

Diversity Order of Joint Detection in Distributed Radar Networks

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Abstract—In recent work we introduced the notion of diversity in distributed radar systems and we evaluated the diversity order of fully distributed networks. Our long-term goal is to analyze the trade-off between distributed detection and detection using co-located antennas. In this paper, we extend our earlier analysis and we evaluate the diversity of joint detection for symmetric noise-limited systems. The Neyman-Pearson (NP) statistic for joint detection is shown to follow a Gamma distribution. We prove that the diversity order of large networks is on the order of \sqrt{K} , thus outperforming the logarithmic diversity order of fully distributed detection. We finally provide simulations validating our theoretical analysis.

I. INTRODUCTION

In a distributed setting, optimal detection requires all the sensors to transmit exact information (likelihood ratios for example) to a fusion center which makes the final decision regarding the presence or absence of a target. However, in practice this is not possible due to various considerations, most notable of which is the limited communication bandwidth available between the sensors and the fusion center. In [1] Tenney *et al.* introduced distributed detection and suggested a (sub-optimal) detection scheme where sensors generate binary decisions which they transmit to the fusion center. The fusion center then combines these local decisions into a final system-wide decision. In the decade following the publication of this work, distributed detection became an active area of research, and researchers studied various optimality criteria, network topologies, and investigated system performance under various assumptions regarding the dependence of the received signals at the sensors [2].

On the other extreme from optimal joint detection lies fully distributed detection where each sensor transmits a 1-bit decision indicating the absence or presence of a target. Within this category, there are three data fusion schemes: the OR rule wherein a target is declared present if at least one sensor declares a target present; the AND rule wherein a target is declared present only if all sensors detect a target; and the MAJ (or majority) rules where a target is declared present if *some* majority of the sensors detect a target. Note that the OR, AND and MAJ fusion rules are of the form “ n out of K ”. At one extreme, we have $n = 1$ (OR rule); at the other, $n = K$ (AND rule) with the MAJ rules in between. We finally note that despite the fact that we often assume that the fusion rule

is fixed, the fusion center might switch between these “ n out of K ” rules depending on channel characteristics, which leads to better performance.

In between joint and fully distributed detection lies multi-bit detection. Each sensor transmits M bits indicating its best estimate regarding the presence of the target. In [3] and [4], the authors maximize the efficacy in order to design optimum quantizers for distributed signal detection. Lee *et al.* applied quantization to the distributed detection problem under the constant false alarm rate criterion [5], and proved that near-optimal performance can be achieved with a small number M of bits transmitted by each sensor.

Consider a radar detection system with K distributed sensors each possessing N antennas. Without a formal measure of performance, it is unclear what the impact of varying N or K is on system performance. In [6] we introduced the notion of a *diversity order* in distributed radar networks. Our newly introduced definition not only enables the comparison of various distributed radar apertures, but also provides intuition into the design of distributed sensing systems.

The notion of a diversity order was first introduced in multiple-input multiple-output (MIMO) wireless communications. The diversity order is defined as the slope, in a log-log plot, of either the bit error rate or outage probability versus signal-to-noise ratio (SNR) curve in Rayleigh fading at asymptotically high SNR [7]. The diversity order captures the number of independent paths over which the data is received. Furthermore, even though this is a high-SNR concept, wireless systems usually reach this asymptotic behavior at practical SNR levels. The notion of diversity order has played a crucial role in the development and design of MIMO wireless systems. On the other hand, radar systems invariably deal with low levels of SNR and an asymptotic definition is not useful. In [6] we defined the diversity order of a symmetric system as the slope of the probability of detection (P_D) versus SNR curve at $P_D = 0.5$. This provides a convenient, consistent and useful definition for system evaluation and design.

The complexity in analyzing the diversity order of distributed radar systems arises from the interplay between the probability of detection and the probability of false alarm, P_F . Unlike wireless communications, and for a fair comparison, P_D must be obtained for a constant P_F . However, both P_D

and P_F depend on the detection thresholds chosen at each sensor. As we will see, due to this interplay between P_D and P_F , both performance and the resulting diversity order are strongly dependent on the chosen fusion rule.

