Joint Domain Localized Adaptive Processing with Zero Forcing for Multi-Cell CDMA Systems

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Abstract— **An integrated beamforming (spatial processing) and multiuser detection (temporal processing) scheme is an effective approach to increase system capacity, but is also impractical due to the high associated computational costs. The authors previously proposed the Joint Domain Localized (JDL) processing which achieves significantly lower computational cost and faster convergence rate in terms of number of training symbols. This paper makes justifies the choice of the transformation matrix that is the basis for the JDL algorithm. Building on the JDL processing, we also introduce a new processor that combines the JDL processing and zero forcing for multi-cell uplink CDMA systems. The simulations show that this approach achieves better performance and faster convergence rate than the JDL algorithm as well as the reduced rank and iterative schemes introduced by other researchers. If restricted by short training sequences, it even outperforms the theoretically optimal processor.**

I. Introduction

Space division multiple access (SDMA) and temporal multiple access techniques such as code division multiple access (CDMA) can be combined to increase system capacity without sacrificing bandwidth [1]. However, both SDMA and CDMA are interference limited. Interference suppression is required to achieve the full potential of these multiple access schemes. In this regard, joint domain adaptive processing that integrates receive beamforming (spatial processing) and multiuser detection (temporal processing) in CDMA systems outperforms all possible linear processing based on the minimum mean squared error (MMSE) criterion [2], [3]. Unfortunately, this jointly optimal processor is impractical due to its high computational cost and data inefficient in terms of the required training symbols.

To overcome the drawbacks of the jointly optimal MMSE (OMMSE) processor, researchers have proposed sub-optimal schemes with fewer adaptive unknowns [4]–[6]. However, these schemes remain computationally intensive and complex. Recently, more practical reduced rank filters have been introduced, such as the Multistage Weiner filter (MSWF) [7], [8] and the iterative constrained optimal MMSE (COMMSE) processor [2]. The MSWF obtains the adaptive weights by a multistage decomposition. It provides satisfactory results but with relatively high complexity and slow convergence rate. With iterative cascaded spatial and temporal processing, the COMMSE filter only yields additive gains while the OMMSE processor yields multiplicative gains. In [9], a joint domain localized (JDL) processor with multiplicative gains, but low computational load and fast convergence was proposed. If restricted by short training sequences, this processor also achieves better performance than the theoretically optimal processor.

The JDL scheme in [9] transforms the data received at an antenna array to a "beamspace". However, there the transformation is chosen in an ad hoc manner. This paper justifies the choice of transformation matrix. This choice is a crucial aspect of the algorithm and the justification presented here strengthens its theoretical foundations.

The original JDL algorithm also assumes knowledge of all users' channels in a single-cell CDMA system. However, in a more practical multi-cell system, unknown inter-cell interference reduces the performance gains arising from joint domain processing. To deal with inter-cell interference, we introduce a new adaptive processing scheme, designated JDL-Z, that integrates JDL processing with zero forcing. The JDL-Z algorithm assumes knowledge of channels and spreading codes of intra-cell users only. In multi-cell scenarios, this new processor has better performance and faster convergence rate compared to the original JDL, MSWF and COMMSE processors. The JDL-Z scheme retains most of the benefits of the JDL approach, especially its excellent performance with short training sequences. This scheme also serves to introduce a framework wherein adaptive processing (zero-forcing) is followed by another stage of adaptive processing (MMSE in a transform domain).

Section II presents the background required for the JDL-Z algorithm, including the model for the direct sequence (DS) CDMA system used in this paper. It also briefly reviews the original JDL algorithm, followed by an analysis of the choices of the transformation matrix in Section III. Section IV presents a detailed description of the JDL-Z algorithm. Section V presents simulations to illustrate the efficacy of the new algorithm. The paper ends with some conclusions in Section VI.

