Optimal Relay Assignment and Power Allocation in Selection Based Cooperative Cellular Networks

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***Abstract*—We consider a system with a single base station communicating with multiple users over orthogonal channels while being assisted by multiple relays. Several recent works have suggested that selection, i.e., a single relay helping the source, is the best option in terms of the resulting complexity and overhead. However, in a multiuser setting, optimal relay assignment is a combinatorial problem. In this paper, using the sum rate as our design metric, we develop a convex optimization problem that provides an extremely tight upper bound on performance. We also provide a heuristic to find a close-to-optimal relay assignment. Simulation results using realistic channel models demonstrate the efficacy of the proposed scheme.**

I. Introduction

Traditionally, spatial diversity in a wireless communication system is harnessed by having multiple antennas at the transmitter and/or receiver. However, constraints on space, power and cost might not permit the use of multiple antennas at the transceivers. In such a situation, multiple distributed transceivers, each with a single antenna, can cooperate with one another to form virtual antenna arrays, and mimic a multi-antenna system. Such a distributed system, unlike multiple co-located antennas, can also help address large-scale fading.

First introduced by Sendonaris et al. in [1], wherein mobile users relay for one another, many cooperation schemes have been studied till date [1–4]. In networks with multiple relays, the traditional strategy has been to let all the relays forward their messages to the destination. The work in [2] and [3] proposed repetition-based cooperation schemes including fixed amplify-and-forward (AF) and decode-and-forward (DF) using orthogonal channels (time/frequency slots). However, having relays transmit on orthogonal bands is bandwidth inefficient. An alternative proposed is to use distributed space-time codes (DSTC), but this requires symbol level synchronization which is difficult to implement over a distributed network. It has recently been shown that all the benefits of cooperative diversity can be achieved with minimum overhead if a single 'best' relay cooperates with the source. This scheme is referred to as selection cooperation [5], [6]. Relay selection has received significant attention recently [5–9].

In the case of a single source-destination pair, choosing the best relay is fairly straightforward and solved for both DF [6] and AF [7] relaying. In both cases, the best relay is the one that contributes the most to the output signal-to-noise ratio (SNR). The selection gets significantly more complicated in the more practical case of multiple information flows. To stay within its power budget, a relay must then divide its available power between all flows it supports. Consequently, a relay that is best for a single flow may not remain the best overall. Relay selection then becomes a combinatorial problem. In [6], the authors present ad hoc approaches to approximate the optimal solution with limited complexity. That work did not address the issue of optimal power allocation across flows.

Relaying, in the context of cellular networks that has received limited attention [10]. In this paper, we develop relay selection as a by-product of a rate-maximization problem. We try to answer the question, *what relay assignment and power allocation scheme maximizes the sum rate to all the users*? Since solving this original problem has exponential complexity, we derive an upper bound on this sum rate. The formulation is a convex optimization problem. We use the resulting Karush-Kuhn-Tucker (KKT) conditions to illustrate why the bound is extremely tight. The bound also leads to a solution of the joint selection and power allocation problem. Unlike previous works in this area, we answer this question in the context of a cellular network that has relays installed to aid the users with a poor link to the base station (BS).

The rest of the paper is organized as follows. In Section II, we describe the system model in some detail. We then formulate the optimization problem and the upper bound to the sum rate in Section III. In Section IV, we show through simulations that this bound is extremely tight. The paper wraps up with some conclusions in Section V.

II. System Model

Consider a cellular network with a single BS, communicating with K users, and assisted by J relays, as shown in Figure 1. Each of the users is assigned an orthogonal channel, over which the BS to user and the relay to user communication take place. The users are frequency division multiplexed, although the results here also apply to the case with time division multiplexing. The relays in the system are fixed wireless terminals installed solely to aid the BS-user communication. The relays use the DF protocol with the same codebook as the transmitter. The communication between the BS and a user happens over two time slots. In the first time slot the BS transmits, while the relays and the user try to decode the message. In the second time slot, one of the relays, chosen *a priori*, re-encodes and then transmits the information it has decoded in the first time slot. The user uses the messages received in the two time slots to decode the transmitted information.

