will not dwell into the details of the optimization problem, and the reader is referred to [7] for more details.

C. Neyman-Pearson Test for Distributed Detection

We will now present the NP test for distributed systems. In the derivation below we assume an arbitrary Gaussian noise covariance matrix denoted by $\mathbf{R}_n$. The covariance matrix of the signal is denoted by

$$\mathbf{S}_k = A^2 \mathbf{s}_k \mathbf{s}_k^H. \tag{5}$$

Given a Swerling type-II target model and under both hypotheses, the received vector is complex Gaussian. Thus, under the null hypothesis, H_0, and due to the independence assumption,

$$\Pr\{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_K|H_0\} = \prod_{k=1}^K \frac{1}{\pi^N |\mathbf{R}_n|} e^{-\mathbf{z}_k^H \mathbf{R}_n^{-1} \mathbf{z}_k}. \tag{6}$$

Similarly, under the target-present hypothesis, H_1,

$$\Pr\{\mathbf{z}_1, \ldots, \mathbf{z}_K|H_1\} = \prod_{k=1}^K \frac{1}{\pi^N |\mathbf{R}_n + \mathbf{S}_k|} e^{-\mathbf{z}_k^H (\mathbf{R}_n + \mathbf{S}_k)^{-1} \mathbf{z}_k}. \tag{7}$$

The NP test is a likelihood ratio of the form:

$$\Lambda(\mathbf{z}_1, \ldots, \mathbf{z}_K) = \frac{\Pr\{\mathbf{z}_1, \ldots, \mathbf{z}_K|H_1\}}{\Pr\{\mathbf{z}_1, \ldots, \mathbf{z}_K|H_0\}} \tag{8}$$

and it can be shown that the NP test leads to the following test-statistic [8]:

$$\zeta = \sum_{k=1}^K \frac{A^2 |\mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{z}_k|^2}{1 + A^2 \mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{s}_k}. \tag{9}$$

Note that the numerator is exponentially distributed under both hypotheses; this is due to the fact that it is a scaled version of the power of a complex Gaussian random variable. Also note that the denominator is independent of the received vector. Another interesting observation is that the numerator is proportional to the output of the adaptive processor using the optimal weight vector $\mathbf{w} = \mathbf{R}_n^{-1} \mathbf{s}_k$ [9].

The NP test also implies that each sensor should use the most powerful test, which is the NP test itself [6]. Consequently, each sensor individually performs a test of the form of Eqn. (9), which in the case of sensor k, reduces to:

$$\zeta_k = \frac{A^2 |\mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{z}_k|^2}{1 + A^2 \mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{s}_k} \gtrless T_h^{(k)}, \tag{10}$$

where $T_h^{(k)}$ is a threshold to be determined in order to maintain the desired probability of false alarm at the sensor.

We will drop the subscripts for convenience. Under the null hypothesis, the received vector $\mathbf{z}$ is the zero-mean complex Gaussian noise vector, and the statistic is exponentially distributed with mean:

$$\lambda_0 = E\{\zeta|H_0\} = \frac{A^2\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s}}{1 + A^2\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s}}. \quad (11)$$

Consequently, the probability of false alarm at sensor k becomes:

$$P_f^{(k)} = \Pr(1|H_0) = \Pr(\zeta > T_h^{(k)}|H_0) = e^{-T_h^{(k)}/\lambda_0}. \quad (12)$$

Similarly, under the target-present hypothesis, the received vector is of the form:

$$\mathbf{z} = \alpha\mathbf{s} + \mathbf{n}, \quad (13)$$

and the statistic for the Swerling type-II model is also exponentially distributed with mean:

$$\lambda_1 = E\{\zeta|H_1\} = \frac{A^2\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s} + A^4|\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s}|^2}{1 + A^2\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s}}, \quad (14)$$

and the probability of detection at each sensor k is:

$$P_d^{(k)} = \Pr(1|H_1) = \Pr(\zeta > T_h^{(k)}|H_1) = e^{-T_h^{(k)}/\lambda_1}. \quad (15)$$

In this paper, and without loss of generality, we assume that $A^2 = 1$. Furthermore, it can be easily shown that under the noise-limited assumption and using Eqn. (2) we have:

$$\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s} = N\gamma. \quad (16)$$

where $\gamma = \sigma^{-2}$ is the local SNR. Plugging Eqn. (16) into Eqns. (11) and (14) we have the following identities:

$$\lambda_0 = \frac{N\gamma}{1 + N\gamma} \quad (17)$$

$$\lambda_1 = N\gamma \quad (18)$$

Finally, in this paper we consider the theoretical case of a symmetric system with each sensor receiving equal power on average. This symmetry assumption follows from the lack of any *a-priori* knowledge regarding the presence of a target, and the characteristics (position and velocity) of the target itself. This follows the same lines as the reasoning presented in the study of diversity in wireless communications [2]. This assumption is particularly useful as it implies that the probabilities of false alarm at the sensors are all assumed to be equal, which significantly simplifies the analysis as will be shown below.