II. Preliminaries

A. System Model

Consider a multi-cell synchronous CDMA uplink system where each base station is equipped with N receive antennas. Within the cell of interest, the base station receives K users' signals including M intra-cell and P inter-cell users. Each user is assigned a random short spreading code with processing gain G. Assuming the channels are slow and flat, the uplink signal within a single symbol period, $0 < t \leq T_s$, at the receive antenna array is an N-dimensional vector given by

$$\mathbf{x}(t)=\sum_{k=1}^{K}\sum_{j=0}^{G-1}a_k b_k s_k^j \psi(t-jT_c)\mathbf{h}_k+\mathbf{n}(t), \tag{1}$$

where a_k, b_k and $\mathbf{h}_k$ are the received amplitude, data symbol and an N-dimensional channel of user k respectively. The sequence $\{s_k^j=\pm 1,\ j=0,\ldots,G-1\}$ represents the length-G spreading code $\mathbf{s}_k$ of user k with chip waveform $\psi(t)$, lasting chip period T_c. Both temporal and spatial signatures of the users are assumed to have unit energy, i.e., $\int_0^{T_c}|\psi(t)|^2dt=1/G$ and $\mathrm{E}\left\{\mathbf{h}_k^H\mathbf{h}_k\right\}=1$ for $\forall k$, where the superscript ^{H} denotes the Hermitian and E the expectation operators. The receiver noise, $\mathbf{n}(t)$, is modelled as white and Gaussian.

After chip matched-filtering $\mathbf{x}(t)$, the received spatio-temporal data signal at the base station for a symbol period is an NG-dimensional vector $\mathbf{x}$, given by

$$\mathbf{x}=\sum_{i=1}^{K}a_k b_k(\mathbf{s}_k\otimes\mathbf{h}_k)+\mathbf{n}=\sum_{k=1}^{K}a_k b_k\mathbf{z}_k+\mathbf{n}, \tag{2}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{z}_k$ is the spatio-temporal channel (spatio-temporal signature) of user k. In this paper, the processor is assumed to have knowledge of the spatial and temporal channels, $\mathbf{h}_k$ and $\mathbf{s}_k$, of all M intra-cell users, but have no knowledge of the channels of the P inter-cell (interfering) users.

B. Joint Domain Localized Processing

Since the JDL algorithm is designed for single-cell systems, the channels for all K users are assumed known. The JDL algorithm is a two-stage beamspace-based scheme. The first stage transforms the spatio-temporal received signal, $\mathbf{x}$, to spatio-temporal "beamspace" by correlating the received signals with η selected spatio-temporal "beams". We define the amplitude-weighted correlation between the spatio-temporal channels of user i and j as

$$\rho_{i,j}=a_i^* a_j\mathbf{z}_i^H\mathbf{z}_j, \tag{3}$$

where * denotes complex conjugation.

In the original JDL algorithm [9], the η spatio-temporal "beams" are formed by correlating the received signal with its own spatio-temporal signature and $(\eta-1)$ signatures of the most interfering users - the $(\eta-1)$ users with the largest correlation as defined in (3). Note that the receiver requires an estimate of the amplitudes of the received signals. If these amplitudes are not available, the correlation is defined using the spatio-temporal channels $\mathbf{z}_i$ and $\mathbf{z}_j$ only.

This first JDL stage provides some interference suppression by decorrelating the signals of the interferers. The beamspace data can be obtained using a transformation matrix, $\mathbf{T}$. The transformation process is given by

$$\tilde{\mathbf{x}}=\mathbf{T}^H\mathbf{x}, \tag{4}$$

where the tilde ($\tilde{}$) denotes the beamspace domain.