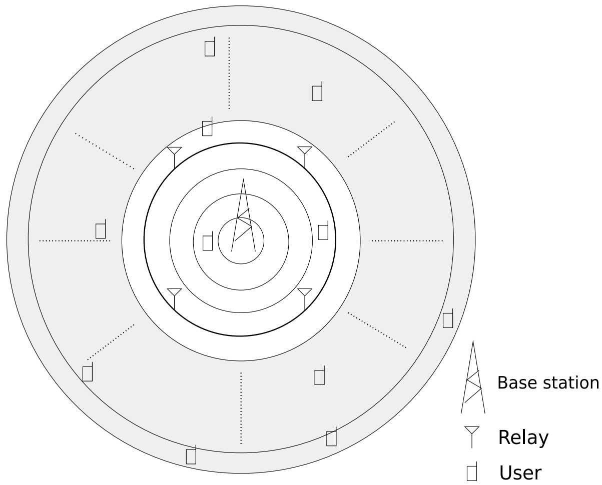

Fig. 1. A relay aided cellular network

Suppose user-k (denoted as d_k) is allotted to relay-j (r_j). For a system as described above, the maximum rate at which the BS can communicate with the receiver with the help of the relay is [6]

$$I_{d_k} = \min\left\{I_{sr_j}, I_{sr_jd_k}\right\}, \quad (1)$$

$$I_{sr_j} = \frac{1}{2}\log_2\left(1+\mathsf{SNR}_s|h_{sr_j}|^2\right), \quad (2)$$

$$I_{sr_jd_k} = \frac{1}{2}\log_2\left(1+\mathsf{SNR}_s|h_{sd_k}|^2+\mathsf{SNR}_r\alpha_{jk}|h_{r_jd_k}|^2\right), \quad (3)$$

where SNR_s and SNR_r are, respectively, the ratios of the transmit power at the BS (denoted as s) and the relay to the noise power at the receiver. h_{sr_j} is the channel between the BS and relay-j, denoted by r_j, similarly $h_{r_jd_k}$ is the channel between relay r_j and destination d_k. Finally, α_{jk} is the fraction of the total relay power used to communicate with user k. The factor of $\frac{1}{2}$ accounts for the fact that the BS-user communication happens over two time slots. I_{sr_j} is the rate at which the source can communicate with relay-j, while $I_{sr_jd_k}$ is the maximum rate at which the source can communicate to user-k with the help of relay-j. Equation (1) ensures that both the relay and the user can decode the message.

The channels between the BS, relays and users are modeled using the COST-231 model as recommended by the IEEE 802.16j working group [11]. The model includes the path loss, large-scale fading (modeled as a log-normal variable) and Rician small-scale fading (if a dominant component is available in a specific link). The strength of the dominant component is higher for the BS-relay links (assuming that the BS and relays are placed at some height above the ground) and lower for the relay-user and BS-user links. The existence of this dominant component is crucial since it suggests that all relays would be able to decode a source codeword, i.e., the factor limiting the rate is the second term, $I_{sr_jd_k}$ in (3).

III. Problem formulation and solution

In the model described in the previous section, every user was allotted one of the J relays. This paper deals with optimizing this allocation with the sum rate to all the users as the metric to compare different relay assignment schemes. Hence, a relay assignment is considered optimal if it maximizes

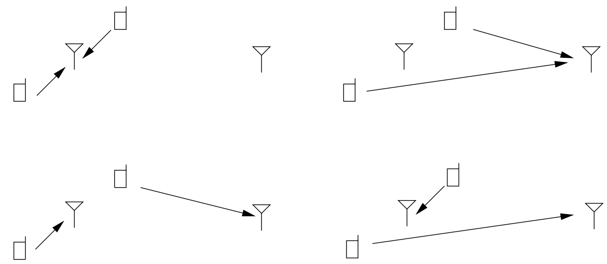

Fig. 2. The four different relay assignments possible

$$\sum_{k=1}^{K} I_{d_k} = \sum_{k=1}^{K}\min\left(I_{sr(d_k)}, I_{sr(d_k)d_k}\right), \quad (4)$$

where, $r(d_k)$ is the relay assigned to user-k.

In practice, the number of users, K, would be much larger than the number of relays, J. Hence, a single relay may be required to support multiple users, and to meet its power constraint, it must divide its power amongst these users. Thus, our objective is now to find the relay assignment scheme which maximizes the sum rate given by (4), and distributing powers at each of the relays amongst the users it supports to maximize the sum rate.

