Beam-forming by mutual coupling effects of parasitic elements in antenna arrays

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An innovative way of beam-forming is described using mutual coupling effects of parasitic elements in an antenna array. It is shown that null and mainbeam placement can be achieved by varying the reactances at the center of the dipoles.

1. Introduction

Traditional beam-forming is achieved by varying the complex gain of an antenna-array [1]. The electromagnetic waves radiated from different segments of the array combine in-phase (and out of phase) to form the major lobes (and nulls) at desired directions. Controlling the complex gain is expensive, as it requires sophisticated RF circuitry for each element. As a result such antennas are available only for specialized applications such as in the military. A cost-effective solution for mobile communication is to exploit the electromagnetic mutual coupling between elements to achieve beam-forming [2].

This paper outlines the design of a five-element wire dipole antenna array. The center element is fed by a source and the remaining four elements are placed equidistant to each other on a circle centered around it. Variable impedances on the four parasitic elements are controlled to place nulls and main beams at desired locations in the far-field radiation pattern.

2. Theory

The radiation pattern of a single center-fed vertical wire dipole is constant (in all directions) in the horizontal plane. If another vertical wire dipole (passive) is brought to its vicinity, altering its center impedances will change the current pattern in each element. It is anticipated that by tuning these impedances it will be possible introduce nulls and main beams at desired locations.

Rigorous formulation of the problem is possible by using Maxwell's equations to obtain the tangential electric field on the surface of the dipoles:

$$E_{surface} = -\frac{j\omega\mu}{4\pi}\int I'\frac{e^{-jkR}}{R}dz' - \frac{j}{4\pi\varepsilon\omega}\frac{\partial}{\partial z}\int\frac{\partial I'}{\partial z'}\frac{e^{-jkR}}{R}dz' \qquad (1)$$

where primed quantities denote source points, unprimed ones are field points and R is the distance between them. This equation may be solved using the method of moments [3]. Rectangular basis and trial functions of unit height and width Δ are used. The Method of Moments reduces the integral equation to the matrix equation:

$$\mathbf{V'}_{M.N\,x1} = \mathbf{Z}_{MN\,x\,MN}\,\mathbf{I}_{MN\,x1} \qquad (2)$$

where M is the number of elements in the array and N is the number of subsections per element. The vector **I** is the current and **V'** is the voltage across each subsection. As the tangential electric field on the surface of a conductor is zero; **V'** is all zeros except for the entries corresponding to the air-gap regions at the center of each dipole. These non zero terms are of the form V_i-z_iI_i with the subscript referring to the center of the i'th dipole in the array and z_i being the variable impedance at that point. For the array described in this paper, only one element is excited by a source. Hence, only V_1, corresponding to the active element, is non-zero. All of the above, allows equation (2) to be expressed as:

$$\mathbf{V} = (\mathbf{Z}+\mathbf{Z_L})\mathbf{I} \quad \text{or} \quad \mathbf{I} = (\mathbf{Z}+\mathbf{Z_L})^{-1}\mathbf{V} \qquad (3)$$

where $\mathbf{Z_L}$ is a diagonal matrix isolating the z_iI_i terms from **V'** leaving **V** with only one non-zero term corresponding to V_1 – i.e. the single excitation voltage in the array.