In the second stage, localized beamspace data $\tilde{\mathbf{x}}$ are adaptively combined, in terms of MMSE, to produce a soft decision statistic for the information symbol. The residual interference and noise are further suppressed in this stage. The MMSE weights are found in beamspace by

$$\tilde{\mathbf{w}}=\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{v}}, \tag{5}$$

$$\tilde{\mathbf{R}}=\mathrm{E}\{\tilde{\mathbf{x}}\tilde{\mathbf{x}}^H\}, \tag{6}$$

$$\tilde{\mathbf{v}}=\mathrm{E}\{\tilde{\mathbf{x}}d^*\}, \tag{7}$$

where d is the desired information symbol. The final soft decision statistic to determine the transmitted symbol is

$$y=\tilde{\mathbf{w}}^H\tilde{\mathbf{x}}. \tag{8}$$

III. Choice of Transformation Matrix

The most important issue with the JDL algorithm is the choice of transformation matrix. In [9] this matrix is chosen in an ad hoc manner, based on decreasing values of the correlation in (3). Here we justify this choice.

A. Optimal method based on MMSE

The optimal choice of η spatio-temporal channels would minimize the final mean squared error (MSE). Since [6]

$$\mathrm{MSE}=1-\tilde{\mathbf{w}}^H\tilde{\mathbf{R}}\tilde{\mathbf{w}}=1-\tilde{\mathbf{v}}^H\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{v}}, \tag{9}$$

the η spatio-temporal channels should be chosen such that $\tilde{\mathbf{v}}^H\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{v}}$ is maximized. However, this would entail evaluating $\tilde{\mathbf{v}}^H\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{v}}$ for all ${}^KC_\eta=K!/(K-\eta)!\eta!$ possible combinations of η spatio-temporal channels. Clearly this brute force method is highly computationally intensive and impractical.

B. Suboptimal iterative method based on MMSE

Since the optimal choice of $\mathbf{T}$ is impractical, we introduce a MMSE-based, recursive, suboptimal method which has lower computation load. The η columns of $\mathbf{T}$ are chosen iteratively, based on the MMSE criterion in (9). The first column of $\mathbf{T}$ is chosen by direct evaluation of $\tilde{\mathbf{v}}^H\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{v}}$. Since this corresponds to $\eta=1$, $\tilde{\mathbf{R}}$ is only a number. For the other $(\eta-1)$ columns of $\mathbf{T}$, one spatio-temporal channel is chosen in each iteration based on the previously chosen channels in the MMSE sense. Without loss of generality, user 1 is the desired user.

Proposition: Given $\mathbf{T}_n=[\mathbf{t}_1,\mathbf{t}_2,\ldots,\mathbf{t}_n]$, the best choice of $\mathbf{t}_{n+1}$, to form $\mathbf{T}_{n+1}=[\mathbf{t}_1,\mathbf{t}_2,\ldots,\mathbf{t}_n,\mathbf{t}_{n+1}]$ in MMSE sense is given by

$$\mathbf{t}_{n+1}=\arg\max_{\mathbf{z}_i}\frac{|\tilde{\mathbf{z}}_1^H\tilde{\mathbf{R}}_n^{-1}\mathbf{T}_n^H\mathbf{R}\mathbf{z}_i-\zeta_{1,i}|^2}{\mathbf{z}_i^H(\mathbf{R}-\mathbf{R}\mathbf{T}_n\tilde{\mathbf{R}}_n^{-1}\mathbf{T}_n^H\mathbf{R})\mathbf{z}_i}, \tag{10}$$

where $\mathbf{R}=\mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\}$ and $\zeta_{i,j}=\mathbf{z}_i^H\mathbf{z}_j$. $\tilde{\mathbf{R}}_n$ and $\tilde{\mathbf{z}}_1$ are the transformed autocorrelation matrix of the received signal and transformed $\mathbf{z}_1$ with the transformation matrix $\mathbf{T}_n$.

Proof: See Appendix.