To formulate a tractable problem, in this paper we investigate a simplified version of the above problem. As mentioned earlier, in a cellular network, the data rate bottleneck is the compound source-relay-destination channel, the second term in (3). Essentially, we assume that

$$I_{sr_j} > I_{sr_jd_k} \quad \forall j,k. \quad (5)$$

In Section IV, we examine the validity of this assumption. Using (5), the sum rate to all the users reduces to $\sum_{k=1}^{K} I_{sr(d_k)d_k}$. Note that in spite of the assumption, the solution is not immediate. The fact that the relays divide their power amongst the users they support makes the problem complex. To illustrate this point, consider the case with two users and two relays as shown in Figure 2. Depending on the channel coefficients, any of the four assignment schemes could yield the maximum sum rate. The problem at hand is, therefore, an integer programming problem.

We will now state the problem formally. For the sake of brevity, let c_k represent $\mathsf{SNR}_s|h_{sd_k}|^2$ and p_{jk} represent $\mathsf{SNR}_r|h_{r_jd_k}|^2$, $j=1,2,\ldots,J$. Let α_{jk} be the fraction of the power of relay-j used to communicate to user-k. The optimization problem maximizes the sum rate to all the users subject to two constraints: only a single relay helps each user and each relay must meet a power constraint. The formal optimization problem is, therefore,

$$\max_{\{\alpha_{jk}\}} R = \max_{\{\alpha_{jk}\}} \sum_{k=1}^{K} \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk} \right) \quad (6)$$

$$\text{such that} \quad \forall k, \quad \alpha_{jk}\alpha_{lk} = 0,\ j \neq l, j, l \in \{1, 2, \ldots, J\} \quad (7)$$

$$\sum_{k=1}^{K} \alpha_{jk} \leq 1 \qquad \forall \ \ j, \quad (8)$$

$$\alpha_{jk} \geq 0, \quad (9)$$

where the objective function assumes the relay uses the same codebook as the source. Equation (7) enforces the selection rule allowing only one α_{jk} term to be non-zero for all relays. The remaining two constraints force the power allocated to be positive, but not exceed a chosen threshold. Note that we cannot use the usual gradient based methods to maximize the objective function in (6). Furthermore, an inherent assumption is that the BS has knowledge of the parameters that define the problem.

The solution to the optimization problem in (6)-(9) is complicated by the constraint in (7). There are a total of J^K possible relay assignments (each of the users can be assigned to any of the relays) and each of these schemes must be tested. Once the relay assignment is done, the well known water-filling algorithm can be used to allocate power at the relays to maximize the sum rate. Hence, an exhaustive search would involve the solving of J^K water-filling problems, and finding the maximum among them. Clearly, this is impossible for realistic values of J and K. We therefore explore alternative approximate formulations.

A. An upper bound to the maximum sum rate

The objective function of the optimization problem in (6)-(9) is concave and the constraints, other than the one in (7), are affine. Our strategy to solve the optimization problem in hand is to ignore the constraints given in (7) and maximize the sum rate subject to the power constraints alone:

$$\max_{\{\alpha_{jk}\}} \sum_{k=1}^{K} \frac{1}{2} \log_2 \left(1 + c_k + \sum_{j=1}^{J} p_{jk}\alpha_{jk} \right), \quad (10)$$

$$\text{such that} \quad \sum_{k=1}^{K} \alpha_{jk} \leq 1 \qquad \forall \ \ j, \quad (11)$$

$$\alpha_{jk} \geq 0. \quad (12)$$

Since we ignore a constraint, the solution so obtained will be an upper bound to the maximum sum rate achieved by selection. In the next section we illustrate why this upper bound is tight and in most cases is, in fact, the exact solution to the original optimization problem. Furthermore, the optimum power allocation vectors to the simplified maximization problem serve as a heuristic to the joint relay assignment and power allocation problem.

Note that since the optimization problem is now concave, solving this simplified problem is fairly straightforward. For example, interior point methods, discussed in [12] can be used to solve the problem. The computational complexity involved in solving the optimization problem is polynomial in K and J, and the problem is, hence, tractable for practical values of K and J.