The work in [6] focuses on the definition of diversity and the analysis of the diversity order of the OR and AND rules. In this paper we extend the work of [6] to include a diversity analysis of joint (optimal) detection, which we prove to be of the form of maximal ratio combining (MRC). The Neyman-Pearson test statistic follows a Gamma distribution, and the probabilities of false alarm and detection are determined accordingly. We show that for large K , the diversity order of joint detection is on the order of \sqrt{K} .

This paper is organized as follows: Section II introduces the system model under consideration and provides a brief background on distributed detection including the NP test and the corresponding receiver operating characteristics (ROC). Section III briefly reviews the notion of diversity order in distributed radar systems. In Section IV we analyze joint detection and we evaluate its diversity order in Section V. The paper ends with some conclusions and suggestions for future work in Section VI.

II. SYSTEM MODEL AND BACKGROUND

In this section we present our system model and a brief overview of the available literature regarding distributed detection. We will also recall the main results of [6]. We begin with our system model.

A. System Model

The overall system comprises K distributed sensors attempting to detect the presence of a target in a certain region in space. Each sensor possesses N co-located antennas. The model focuses on a noise-limited scenario; interference and adaptive suppression of interference is briefly discussed in section VI. The k -th sensor receives a data vector of the form:

$$\mathbf{z}_k = \begin{cases} \alpha_k \mathbf{s}_k + \mathbf{n}_k, & \text{if target is present} \\ \mathbf{n}_k, & \text{if target is absent} \end{cases}, \quad (1)$$

where \mathbf{s}_k is the space-time target steering vector corresponding to the target look direction and velocity, α_k is the complex-valued amplitude, and \mathbf{n}_k is the additive interference and noise vector. The target is modelled as a Swerling type-II and consequently, $\{\alpha_k\}_{k=1}^K$ are independent and identically distributed (i.i.d.) drawn from a zero mean complex Gaussian random process whose variance determines the SNR.

Each sensor k transmits a decision u_k to a fusion center, which makes the final decision u_0 indicating the presence (hypothesis H_1) or absence (hypothesis H_0) of a target in the region of space monitored by the sensors. We will adopt the convention that 1 symbolizes H_1 and 0 symbolizes H_0 . \mathbf{u} is the length- K vector of the decisions of the sensors.

In this work, we assume that, given the hypothesis, the observations at the sensors are statistically independent. We also assume that the noise statistics are known, and the corresponding ROC can be derived accordingly. Finally, we

assume that the fusion center receives the data from the local sensors without error. The reader is referred to [8] and the references therein for a summary on channel-aware distributed detection.

B. Distributed Detection

Tenney *et al.* analyzed the problem of distributed detection under the Bayesian criterion [1]. In radar applications, we are particularly interested in preserving a constant false alarm rate (CFAR). In [9] the authors prove that the optimal detection rule under CFAR is a Neyman-Pearson test at both the fusion center and the local sensors. At the fusion center, the NP test is of the form:

$$u_0 = \begin{cases} 1, & \text{if } \Pr(\mathbf{u}|H_1) \geq t_0 \Pr(\mathbf{u}|H_0) \\ 0, & \text{if } \Pr(\mathbf{u}|H_1) < t_0 \Pr(\mathbf{u}|H_0) \end{cases}, \quad (2)$$

where t_0 is a global threshold to be determined according to the required false alarm probability (P_F). By the monotonicity of the optimum fusion rule established in [10], and knowing that the NP test is the most powerful test [2], the local tests at the sensors are also NP tests defined as:

$$u_k = \begin{cases} 1, & \text{if } \Pr(y_k|H_1) \geq t_k \Pr(y_k|H_0) \\ 0, & \text{if } \Pr(y_k|H_1) < t_k \Pr(y_k|H_0) \end{cases}, \quad (3)$$

where y_k is the output of the processor at the k -th sensor and $\{t_k\}_{k=1}^K$ are the local thresholds to be determined. The problem is reduced to maximizing the probability of detection $P_D = \Pr(u_0 = 1|H_1)$ under the constraint that the global probability of false alarm $P_F = \Pr(u_0 = 1|H_0)$ is held constant. This problem is non-convex in general and no global optima are guaranteed by the optimization process. We will not dwell into the details of the optimization problem, and the reader is referred to [11] for more details.