The η spatio-temporal channels that form the columns of the transformation matrix $\mathbf{T}$ are therefore found by the following recursive scheme:

1) Initialization:

$$\mathbf{t}_1=\arg\max_{\mathbf{z}_i}\tilde{\mathbf{v}}^H\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{v}},\qquad 1\le i\le M \tag{11}$$

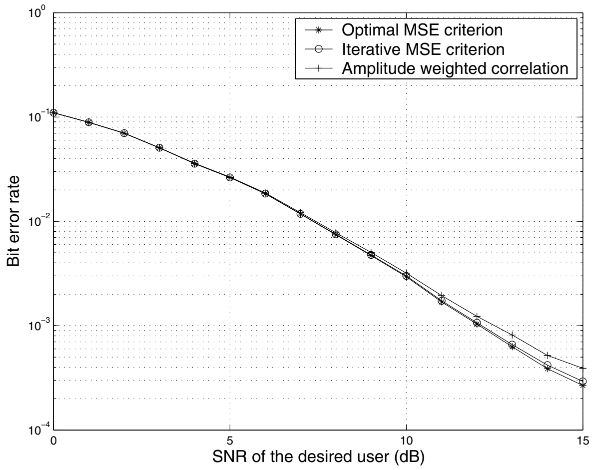

Fig. 1. Performance comparison of using different choices of transformation matrices.

where ($\tilde{}$) represents the transformed domain with $\mathbf{T} = \mathbf{z}_i$. Note that at this stage, $\tilde{\mathbf{R}}$ is a single number.

2) For $n = 1, \ldots, \eta - 1$, set $\mathbf{T}_{n+1} = [\mathbf{T}_n \;\; \mathbf{t}_{n+1}]$ where $\mathbf{t}_{n+1}$ is chosen using the criterion in (10).

The computational advantage of the iterative method over the optimal method is clear. The iterative method requires only η matrix inversions while the optimal method requires ${}^K C_\eta = K!/(K-\eta)!\eta!$ inversions of an $\eta \times \eta$ matrix. On the other hand, the iterative method is suboptimal and is not guaranteed to provide better results than other suboptimal approaches.

There are, therefore, three proposed choices of the transformation matrix $\mathbf{T}$. The matrix can be chosen optimally using the technique in Section III-A or sub-optimally using (3) in Section II-B or (10) in Section III-B. Clearly, the sub-optimal approach in Section II-B, using the η largest values of (3), has lowest, almost trivial, computation load.

Fig. 1 illustrates the performance comparison using constructions of $\mathbf{T}$ by these three methods. The system contains N=4 receive antennas, K=8 equal power known users and G=8. The LPR size, η, is set to 5. From the figure it appears that the simple, though ad-hoc, approach in [9], using the η largest amplitude-weighted correlations in (3), is a good choice for $\mathbf{T}$ because it achieves a close-to-optimal performance with much lower computation than the other two methods.

IV. JDL processing using Zero Forcing

This section describes an extension of the JDL algorithm that includes zero-forcing. The extension is based on the realization that matched filtering in the first stage effectively obtains a non-adaptive estimate of the desired signal. The JDL algorithm then introduces the adaptive second MMSE stage by combining η non-adaptive estimates to suppress the residual interference in the non-adaptive estimates. Here, the non-adaptive first stage is replaced by adaptive zero-forcing [10], however at the cost of higher computational complexity. The choice of zero-forcing in the first stage is just one of many possibilities, used to illustrate the concept of adaptive processing followed by another stage of adaptive "beamspace" processing to suppress residual interference. Since zero-forcing can eliminate all known intra-cell interference in the first stage, this processor performs very well in multi-cell environments.

A multi-cell CDMA system is used with M known intra-cell and P unknown inter-cell users. In the first stage, the transformation matrix, $\mathbf{T}$, is constructed using zero forcing to eliminate the known intra-cell interference. The transformation matrix consists of η zero forcing weight vectors at which unity responses are formed at the desired and $(\eta-1)$ most correlated intra-cell spatio-temporal channels.