B. Tightness of the bound

The previous section simplifies the original optimization problem into a simpler problem that serves as an upper bound. Using the case of two relays we now show why this upper bound is tight. With only $J = 2$ relays, the optimization problem is,

$$\min_{\{\alpha_{1k},\alpha_{2k}\}} -R = \min_{\{\alpha_{1k},\alpha_{2k}\}} -\sum_{k=1}^{K} \frac{1}{2} \log_2 \left(1 + c_k + p_{1k}\alpha_{1k} + p_{2k}\alpha_{2k} \right) \quad (13)$$

$$\text{such that} \sum_{k=1}^{K} \alpha_{jk} - 1 = 0,\ j = 1, 2; \quad -\alpha_{jk} \leq 0\ . \quad (14)$$

The Lagrangian of the minimization problem is given by

$$\mathcal{L}(\{\alpha_{1k}, \alpha_{2k}\}\,;\{\lambda_k^1\}, \{\lambda_k^2\}, \nu_1, \nu_2) = -R - \sum_{k=1}^{K} \lambda_k^1 \alpha_{1k} - \sum_{k=1}^{K} \lambda_k^2 \alpha_{2k} + \nu_1 \left(\sum_{k=1}^{K} \alpha_{1k} - 1 \right) + \nu_2 \left(\sum_{k=1}^{K} \alpha_{2k} - 1 \right), \quad (15)$$

where λ_k^1 and $\lambda_k^2,\ k = 1, 2, \ldots, K$ are the Lagrange multipliers associated with the constraint on positive power, and ν_1 and ν_2 are the Lagrange multipliers associated with the constraint on the total power at the two relays. Any solution to the optimization problem satisfies the KKT conditions, which are

$$\frac{p_{1k}}{1 + c_k + \sum_{i=1}^{2} p_{ik}\alpha_{ik}} + \lambda_k^1 = \nu_1,\ \lambda_k^1 \alpha_{1k} = 0,\ \lambda_k^1 \geq 0, \quad (16)$$

$$\frac{p_{2k}}{1 + c_k + \sum_{i=1}^{2} p_{ik}\alpha_{ik}} + \lambda_k^2 = \nu_2,\ \lambda_k^2 \alpha_{2k} = 0,\ \lambda_k^2 \geq 0. \quad (17)$$

Now suppose for some $i \in \{1, 2, \ldots, K\}$, α_{1i} and α_{2i} are both non-zero, then the conditions $\lambda_i^1 \alpha_{1i} = 0$ and $\lambda_i^2 \alpha_{2i} = 0$ dictate that λ_i^1 and λ_i^2 are both zero. From the KKT conditions, it follows that

$$\frac{\nu_1}{p_{1i}} = \frac{\nu_2}{p_{2i}}. \quad (18)$$

Similarly, if α_{1j} and α_{2j} are both non-zero for some $j \in \{1, 2, \ldots, K\}$, then

$$\frac{\nu_1}{p_{1j}} = \frac{\nu_2}{p_{2j}}. \quad (19)$$

Equations (18) and (19) cannot simultaneously be true (unless $p_{1i}/p_{2i} = p_{1j}/p_{2j}$, which occurs with probability zero). This implies that *when the power is optimally allocated,* $K - 1$ *of the* $(\alpha_{1k}, \alpha_{2k})$ *pairs have only one non-zero entry, and* at most *one of the* K *pairs has two non-zero entries*. This indicates that the solution obtained by ignoring (7) comes quite close to the solution to the original optimization problem in (6)-(9).

For the case of three relays, writing down the KKT conditions and analyzing them in a manner similar to the previous case; suppose for some i, (α_{1i}, α_{2i}, α_{3i}) are all non-zero, then,

$$\frac{\nu_1}{p_{1i}} = \frac{\nu_2}{p_{2i}} = \frac{\nu_3}{p_{3i}}. \tag{20}$$

This dictates that in all other triplets $(\alpha_{1k}, \alpha_{2k}, \alpha_{3k})$, at least two of the entries has to be zero.

Now consider the case when for no i, (α_{1i}, α_{2i}, α_{3i}) are all non-zero. Without loss of generality, suppose for some j, α_{1j} and α_{2j} are non-zero, and for some k, α_{2k} and α_{3k} are non-zero, then,

$$\frac{\nu_1}{p_{1j}} = \frac{\nu_2}{p_{2j}}, \qquad \frac{\nu_2}{p_{2k}} = \frac{\nu_3}{p_{3k}}. \tag{21}$$

These two equations imply that in all other three-tuples $(\alpha_{1k}, \alpha_{2k}, \alpha_{3k})$, only one of the entries is non-zero. This is because, if for some l, α_{1l} and α_{3l} are non-zero, then, (21) would imply, $p_{1l}/p_{3l} = p_{2j}p_{1i}/p_{3j}p_{2i}$, which occurs with probability zero. Hence, for the case of three relays, at most two of three-tuples can have more than one non-zero entry.