III. Diversity for Distributed Networks

Diversity played a monumental role in the development of current MIMO wireless communications systems. In this context, diversity is defined as the slope in a log-log scale of the probability of error (P_e) versus SNR curve for asymptotically high SNR. We note that wireless systems do achieve this behavior at reasonable SNR levels. From its definition, diversity order captures performance gains for incremental gains in SNR. In addition, it reflects the number of independent paths in the systems. This lead to the characterization of a Diversity-Multiplexing tradeoff which is a tradeoff between rate and reliability: we can either send more data streams or send data more reliably.

A similar definition for diversity in distributed networks is hindered by three main considerations. First, radar systems invariably deal with SNR levels that are much lower than acceptable SNR ranges in wireless communications, hence an asymptotic definition might not be convenient. In addition, while P_e ranges of 10^{-3} to 10^{-6} are usually required in wireless systems in order to guarantee a certain Quality of Service, such ranges (for the probability of miss) are rarely of interest in the radar context. Finally, and proceeding with the latter idea, radar engineers are mainly interested in the 'rising' portion of the P_D curve as the steepness of this portion shows how fast the radar system goes from the low P_D to the high P_D regime. This being stated, the need arises for a more convenient definition for diversity in distributed radar networks. We chose the following definition:

Definition 3.1: The ***diversity order*** of a distributed radar system is the slope of the P_D curve at $P_D = 0.5$.

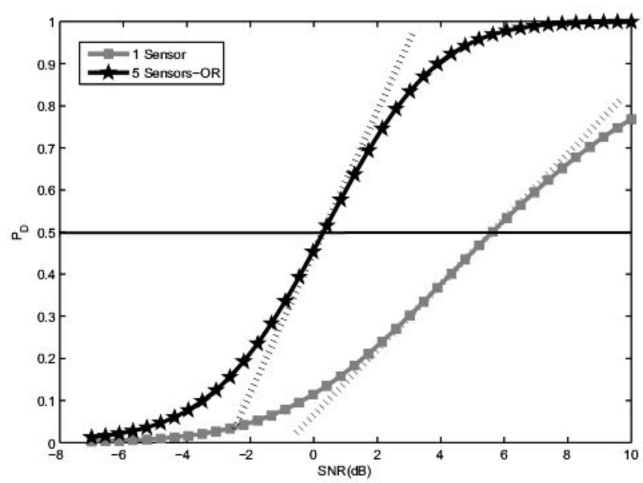

Fig. 1. Diversity as in Definition 3.1.

Figure 1 illustrates this definition by comparing the diversity orders of a single sensor and that of a system comprising 5 sensors. As this definition might seem arbitrary, we will give some reasoning regarding its appropriateness. First, the required probability of detection ($P_D = 0.5$) should be achievable (in fact expected) by most radar systems and for reasonable SNR levels. In addition, since we are mainly interested in the 'rising' portion of the P_D curve, the slope at $P_D = 0.5$ is highly likely to best estimate an average slope over this portion of the curve, and thus being a simple and meaningful measure reflecting the effective performance of the system.

IV. Diversity Order of Fully Distributed Systems

We call a system fully distributed when each sensor transmits a binary decision to the fusion center. The optimal fusion rule is an NP test and can be shown to be a summation of K log-likelihood ratios of the form:

$$\Lambda(\mathbf{u}) = \sum_{i=1}^{n_{H_1}} \log \frac{\Pr(1_i|H_1)}{\Pr(1_i|H_0)} + \sum_{i=1}^{n_{H_0}} \log \frac{\Pr(0_i|H_1)}{\Pr(0_i|H_0)} \qquad (19)$$

where $\Pr(j_i|H_\ell)$ is the probability that sensor i declares hypothesis H_j given hypothesis H_ℓ is true, $j,\ell = 0,1$. For example, $\Pr(1_i|H_1)$ is the probability of detection at sensor i. In addition, n_{H_j} is the number of sensors that declared that hypothesis H_j is true and clearly, $n_{H_0} + n_{H_1} = K$.