To illustrate this construction with an example, if $\eta = 3$ and user 1 is the desired user, we assume $\mathbf{z}_1$, $\mathbf{z}_2$ and $\mathbf{z}_3$ are the desired and the $(\eta-1)$ most correlated intra-cell spatio-temporal channels, as defined by (3). The transformation matrix is then given by,

$$\mathbf{T} = [\mathbf{t}_1 \quad \mathbf{t}_2 \quad \mathbf{t}_3], \tag{12}$$

$$\text{where} \quad \mathbf{Z}_{1:M}^H \mathbf{t}_i = \mathbf{0}_i, \qquad i = 1, 2, 3. \tag{13}$$

The matrix $\mathbf{Z}_{1:M} = [\mathbf{z}_1 \; \mathbf{z}_2 \; ... \; \mathbf{z}_M]$ contains the channels of all M intra-cell users and $\mathbf{0}_i$ is a length-M vector of zeros with a single one in the i-th position. The weights, $\mathbf{t}_i$, therefore have unity response to the i-th spatio-temporal channel, and null out all other users' channels. This stage eliminates all intra-cell interference. As in (4), the transformed received signal $\tilde{\mathbf{x}}$ is defined as

$$\tilde{\mathbf{x}} = \mathbf{T}^H \mathbf{x}. \tag{14}$$

The size of the localized processing region (LPR), η beams in joint domain, is an implementation issue, representing a trade-off between computation load and performance illustrated in Section V.

In the second stage, the zero-forced beamspace data $\tilde{\mathbf{x}}$ is adaptively combined in the sense of MMSE. This stage of the JDL-Z algorithm is the same as the JDL algorithm where (5)-(7) are used to obtain adaptive weights in the transform domain. This stage suppresses residual interference and noise. The use of a second stage eliminates the problem of noise enhancement associated with zero forcing. In practice, the MMSE weights may be obtained using sample matrix inversion (SMI) [11] by finding the sample-averaged estimate of $\tilde{\mathbf{R}}$ or using a training-based scheme such as the Least Mean Squares (LMS) algorithm. As in (8), the final soft decision of the JDL-Z algorithm is

$$y = \tilde{\mathbf{w}}^H \tilde{\mathbf{x}}. \tag{15}$$

At first glance, the JDL-Z algorithm appears to be extremely computationally intensive. It appears to require η zero-forcing steps for each user of interest. However, in practice, the overall computation load is not much greater than that of zero-forcing. After all, all inter-cell users are "desired" and their zero-forced estimates would be obtained in any case. The additional computation load of the JDL-Z algorithm is just that associated with finding the η adaptive unknowns in $\tilde{\mathbf{w}}$. Finally, it must be noted that the choice of zero-forcing is somewhat arbitrary.

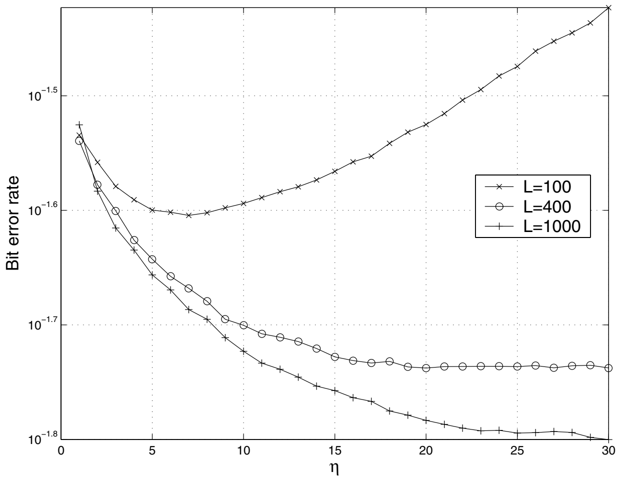

Fig. 2. Bit error rate of JDL-Z algorithm using SMI with 100, 400 and 1000 training bits.