On the other hand, optimal detection systems jointly process the signals received at all K sensors, a scheme which can be regarded as maximal ratio combining (MRC) [12]. The diversity order of such a system will be the main subject for this paper.

C. Neyman-Pearson Test for Distributed Detection

We will now develop the NP test for distributed systems. Given the Swerling type-II target model, under both hypotheses, the received vector is complex Gaussian [13]. The derivation below uses an arbitrary Gaussian noise covariance matrix denoted as \mathbf{R}_n ; and the covariance matrix of the signal is denoted by

$$\mathbf{S}_k = A^2 \mathbf{s}_k \mathbf{s}_k^H, \quad (4)$$

where $A^2 = E\{|\alpha_k|^2\}$ is the mean target power and α_k is the complex amplitude of the received signal space-time steering vector \mathbf{s}_k . Here $E\{\cdot\}$ represents the statistical expectation operator.

Under the null hypothesis, H_0 , and due to the independence assumption,

$$\Pr\{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_K | H_0\} = \prod_{k=1}^K \frac{1}{\pi^N |\mathbf{R}_n|} e^{-\mathbf{z}_k^H \mathbf{R}_n^{-1} \mathbf{z}_k}. \quad (5)$$

Similarly, under the target-present hypothesis, H_1 ,

$$\Pr\{\mathbf{z}_1, \dots, \mathbf{z}_K | H_1\} = \prod_{k=1}^K \frac{1}{\pi^N |\mathbf{R}_n + \mathbf{S}_k|} e^{-\mathbf{z}_k^H (\mathbf{R}_n + \mathbf{S}_k)^{-1} \mathbf{z}_k}. \quad (6)$$

The likelihood ratio corresponding to the NP test is of the form:

$$\Lambda(\mathbf{z}_1, \dots, \mathbf{z}_K) = \frac{\Pr\{\mathbf{z}_1, \dots, \mathbf{z}_K | H_1\}}{\Pr\{\mathbf{z}_1, \dots, \mathbf{z}_K | H_0\}} \quad (7)$$

and it can be shown that the NP test leads to the following test-statistic [13]:

$$\zeta = \sum_{k=1}^K \frac{A^2 |\mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{z}_k|^2}{1 + A^2 \mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{s}_k}. \quad (8)$$

Note that the numerator is exponentially distributed under both hypotheses and that the denominator is independent of the received vector. Another important observation is that the numerator is proportional to the output of the adaptive processor using the optimal weight vector $\mathbf{w} = \mathbf{R}_n^{-1} \mathbf{s}_k$ [14].

The NP test also implies that each sensor should use the most powerful test, which is the NP test itself [2]. Consequently, each sensor individually performs a test of the form of Eqn. (8), which in the case of sensor k , reduces to:

$$\zeta_k = \frac{A^2 |\mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{z}_k|^2}{1 + A^2 \mathbf{s}_k^H \mathbf{R}_n^{-1} \mathbf{s}_k} \geq T_h^{(k)}, \quad (9)$$

where $T_h^{(k)}$ is a threshold to be determined in order to maintain the desired probability of false alarm.

We will drop the subscripts for convenience. Under the null hypothesis, the received vector \mathbf{z} is the zero-mean complex Gaussian noise vector, and the statistic is exponentially distributed with mean:

$$\lambda_0 = E\{\zeta | H_0\} = \frac{A^2 \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}}{1 + A^2 \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}}. \quad (10)$$

Consequently, the probability of false alarm at sensor k becomes:

$$P_f^{(k)} = \Pr(1 | H_0) = \Pr(\zeta > T_h^{(k)} | H_0) = e^{-T_h^{(k)}/\lambda_0}. \quad (11)$$

Similarly, under the target-present hypothesis, the received vector is of the form:

$$\mathbf{z} = \alpha \mathbf{s} + \mathbf{n}, \quad (12)$$

and the statistic for the Swerling type-II model is also exponentially distributed with mean:

$$\lambda_1 = E\{\zeta | H_1\} = \frac{A^2 \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s} + A^4 |\mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}|^2}{1 + A^2 \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}}, \quad (13)$$

and the probability of detection at each sensor k is:

$$P_d^{(k)} = \Pr(1 | H_1) = \Pr(\zeta > T_h^{(k)} | H_1) = e^{-T_h^{(k)}/\lambda_1}. \quad (14)$$

The problem reduces to maximizing the total probability of detection P_D keeping the total false alarm probability P_F constant.