Generalizing it to the case of J relays, at most $J-1$ of the J-tuples $(\alpha_{1k}, \alpha_{2k}, \ldots, \alpha_{Jk})$ can have more than one non-zero entry. This indicates that if $K > J-1$, then a large fraction of the users are guaranteed to receive power from only one relay. Note that, in practice, one would expect the $K \gg J$ and this condition to be easily satisfied.

C. Optimal relay assignment and a lower bound

Solving the simplified optimization problem in (10)-(12) yields an upper bound on the achievable sum rate, but does not solve the original problem of assigning users to a single relay. This is because some of the users are assigned power from multiple relays. A simple heuristic, then, is to assign each such user to the relay with the maximum allocated power, i.e.,

$$r(d_k) = r_m \quad \text{if} \quad \alpha_{mk} = \max_j \{\alpha_{jk}\}. \tag{22}$$

Once this relay assignment is done, J water-filling problems can be solved for the power distribution at each of the relays. However, we can avoid a second round of water-filling by re-using the power allocation vectors derived from the earlier step. Note that in such a case, there could be some power left over at some of the relays. We will see in the next section that both the upper and lower bounds are extremely tight to the point of being indistinguishable in realistic settings.

An interesting aside is that this heuristic also provides a *lower bound* on the original optimization problem of (6)-(9). This is because any feasible solution can only be as good or worse than the optimal solution.

In summary, in this section we have developed, in the context of relay-assisted cellular communications, the power optimization problem that maximizes the sum rate to multiple users wherein each user is assigned to a single relay. The associated solution has exponential complexity and we formulated a simplified problem that serves as an upper bound and a related heuristic that also serves as a lower bound.

TABLE I
PARAMETERS USED IN COST231 MODEL

Parameter	Value chosen	Parameter	Value chosen
BS height	50m	Rooftop height	30m
Relay height	50m	User height	1.5m
Frequency	1GHz	Road orientation	90 degrees
Building spacing	50m	Street width	12m
Transmit power	20dBm	Noise power	-120dBm

TABLE II
PERCENTAGE OF LOCATIONS WHERE (5) IS SATISFIED

Distance from the BS (m)	% locations	Distance from the BS (m)	% locations
0-100	93.591	500-600	99.943
100-200	99.642	600-700	99.963
200-300	99.815	700-800	99.977
300-400	99.309	800-900	99.989
400-500	99.482	900-1000	99.992

IV. Numerical Results and Discussion

In this section, we verify the validity of the assumption in (5) and present the results of simulations to illustrate the tightness of the two bounds developed earlier. The simulations are implemented using the COST-231 channel model. In interests of brevity, the details of the model are not presented here. The details are available in [11].

To verify the assumption in (5), we consider a circular cell, centered at a BS, of radius one kilometer with four relays positioned at $(\pm 200\sqrt{2}m, \pm 200\sqrt{2}m)$, i.e., on a ring of radius 400m. 3×10^6 user locations in the cell are randomly generated. For each location, independent channels are generated using the COST-231 channel model. The parameters used in the model are listed in Table I. As shown in Fig. 1, we divide the cell into annular rings of radius 100 meters. In Table II we list the percentage of number of locations where (5) is valid. It is evident from the table that the assumption we make is valid whenever the user is farther than 300m from the BS. Essentially, for all user locations of interest, i.e., areas where users have a relatively weak channel to the BS, the assumption is valid.