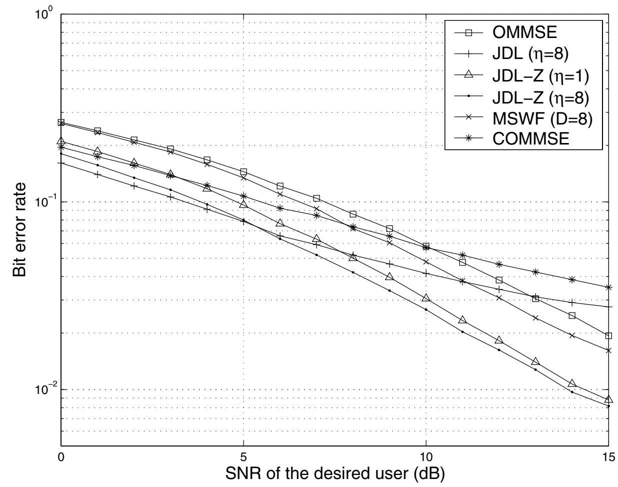

Fig. 3. Performance comparison, N=4, M=30, P=80, G=12.

Any adaptive processing scheme that produces a soft-statistic may be used. The JDL-Z framework allows for these statistics to be combined adaptively to further reduce interference.

V. Simulation Results

This section analyses the effect of LPR sizes on performance of the JDL-Z algorithm. It also compares the performances of JDL-Z, JDL, OMMSE, MSWF and COMMSE algorithms in terms of bit error rate (BER) and convergence rate. The first example shows the algorithm performance with different LPR sizes. The second and third examples illustrate the performance advantages and faster convergence of JDL-Z compared to OMMSE and other reduced-rank algorithms.

In all examples presented below, the receiver is assumed to know the signatures of the intra-cell users only. The weights $\tilde{\mathbf{w}}$ are estimated by SMI. Slow, flat and uncorrelated Rayleigh fading channels are modelled as constant over the time period used to estimate the covariance matrix and vector, $\tilde{\mathbf{R}}$ and $\tilde{\mathbf{v}}$, in each simulation. BPSK is used for data modulation.

A. Example 1: Effect of different η

This example shows that JDL-Z algorithm is not very sensitive to different η sizes. The example simulates a multi-cell DS-CDMA uplink system with $N = 4$ receive antennas, $M = 30$ intra-cell and $P = 80$ inter-cell users and processing gain $G = 12$. The SNR of the intra-cell users are all 10dB. 30 inter-cell interferers have SNR of -2dB, while the others have SNR of -10dB. The performance of the JDL-Z algorithm with varying η is shown in Fig. 2. The estimated weights are obtained by SMI using $L = 100, 400$ and 1000 training bits.

Fig. 2 shows that the performance of the JDL-Z algorithm depends on η as well as the number of training bits. As expected, with a large number of training symbols, the performance improves with larger η. However, with a short training sequence, the performance can worsen with increasing η. This occurs because, with few training symbols, the η adaptive weights cannot be properly estimated. Note that, in this example, using a second stage can potentially halve the bit error rate as compared to using zero-forcing alone ($\eta = 1$). A simulation such as this may also be used to find the optimal value of η given the number of training bits.

B. Example 2: Heavily loaded system

The second example simulates a heavily loaded multi-cell CDMA system with N=4 receive antennas, M=30 equal power intra-cell and P=80 inter-cell users, each with processing gain G=12. Of the 80 inter-cell interferers, 30 are at a power level -12dB and 50 are at -20dB with respect to the desired user. The number of training bits is $L = 100$. At each SNR of the desired user, the BER is calculated using 10^4 simulations with 1000 bits detected in each simulation. Fig. 3 compares the BER of two implementations of JDL-Z (η = 1 and 8), JDL (η=8), OMMSE, MSWF (D=8 stages) and COMMSE algorithms. The figure shows that JDL-Z processing achieves extremely good performance. Both JDL-Z and JDL algorithms outperform the OMMSE algorithm as well as other reduced rank algorithms including MSWF and COMMSE.