We consider the theoretical case of a symmetric, noise limited system, i.e., $\mathbf{R}_n = \sigma^2 \mathbf{I}$, with each sensor receiving equal power on average. The symmetry assumption follows from the lack of any *a-priori* information regarding the presence of a target, and the characteristics (position and velocity) of the target itself. This follows the same lines as the reasoning presented in the study of diversity in wireless communications [7]. This assumption is particularly useful as it implies that the probabilities of false alarm at the sensors are all assumed to be equal, which significantly simplifies the analysis as will be shown below.

III. DIVERSITY ORDER FOR DISTRIBUTED NETWORKS

In [6], we introduced the notion of diversity in distributed networks and we evaluated the diversity order of fully distributed detection schemes where each sensor sends a 1-bit binary decision to the fusion rule. This section briefly reviews the notions and main results of [6].

In [6], we proposed the following definition of diversity in distributed networks:

Definition 3.1: The **diversity order** of a symmetric distributed radar system is the slope of the P_D curve at $P_D = 0.5$.

The main intuition behind this definition is that this slope at $P_D = 0.5$ is most likely to best estimate the slope along the ‘rising’ part of the P_D versus SNR curve and also captures the behaviour at realistic SNR levels.

Without loss of generality, in the following discussion we assume that $A^2 = 1$, and thus the input SNR is

$$\gamma = \frac{A^2}{\sigma^2} = \frac{1}{\sigma^2}. \quad (15)$$

Given N array elements at each sensor and under the null hypothesis,

$$\lambda_0 = E\{\zeta | H_0\} = \frac{N\gamma}{1 + N\gamma}. \quad (16)$$

Similarly, under the target-present hypothesis,

$$\lambda_1 = E\{\zeta | H_1\} = N\gamma. \quad (17)$$

Once the probability of false alarm at each sensor, $P_f^{(k)}$, is known, Eqn. (11) can be inverted to obtain the threshold at each sensor.

Finally, we evaluated the diversity order of various combining schemes, including the OR rule, AND rule and the various MAJ rules. We proved the following results [6]:

Theorem 3.2: For a symmetric noise-limited system using the “1 out of K ” (OR) fusion rule, and for large K , the slope at $P_D = 0.5$ increases as $N \ln K$.

Theorem 3.3: For a symmetric noise-limited system using the “ K out of K ” (AND) fusion rule, and for large K , there is no improvement when K is increased.

We also showed that no generalization can be made regarding the performance of the MAJ rules. However, our analysis showed that the OR rule, with only logarithmic gains, is best fitted for our theoretical analysis.

Finally, one should not read too much into this notion of diversity order. As in wireless communications, our definition of diversity order does not address *where*, as a function of SNR, is the bend in the P_D curve. Various systems may achieve the same diversity order but still have very different performance. We will draw attention to this later when comparing the OR rule with optimal joint detection. Furthermore, note that these results pertain to the Swerling type-II target model, and other models might lead to different results.

IV. OPTIMAL JOINT DETECTION

The analysis above focused on the various popular schemes for distributed detection where each sensor communicates a single binary decision to a fusion center. The other end of the performance (complexity, and bandwidth requirement) spectrum would be a perfect joint detection procedure wherein the signals from all sensors are processed jointly.