However sub-optimal, most distributed systems adopt a fixed fusion rule while optimizing the local rules accordingly. More specifically, the fusion center adopt one of the "n out of K" rules, with $n = 1$ corresponding to the OR rule, $n = K$ to the AND rule and the MAJ rules lie in between. In this section we will analyze the diversity order of these detection schemes.

A. Diversity Order for OR Detection

The fusion center adopts the OR ("1 ouf of K") rule if it declares target present when *at least* 1 sensor detects the target. Consequently, a target is missed only if all sensors miss, and the probability of detection is of the form:

$$P_D = 1 - (1 - P_d^{(k)})^K \qquad (20)$$

where $P_d^{(k)}$ is the local probability of detection. We invert Eqn. (12) to get the threshold:

$$T_h^{(k)} = -\ln P_f^{(k)} \lambda_0 = -\ln P_f^{(k)} \left(\frac{N\gamma}{1+N\gamma} \right). \qquad (21)$$

The total probability of detection reduces to:

$$P_D = 1 - (1 - e^{\frac{\ln P_f^{(k)}}{1+N\gamma}})^K \qquad (22)$$

Since we are interested in the slope of this curve, we differentiate with respect to γ and we get the following expression:

$$\frac{dP_D}{d\gamma} = \frac{-K \times N \times \ln(P_f^{(k)}) e^{\frac{\ln P_f^{(k)}}{1+N\gamma}} (1 - e^{\frac{\ln P_f^{(k)}}{1+N\gamma}})^{K-1}}{(1+N\gamma)^2} \qquad (23)$$

Theorem 4.1: For a noise-limited system using the "1 out of K" (OR) fusion rule, and for large K, the slope at $P_D = 0.5$ increases as $N \ln K$.

Proof: For $P_D = 0.5$,

$$P_D = 1 - (1 - e^{\frac{\ln P_f^{(k)}}{1+N\gamma}})^K = 0.5, \qquad (24)$$

$$e^{\frac{\ln P_f^{(k)}}{1+N\gamma}} = 1 - 0.5^{\frac{1}{K}}, \qquad (25)$$

Consequently,

$$1 + N\gamma = \frac{\ln P_f^{(k)}}{\ln(1 - 0.5^{\frac{1}{K}})}. \qquad (26)$$

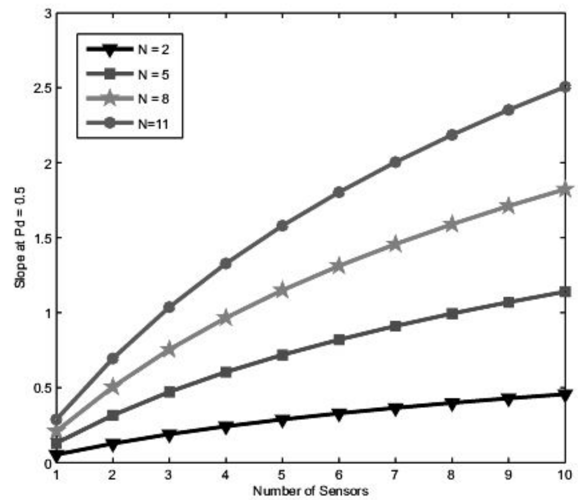

Fig. 2. Diversity Order at $P_D = 0.5$ for the OR rule

Using Eqns. (25) and (26) in conjunction with Eqn. (23) and noting that for the OR fusion rule:

$$P_f^{(k)} \approx \frac{P_F}{K}, \qquad (27)$$

we get the following expression for the slope of the P_D curve at $P_D = 0.5$:

$$\frac{dP_D}{d\gamma} = \frac{-NK \ln^2 \left(1 - 0.5^{\frac{1}{K}}\right)\left(0.5^{1-\frac{1}{k}} - 0.5\right)}{\ln P_F - \ln K}, \qquad (28)$$

and the limit:

$$\lim_{K\to\infty} \frac{dP_D}{d\gamma} \frac{1}{\ln K} = N \frac{\ln 2}{2}. \qquad (29)$$

which is a constant, and the proof is concluded by noting that the limits of both the numerator and denominator exist at infinity. ■

Figure 2 plots the diversity order as a function of K and N. The figure shows that the logarithmic behavior is also apparent for low (and thus practical) values of K and not only in the limit thus validating our theoretical analysis.

B. Diversity Order for AND Detection

At the other extreme from the OR rule, if the fusion center adopts the "K out of K" (AND) rule, the following expressions hold:

$$P_f^{(k)} = \sqrt[K]{P_F}, \qquad (30)$$

$$P_m^{(k)} = 1 - e^{\frac{\ln P_f^{(k)}}{1+N\gamma}}, \qquad (31)$$

$$P_D = (1 - P_m^{(k)})^K. \qquad (32)$$

Combining Eqns. (30), (31) and (32) we get:

$$P_D = e^{\frac{\ln P_F}{1+N\gamma}} = (P_f^{(k)})^{\frac{1}{1+N\gamma}}. \qquad (33)$$

which is independent of K, and we conclude that for the AND case, we have *no* improvement in diversity order when we add sensors to the network.