C. Example 3: Convergence rate comparison

This example illustrates the faster convergence of the JDL-based algorithms compared to OMMSE and other reduced-rank algorithms. Consider a system with N=4 receive antennas, M=20 equal power and P=60 users, each with processing gain G=16. Of the 60 inter-cell interferers, 20 are at a power level of -12dB and 40 at -20dB with respect to the users in the cell of interest. Fig. 4 plots the output signal-to-interference-plus-noise ratio (SINR) of the JDL, JDL-Z, MSWF, theoretically optimal OMMSE and COMMSE algorithms as a function of the number of training bits used. The JDL and JDL-Z algorithms use $\eta = 10$ while the MSWF uses $D = 10$ stages.

The figure shows that the JDL-Z, OMMSE and MSWF algorithms converge to the optimal SINR, but that OMMSE

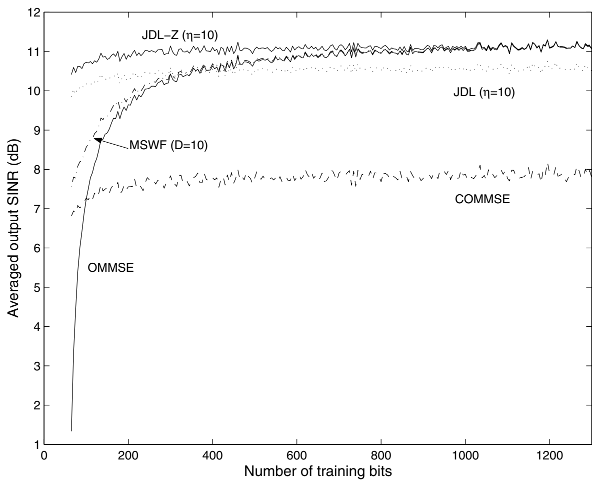

Fig. 4. Convergence rate comparison of JDL-Z, JDL and OMMSE.

scheme has the slowest convergence rate as it has the largest number of adaptive weights ($NG = 64$ weights). JDL-Z also converges faster than the MSWF algorithm. With the same number of adaptive weights ($\eta = 10$), JDL-Z converges slightly faster than JDL. This comes at the cost of significant additional computational complexity in the first zero-forcing stage. Therefore, JDL-Z would be most effective in relatively fast fading channels.

VI. Conclusions

This paper builds on the previously developed two-stage beamspace-based joint domain localized adaptive processing algorithm [9]. That algorithm depends on a good choice of transformation matrix; a simple choice is justified here. This paper then introduces the JDL-Z algorithm, an extension of JDL which incorporates zero forcing. The non-adaptive first stage in JDL processing is replaced by adaptive zero-forcing. Such a two-stage processor is especially useful for multi-cell CDMA systems effectively suppressing inter *and* intra-cell interference. The use of the second stage eliminates the problem of noise enhancement associated with zero forcing.

The simulations presented show that the new approach outperforms the JDL and other reduced rank filters such as the MSWF [7] and COMMSE [2] filters. If limited by short training sequences, because of a significantly faster convergence rate, JDL-Z performs even better than the fully optimal processor.

Appendix

The proposition in Section III-B states that the best choice, in the MMSE sense, to augment the rank-N transformation matrix $\mathbf{T}_n$ is to choose the spatio-temporal signature vector $\mathbf{z}_i$ given by

$$\mathbf{t}_{n+1} = \arg\max_{\mathbf{z}_i} \frac{|\tilde{\mathbf{z}_1}^H \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i - \zeta_{1,i}|^2}{\mathbf{z}_i^H (\mathbf{R} - \mathbf{R}\mathbf{T}_n \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R}) \mathbf{z}_i}, \tag{16}$$

Proof: We define the following symbols:

$\tilde{\mathbf{R}}_k$ is the transformed autocorrelation matrix of the received signal with $\eta = k$,
$\zeta_{ij} = \mathbf{z}_i^H \mathbf{z}_j$ is the correlation between user i and j,
$\mathbf{T}_k$ is the transformation matrix with $\eta = k$.