A. Joint Detection: An MRC approach

In joint detection, the data fusion center uses the test statistic described in Eqn. (8) to make the global decision. Note that this approach is equivalent to Maximum Ratio Combining (MRC) because each sensor contributes to the statistic proportionately to its received power. Under the Swerling type-II model and the symmetry assumption, the statistic is a summation of K i.i.d. exponential random variables. The sum follows a Gamma distribution of the form [15]:

$$f(x; K, \theta) = x^{K-1} \frac{e^{-x/\theta}}{\theta^K \Gamma(K)}, \quad (18)$$

where θ is the common mean of the K random variables and $\Gamma(K) = (K-1)!$ for integer K .

The probability of false alarm is the probability that the statistic surpasses a threshold T_h to be determined given the null hypothesis. This is the complement of the cumulative distribution function (CDF) of ζ given the null hypothesis, and is given by the upper incomplete Gamma function $\Gamma(K, x)$ defined as follows:

$$\begin{aligned} P_F &= \Pr(\zeta > T_h | H_0) = \Gamma\left(K, \frac{T_h}{\lambda_0}\right), \\ &= \frac{1}{\Gamma(K)} \int_{T_h/\lambda_0}^{\infty} x^{K-1} e^{-x} dx. \end{aligned} \quad (19)$$

Similarly, the probability of detection is defined as follows:

$$\begin{aligned} P_D &= \Pr(\zeta > T_h | H_1) = \Gamma\left(K, \frac{T_h}{\lambda_1}\right), \\ &= \frac{1}{\Gamma(K)} \int_{T_h/\lambda_1}^{\infty} x^{K-1} e^{-x} dx. \end{aligned} \quad (20)$$

A closed form expression to determine the threshold for a chosen probability of false alarm P_F is difficult to obtain and we are reduced to numerical solutions. Given the desired P_F , we perform a line search in order to find the corresponding threshold, T_h using Eqn. (19). We then replace the obtained value of T_h in Eqn. (20) to determine the probability of detection at a specific SNR.

Figure 1 compares the performance of MRC and the OR Rule for 3 and 8 sensors. The SNR gap between MRC and the OR fusion rule increases from around 1.2 dB for 3 sensors to 2.5 dB for 8 sensors. The curves seem parallel and this might hint that MRC and the OR fusion rule have the same diversity order. This is mainly caused by the log-scale on the x-axis. We will show in the following sections that the two schemes behave differently with increasing K .

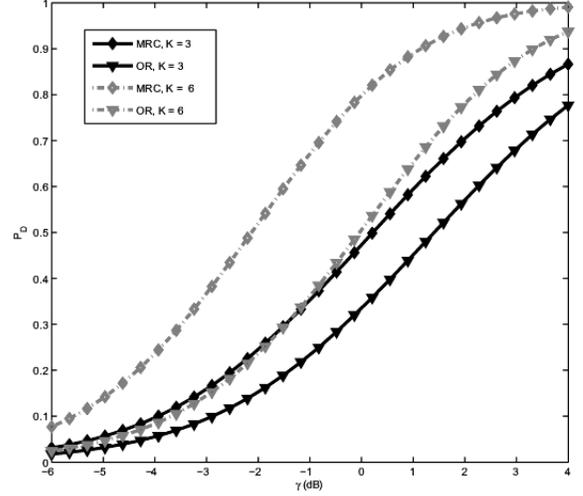


Fig. 1. Comparison of MRC and OR Rule

B. A Gaussian Approximation

Finding a closed-form solution for the slope and diversity order following the analysis above is intractable. However, when the number of sensors grows large, the central limit theorem dictates that the test statistic ζ follows a Gaussian distribution. Under the null hypothesis and the symmetry assumption, each sensor will contribute an exponentially distributed term with mean $\mu_k = \lambda_0$ and variance $\sigma_k^2 = \lambda_0^2$. Hence, by the independence assumption, the statistic ζ will be Gaussian distributed with mean $\mu_\zeta = K\lambda_0$ and standard deviation $\sigma_\zeta = \sqrt{K}\lambda_0$. The probabilities of false alarm and detection are consequently defined as follows:

$$P_F = \Pr(\zeta > T_h | H_0) = Q\left(\frac{T_h - K\lambda_0}{\sqrt{K}\lambda_0}\right) \quad (21)$$

$$P_D = \Pr(\zeta > T_h | H_1) = Q\left(\frac{T_h - K\lambda_1}{\sqrt{K}\lambda_1}\right) \quad (22)$$

where $Q(x)$ is the upper tail of the standard normal distribution and is defined by