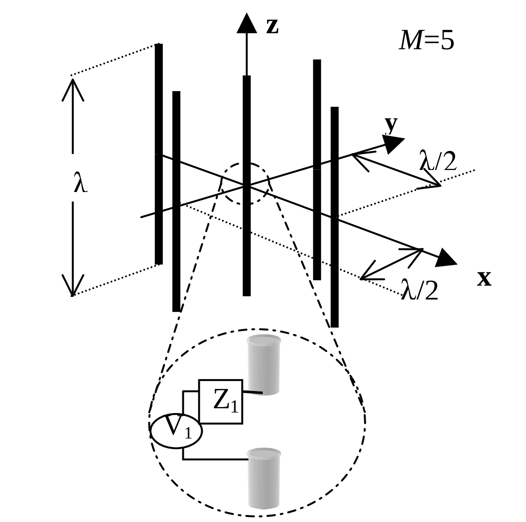

Figure 1- Antenna configuration

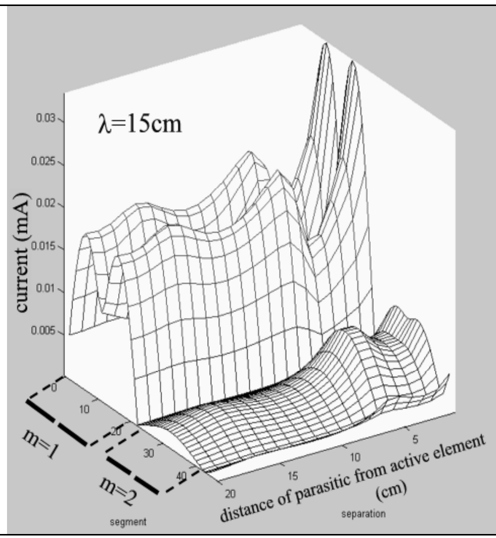

Figure 2 –current magnitude vs. separation

Note that for the antenna configuration shown in figure 1, the far-field radiation pattern is given by:

$$E_\theta = j\omega \frac{\mu}{4\pi}\frac{e^{-jkR}}{R}\Delta \sin(\theta)\left[1+\sum_{m,n} I_{mn} \exp(jk[R_m \sin(\theta)\cos(\phi-\phi_m)+z\cos(\theta)]\right) \qquad (4)$$

where Δ is the width of each subsection, R represents the distance from the far-field point to the center of the active element. For this array, ϕ is the azimuthal angle (in the spherical coordinate system) and ϕ_m is the horizontal angle between the x-axis and the center of the m'th element. In equation (4) R_m represents the horizontal distance of each element from the origin and I_{mn} is the current in the n'th segment of the m'th element in the array

From (4), the radiation intensity in any given direction is a function of the current **I** in the antenna and from (3) **I** itself is a function of center impedances $\mathbf{Z_L}$. The simulations outlined in this paper involve varying only the reactance part of z_i to maximize/minimize (4) in a given direction.

This involves a non-linear minimization problem of (M-1) (4 in the case described) independent variables. The trust-region method [4,5] of non-linear minimization is used to obtain solutions to the problem. At each iteration, the radiation pattern in (4) can be represented by a quadratic approximation:

$$E_\theta(\mathbf{S}) = E_0 + \mathbf{a}^T\mathbf{S} + ½\ \mathbf{S}^T\mathbf{H}\mathbf{S} \qquad (5)$$

where **H** is the Hessian Matrix – a symmetric matrix of second derivatives ($H_{ij}=\partial^2 E_\theta/\partial x_i \partial x_j$) defining the curvature of E_θ in the ellipsoidal neighborhood $\|\mathbf{CS}\| < \delta$ where **C** is a diagonal matrix. In equation (5), $\mathbf{S}=\mathbf{X}-\mathbf{X_0}$ where **X** is the vector of center reactances of the elements in the array and $\mathbf{X_0}$ is the center of the neighborhood being approximated. The gradient of the surface in (5) for all **S** in the ellipsoidal neighborhood can be computed by

$$\mathbf{g} = \mathbf{a} + \mathbf{H}\mathbf{S} \qquad (6)$$

The problem of optimizing (4) in the region $\mathbf{l}<\mathbf{X}<\mathbf{u}$, where **l** and **u** are lower and upper bound vectors respectively on the reactance values, is now optimizing (5) in its neighborhood at each iteration. Equation (5) can be optimized by letting **g** in (6) equal to zero to obtain:

$$\mathbf{S} = -\mathbf{H}^{-1}\mathbf{a} \qquad (7)$$

First, it has to be verified that (7) gives a minimum point (for the minimization problem). Secondly, it can be chosen if and only if it lies in the region $\|\mathbf{CS}\| < \delta$. If that is not the case, then **S** is chosen from the Boundary points $\|\mathbf{CS}\|=\delta$ such that $\mathbf{E}_\theta(\mathbf{X_0}+\mathbf{S})<\mathbf{E}_\theta(\mathbf{X_0})$. The current search point $\mathbf{X_0}$ is updated $\mathbf{X_0}+\mathbf{S}$. If it is either impossible to find an **S** or if it makes the new $\mathbf{X_0}$ exceed the bounds for the reactances, then, δ is decreased. The steps are repeated until convergence to the local minimum is established.