C. Diversity Order for MAJ Detection

If "n ouf of K" sensors are required to declare detection, $n = 2, \ldots, K-1$, the probability of detection is a binomial sum of the form:

$$P_D = \sum_{i=n}^{K} \binom{K}{i} (P_d^{(k)})^i (1 - P_d^{(k)})^{K-i}. \tag{34}$$

Let us denote each element of the summation in Eqn. (34) by $P_{d,\ell}$. We replace $P_d^{(k)}$ by its value from Eqn. (15) and differentiate with respect to γ:

$$\frac{dP_{d,\ell}}{d\gamma} = -\frac{1}{(1+N\gamma)^2} \binom{K}{\ell} e^{\frac{\ell \ln P_f^{(k)}}{1+N\gamma}} (1 - e^{\frac{\ln P_f^{(k)}}{1+N\gamma}})^{K-\ell-1} \times \left((K-\ell) N \ln P_f^{(k)} + KN \ln P_f^{(k)} e^{\frac{\ln P_f^{(k)}}{1+N\gamma}} \right), \tag{35}$$

which is strictly positive for all values of γ. However, even if we assume that $P_f^{(k)}$ is asymptotically increasing with n, $\frac{dP_{d,\ell}}{d\gamma}$ is not monotonic in $P_f^{(k)}$, and no generalization can be made regarding the behavior of the system when the parameter n varies.

We simulated various scenarios to get an idea about the behavior of the probability of detection curve when n varies. For example, for $K = 4$, $n = 2$ leads to the best performance for a certain range of input SNR. However, simulations also show the the OR rule is always a serious competitor, and it most often leads to the best performance. In addition, we note that while $P_f^{(k)}$ can be approximated directly for the OR rule (Eqn. 27), such procedure requires finding the root of a degree-K polynomial which falls in the interval $[0, 1]$, which is not straightforward. In addition, we have proved in recent work that the OR rule is asymptotically optimal among all fusion rule that require a local probability of false alarm that decays as $\frac{1}{K}$. Hence, we can assume that the OR rule is best-fitted for our theoretical analysis.

V. Recent Work

Optimal joint detection and 'optimal' 1-bit detection have been the subject for our recent research. For joint optimal detection, we proved the following main results:

- The NP test statistic follows a Gamma distribution;
- The diversity order grows as $N\sqrt{K}$.

When the fusion center of a fully distributed system uses the optimal NP test instead of a fixed fusion rule, we proved the following results:

- The NP test statistic follows a binomial distribution;
- The diversity order grows as $N\sqrt{K}$.

The details of these proofs will be omitted due to space limitations. However, these results show that distributed systems incur severe performance loss when compared to co-located antennas. This also proves that there is no diversity gain achieved by adding the number of bits transmitted by each sensor.

VI. Conclusion and Future Work

In this paper we have made two main contributions. First, we introduced the notion of diversity order for distributed detection: a simple measure of performance as a function of various system parameters, of which we cite the number of sensors, the antenna array size and the processing scheme. We then analyzed the performance of various distributed detection schemes. We have proved that the gain is *at most* logarithmic for fully distributed detection. For all optimal detection schemes, the gain in diversity order grows as $\sqrt{K}$, which proves that no diversity gain is achieved by enabling the local sensors to transmit more bits. However, as the transmission of additional bits improves performance, we are required to characterize the 'coding gain' (as opposed to diversity gain) achieved by transmitting more bits through the network.

Our results primarily show that most of the gains can be achieved with a small number of sensors. This underlines the importance of geometry in STAP detection. In fact, even in the presence of a large sensor network, choosing a *small* number of sensors 'smartly' (i.e. with a certain geometry) will achieve near-optimum performance and will require much less resources, a result that is in harmony with [8], and which provides additional motivation for the work on geometry and mobility [10].

The main subject for our future work will be the characterization of a Diversity-Multiplexing tradeoff in distributed detection systems. Having defined diversity, we anticipate multiplexing to be the number of targets or range cells, that can be interrogated simultaneously by the network. In this setting, the tradeoff would be to either detect K targets each with reliability r or detect a single target with reliability $r \times \ln K$. However intuitive, any confirmation of the validity of this statement is premature, and will be subject to further scrutiny.

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