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \quad (23)$$

Given the desired probability of false alarm P_F , we invert Eqn. (21) to get the threshold:

$$T_h = Q^{-1}(P_F) \cdot \sqrt{K} \cdot \lambda_0 + K \cdot \lambda_0 = \lambda_0 \cdot \sqrt{K} [\sqrt{K} + Q^{-1}(P_F)]. \quad (24)$$

We combine Eqn. (22) and Eqn. (24) to get the probability of detection at a certain SNR.

Figure 2 reflects the accuracy of this approximation, and shows that when the number of sensors increases, the approximation becomes more accurate. Figure 2 also shows that a large number of sensors is required to achieve a reasonable accuracy. However, as will be shown in the following section, the diversity order of the MRC processing scheme will closely follow this approximation even for small K .

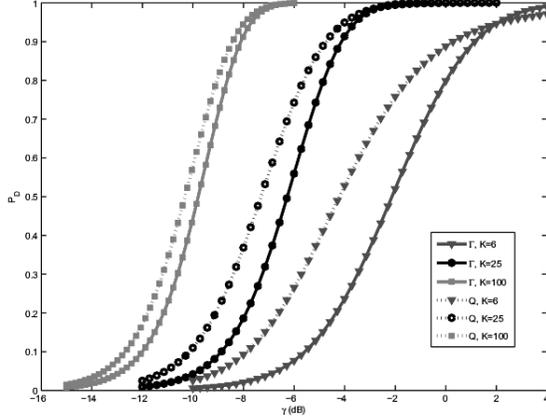


Fig. 2. Accuracy of the Gaussian Approximation

V. DIVERSITY ORDER OF OPTIMAL JOINT DETECTION

Proceeding with the analysis presented in the previous section, we derive the diversity order for MRC joint detection.

Theorem 5.1: For a symmetric noise-limited system using joint MRC detection, and with K large, the slope at $P_D = 0.5$ increases as $N\sqrt{K}$.

Proof: At $P_D = 0.5$,

$$P_D = Q\left(\frac{T_h - K\lambda_1}{\sqrt{K}\lambda_1}\right) = 0.5. \quad (25)$$

Inverting the Q-function, we get:

$$\frac{T_h - K\lambda_1}{\sqrt{K}\lambda_1} = 0. \quad (26)$$

Using Eqn. (24) and with simple algebra

$$1 + N\gamma = \frac{\sqrt{K} + Q^{-1}(P_F)}{\sqrt{K}}. \quad (27)$$

Let t be defined as:

$$t = \frac{T_h - K\lambda_1}{\sqrt{K}\lambda_1} = \frac{[\sqrt{K} + Q^{-1}(P_F)]}{1 + N\gamma} - \sqrt{K}. \quad (28)$$

Using the chain rule and the fundamental theorem of calculus we get:

$$\begin{aligned} \frac{dP_D}{d\gamma} &= \frac{dP_D}{dt} \frac{dt}{d\gamma} \\ &= \frac{N \cdot [\sqrt{K} + Q^{-1}(P_F)]}{\sqrt{2\pi}(1 + N\gamma)^2} e^{-\frac{1}{2} \left(\frac{[\sqrt{K} + Q^{-1}(P_F)]}{1 + N\gamma} - \sqrt{K} \right)^2} \end{aligned} \quad (29)$$