Our next simulation tests the tightness of the upper bound as developed in this paper and the resulting heuristic. Note that this heuristic is our final solution to the joint selection and power allocation problem. Figure 3 plots the upper bound and sum rate achievable by the heuristic (that also acts as a lower bound on the achievable sum rate) for varying values of J and K averaged over many channel realizations. The lower bound has been computed by re-using the power allocation vectors resulting from solving the simplified optimization problem (10)-(12) (Refer Section III-C). The average signal-to-noise ratio is set at 30dB. For a fair comparison, the power allocated to each relay is set to $1/J$, i.e., all curves use the same total power. As is clear from the figure, the upper and lower bounds are indistinguishable. The heuristic, therefore, is an extremely effective solution to the joint selection and power allocation problem. By an exhaustive search, we also find the exact maximum sum rate for the case with $J = 2$ relays and K

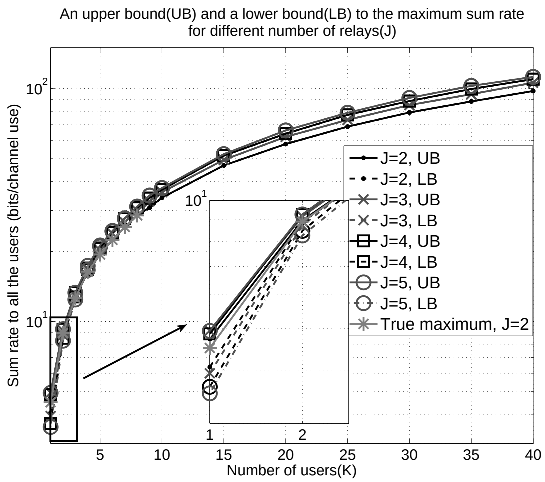

Fig. 3. The proposed upper bound to the maximum sum rate and the heuristic (a lower bound), as a function of the number of users. Note that both the bounds are extremely tight

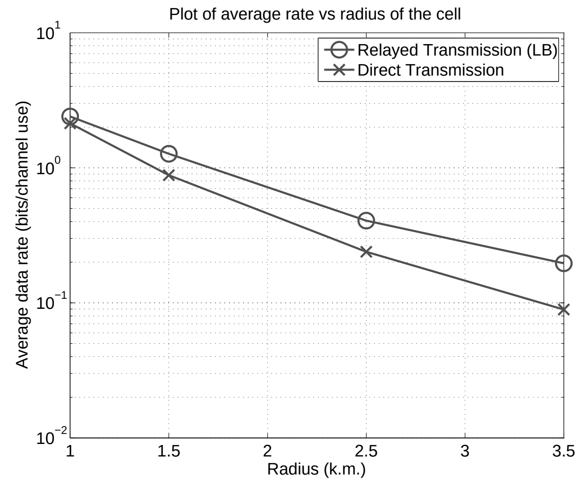

Fig. 4. Average user-rate as a function of the radius of the cell

between one and eight. Note that since each exhaustive search requires solving of J^K water-filling problems, any larger value of J is infeasible. We include the results in the plot.

For our final simulation, we consider a cell of radius $\mathrm{r_{cell}}$, with four relays. The users are uniformly distributed in the outer annular ring, of inner radius $\mathrm{r_{cell}}/2$ and outer radius $\mathrm{r_{cell}}$, the area shaded in gray in Figure 1. The density of users is set to 30/π per square kilometer. We find the average user-rate with and without the relays, and plot it as a function of $\mathrm{r_{cell}}$ in Figure 4. It is evident from the plot that the average data rates in a relay aided cellular network are higher than that of the data rates in the cell without relays. Hence, the coverage area of a cell can be substantially improved by deploying relays, assuming that improvement is commensurate with the cost of deployment. The worst case complexity of the optimal solution would have been on order of 4^{275} water-filling solutions. The simulation hence illustrates the usefulness of the alternative joint strategy developed here.

V. Conclusion

This paper deals with the use of cooperation in a cellular network wherein a base station is assisted by a few dedicated relays. Previous work largely for mesh and sensor networks has shown the importance of *selection*, i.e., each user using only one relay, since this minimizes the overhead due to orthogonal channels. However, in a scenario with multiple data flows, the selection process has been either brute force or ad hoc. Previous work has also largely ignored the problem of power allocation once the selection is achieved. In this paper we have developed an optimization framework to solve the problem of joint selection and power allocation.

The optimization problem uses the achievable sum rate as its figure of merit. Given that the selection problem has exponential complexity, in this paper we formulate an alternative convex optimization problem whose solution provides an upper bound on the sum rate. However, for practical values of number of users, the bound is indistinguishable from the true solution for sum rate. Since this solution can violate the selection condition, a related heuristic is derived that assigns users to the relay which allocates it the maximum power. The resulting lower bound is also extremely tight and indistinguishable from the upper bound. Essentially, we have an efficient solution to the problem at hand.

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