Without loss of generality, we assume that user 1 is the desired user and the LPR size is η.
For $1 \leq n \leq (\eta - 1)$:

Since minimizing mean squared error of the detected symbol is equivalent to maximizing $\tilde{\mathbf{v}} \tilde{\mathbf{R}}_{n+1}^{-1} \tilde{\mathbf{v}}$, the $(n{+}1)th$ column of $\mathbf{T}$, $\mathbf{z}_i$, is chosen such that $\tilde{\mathbf{v}} \tilde{\mathbf{R}}_{n+1}^{-1} \tilde{\mathbf{v}}$ is maximized. We have

$$\tilde{\mathbf{R}}_{n+1}^{-1} = \mathrm{E}\left[\mathbf{T}_{n+1} \mathbf{x}\mathbf{x}^H \mathbf{T}_{n+1}^H\right]^{-1} = \begin{pmatrix} \tilde{\mathbf{R}}_{11} & \tilde{\mathbf{R}}_{12} \\ \tilde{\mathbf{R}}_{12}^H & \tilde{\mathbf{R}}_{22} \end{pmatrix},$$

where

$$\tilde{\mathbf{R}}_{11} = \left[\tilde{\mathbf{R}}_n - \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i (\mathbf{z}_i^H \mathbf{R} \mathbf{z}_i)^{-1} \mathbf{z}_i^H \mathbf{R} \mathbf{T}_n\right]^{-1},$$

$$\tilde{\mathbf{R}}_{12} = \frac{\tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i}{\mathbf{z}_i^H \mathbf{R} \mathbf{T}_n \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i - \mathbf{z}_i^H \mathbf{R} \mathbf{z}_i},$$

$$\tilde{\mathbf{R}}_{22} = [\mathbf{z}_i^H \mathbf{R} \mathbf{z}_i - \mathbf{z}_i^H \mathbf{R} \mathbf{T}_n \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n \mathbf{R} \mathbf{z}_i]^{-1}.$$

After simplification and using the matrix inversion lemma,

$$\begin{aligned}
\tilde{\mathbf{v}}^H \tilde{\mathbf{R}}_{n+1}^{-1} \tilde{\mathbf{v}} \\
&= |a_1|^2 [\tilde{\mathbf{z}}_1^H (\tilde{\mathbf{R}}_n - \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i (\mathbf{z}_i^H \mathbf{R} \mathbf{z}_i)^{-1} \mathbf{z}_i^H \mathbf{R} \mathbf{T}_n)^{-1} \tilde{\mathbf{z}_1} \\
&\quad + \frac{\zeta_{1,i} \mathbf{z}_i^H \mathbf{R} \mathbf{T}_n \tilde{\mathbf{R}}_n^{-1} \tilde{\mathbf{z}}_1 + \zeta_{i,1} \tilde{\mathbf{z}}_1^H \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i - |\zeta_{1,i}|^2}{\mathbf{z}_i^H \mathbf{R} \mathbf{T}_n \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i - \mathbf{z}_i^H \mathbf{R} \mathbf{z}_i}] \\
&= |a_1|^2 \left[\tilde{\mathbf{z}}_1^H \tilde{\mathbf{R}}_n^{-1} \tilde{\mathbf{z}}_1 + \frac{|\tilde{\mathbf{z}}_1^H \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R} \mathbf{z}_i - \zeta_{1,i}|^2}{\mathbf{z}_i^H (\mathbf{R} - \mathbf{R} \mathbf{T}_n \tilde{\mathbf{R}}_n^{-1} \mathbf{T}_n^H \mathbf{R}) \mathbf{z}_i}\right]
\end{aligned} \tag{17}$$

Since the first term is constant, the best choice of $\mathbf{z}_i$ is that given in (16). ■

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