Using Eqns. (27) and (29), we get:

$$\frac{dP_D}{d\gamma}(P_D = 0.5) = \frac{N \cdot K}{\sqrt{2\pi}[\sqrt{K} + Q^{-1}(P_F)]} \quad (30)$$

which behaves as

$$N\sqrt{K} \quad (31)$$

thus concluding the proof. ■

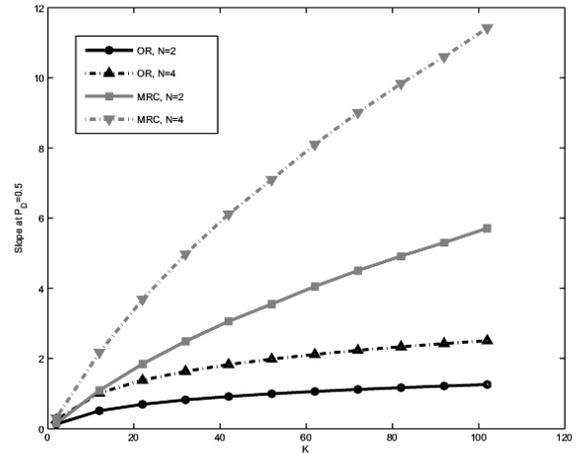


Fig. 3. Comparison of the slope of MRC and OR at $P_D = 0.5$

Again, this behavior is only proved valid in the limit when K is large. In addition, we note the linear gains in the number of antennas N , a result analogous to both fully distributed detection and MIMO systems. Figure 3 compares the slope at $P_D = 0.5$ for joint and distributed detection for various values of the parameter N . Note that the slope values for MRC were calculated numerically from the Gamma distribution, not from the Gaussian approximation. The slope increases much faster for MRC (joint detection) than the OR rule (distributed detection), and we conclude that the behavior of the diversity order as approximated using the central limit theorem is accurate even for small values of K .

A. Discussion

In wireless communications, as the number of antennas approaches infinity, the Rayleigh fading channel is reduced to an AWGN channel and we achieve line-of-sight conditions. In distributed detection, we have shown that a similar process takes place. For a fully distributed detection system (OR rule), which can be interpreted as a Selection Combining

(SC) scheme, the slope increases at $\ln(K)$. When the number of bits per sensor is infinite (MRC), the slope increases as \sqrt{K} . Following the new definitions, and unlike wireless communications, MRC and SC lead to different diversity orders. This is due to the interplay, in radar systems, between the probability of false alarm and the probability of detection.

The diversity order developed here provides a simple and consistent measure of the performance loss arising from using distributed detection (the OR rule - diversity order of $\ln(K)$) instead of joint detection (the MRC - diversity order of \sqrt{K}). Finally, it is interesting to note that neither scheme provides a diversity order of NK - which would arise if we had NK co-located antennas, i.e., the diversity order provides a simple measure of the loss in performance due to distributed detection, even if optimal processing is used.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we evaluated the diversity order of joint (optimal) detection in distributed radar networks. We first recalled the main notions and results of our previous work [6], where we defined the diversity order of a distributed radar system as the slope of the P_D curve at $P_D = 0.5$. We also evaluated the diversity of fully distributed schemes. In this paper, we extended this analysis to joint detection, where each sensor transmits the full likelihood ratio to the fusion center. The central limit theorem enabled us to approximate the NP test statistic by a Gaussian distributed statistic in order to prove that the diversity order of an optimal system is on the order of \sqrt{K} . We finally provided simulations that show that the slope for MRC systems increases much faster than the logarithmic increase of the OR rule, which validates our theoretical results.

One interesting observation, and once again unlike wireless communications, there is no clear-cut way on how to pass from the logarithmic increase of fully distributed schemes to the square-root increase of joint detection. Hence it is of great interest to analyze the performance of partially distributed systems, where local sensors transmit multi-bit likelihood ratios to the fusion center. This will be subject to future investigation.

We will also extend our work to include the more realistic STAP detection settings where the noise covariance matrix is of the form:

$$\mathbf{R}_n = \mathbf{R}_\gamma + \mathbf{R}_j + \mathbf{R}_c \quad (32)$$

where $\mathbf{R}_\gamma, \mathbf{R}_j$ and \mathbf{R}_c are respectively the additive noise, jamming and clutter covariance matrices [14].

Finally, and in harmony with our previous analysis in [6], we emphasize on the fact that our results do not necessarily characterize a $K - \sqrt{K}$ rate-reliability trade-off in distributed radar networks. Our main objective for future research is to delineate such a trade-off similarly to diversity-multiplexing in wireless communications. In the radar context, we envision multiplexing to mean the interrogation of multiple look points simultaneously.

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