3. Results

The theory outlined in the previous section was used to simulate a wire dipole array consisting of 1 active and 4 parasitic elements. All simulations were carried out at a frequency of 2GHz using dipoles of length λ. The analysis is carried out using MATLAB®.

Figure 2 shows a magnitude plot of the current in the active element (m=1) and one of the parasitic elements (m=2) due to symmetric loading of the entire array. All parasitic elements will have identical current distributions due to symmetry in this particular case. As shown, it seems that the strongest current that can be generated in the parasitic elements occurs when they are separated from the active one by a distance of $\lambda/2$. Hence when designing the array, all parasitic element were placed a distance $\lambda/2$ from the active dipole. This was done in anticipation of the fact the current in the parasitic elements must be strong enough to generate fields that cancel those of the active element to form the sharp nulls in the radiation pattern.

Figure 3 shows some of the nulls that were formed at desired locations in the far-field pattern. Optimization was done by varying the load reactances of the passive elements and restricting them to lie between 50000Ω and –50000Ω. Note no attempt has been made to control the mainbeam. Convergence for fig. 3(a) was achieved after 10 iterations and fig. 3(b) after 5 iterations.

Figure 4. shows maxima obtained at desired locations in far-field. The algorithm used is identical to that of the minimization problem, except that the negative of (5) was minimized – which is the equivalent of maximizing the electric field vector. Fig 4 (a) converged after 22 iterations and Fig 4(b) converged after 24 iterations.

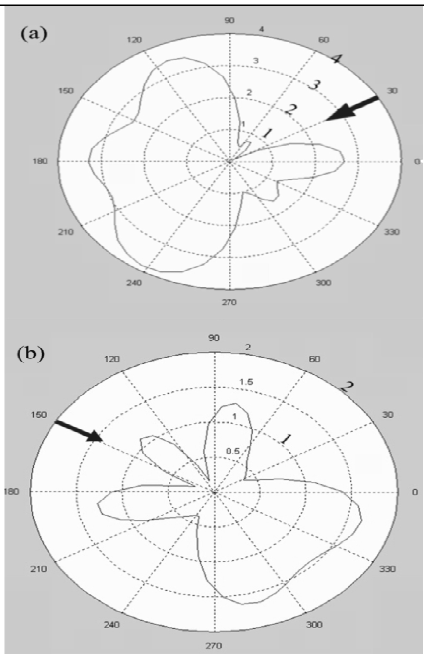

Figure 3 – Nulls at desired locations

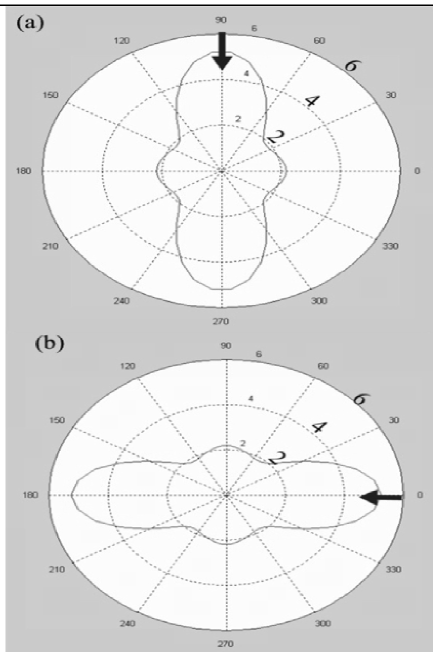

Figure 4 – Main beam at desired locations

4. Conclusion

As demonstrated, a new type of smart antenna can be implemented with beam steering and beam forming capabilities using only one active element. By varying the load impedances of the parasitic elements, nulls and main beams can be placed at desired directions. The advantage of using such an array is that it is more cost effective compared to conventional ones requiring sophisticated RF circuitry to handle their complex gain requirements. The array described in this paper takes advantage of mutual coupling effects between elements. Further investigation of the variation of the beam-pattern with load reactances is needed before beams and nulls can be formed more efficiently and before global optimization can be achieved (over the reactance bounds).

5. References

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