# SUB-CARRIER BASED WEIGHT TECHNIQUE FOR OFDM SYSTEMS

Stipe Moric, Raviraj Adve
Department of Electrical and Computer Engineering
University of Toronto
10 King's College Road,
Toronto, Canada M5S 3G4

***Abstract*— This paper presents a new method to extend adaptive antenna array techniques to mitigate strong co-channel interference in an orthogonal frequency division multiplexing system. The proposed algorithm uses optimal weights determined from classical methods, and known frequency information, to formulate adaptive weights specific to each sub-carrier. Performance improvements are investigated through computer simulations comparing bit error rate (BER) versus signal to noise ratio. These simulations in near-far scenarios illustrate the importance of the proposed technique.**

## I. INTRODUCTION

OFDM has become one of the prime candidates for fourth generation (4G) mobile technology and ad-hoc networks, because of its ability to achieve high-data rates and immunity to frequency-selective fading in multi-path environments [1]. Adaptive antenna arrays have been incorporated into the OFDM model to mitigate co-channel interference in order to increase capacity and decrease the probability of outage without allocating additional spectrum [2], [3]. However, because of limited frequency band allocations, strong co-channel interference caused by wireless LANs, jammers, and the near-far effects of ad-hoc networks will intensity in future OFDM systems [4]. Effective interference suppression is therefore a key issue in practical large-scale OFDM wireless networks. In this regard, combining OFDM with adaptive arrays is a logical choice for 4G mobile systems.

To develop adaptive spatial processing for OFDM systems, researchers have proposed wideband beamforming schemes to mitigate high powered co-channel interference [4], [5]. These techniques generally comprise a narrowband beamformer, reevaluated for each sub-carrier. To reduce the complexity of the wideband beamformer, sub-carrier clustering has been proposed [5]. However, this approach is sub-optimal due to the resultant wandering of adaptive nulls as a function of the cluster frequencies.

This paper presents an accurate algorithm to mitigate strong co-channel interference in OFDM systems with adaptive antenna arrays. Using known frequency bin information and the optimal weights of one narrowband beamformer, individual weights for each sub-carrier are constructed. This frequency sub-carrier based (FSB) technique is significantly less complex when compared to earlier wideband beamforming schemes. As with other work in this area, we assume line of sight communications.

## II. FREQUENCY SUB-CARRIER BASED WEIGHT TECHNIQUE

Consider an $M$-user, $N$ sub-carrier OFDM system with $S$ receive antenna elements. The transmitted time-domain OFDM symbol representation of the $m$-th user, $l$-th sample in the $n$-th block of data can be written as

$$y_{l,m}(n) = \sum_{i=0}^{N-1} x_{i,m}(n) e^{j2\pi(\frac{li}{NT_s}+f_o)t} \tag{1}$$

Where $1/T_s$ is the OFDM symbol rate, $f_o$ is the center frequency of the modulated signal, $t$ is chosen in increments of $T_s$ in order to satisfy the inverse Fast Fourier transform (IFFT) criterion and $x_{i,m}(n)$ is the frequency-domain data symbol representation of the $m$-th user modulated by the $i$-th sub-carrier. The baseband signal in Eqn. (1) can be written in vector form as

$$\mathbf{y}_m(n) = \mathbf{F}^H(n)\mathbf{x}_m(n) \tag{2}$$

where,

$$\mathbf{y}_m(n) = [y_{0,m}, y_{1,m}, ..., y_{N-1,m}]^T \tag{3}$$

$$\mathbf{x}_m(n) = [x_{0,m}, x_{1,m}, ..., x_{N-1,m}]^T \tag{4}$$

$$\mathbf{F}(n) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{(1)(1)}{N}} & \cdots & e^{-j2\pi\frac{(1)(N-1)}{N}+} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{(1)(N-1)}{N}+} & \cdots & e^{-j2\pi\frac{(N-1)(N-1)}{N}} \end{bmatrix} \tag{5}$$

Here, $\mathbf{F}(n)$ is the matrix representation of the baseband Fast Fourier transform (FFT) operation. The frequency bins of IFFT/FFT are known to follow a cyclic pattern, therefore the passband frequencies bins of the $N$ sub-carriers are given by

$$f_n \quad = \quad n\Delta f + f_0, \; n \in [-N/2, N/2-1], \tag{6}$$

where $\Delta f = 1/(NT_s)$ is the minimum orthogonal frequency

spacing between adjacent sub-carriers. Given $M$ users located at angles $\phi_m, m = 0, 1, \ldots M-1$ with respect the array, and assuming an additive white gaussian noise (AWGN) channel, the $l$-th received sample can be written in vector form as

$$\mathbf{v}_l(n) = \mathbf{S}_l(\phi)\mathbf{y}_l^H(n) + \mathbf{n}_l \tag{7}$$

where

$$\mathbf{v}_l(n) = [v_{0,l}\, v_{1,l}\, \ldots v_{S-1,l}]^T \tag{8}$$

$$\mathbf{S}_l(\phi) = \tag{9}$$
$$\begin{bmatrix} 1 & \cdots & 1 \\ e^{j2\pi\frac{f_l}{c}dcos(\phi_0)} & \ldots & e^{j2\pi\frac{f_l}{c}dcos(\phi_{M-1})} \\ e^{j(2)2\pi\frac{f_l}{c}dcos(\phi_0)} & \ldots & e^{j(2)2\pi\frac{f_l}{c}dcos(\phi_{M-1})} \\ \vdots & \ddots & \vdots \\ e^{j(S-1)2\pi\frac{f_l}{c}dcos(\phi_0)} & ... & e^{j(S-1)2\pi\frac{f_l}{c}dcos(\phi_{M-1})} \end{bmatrix}$$

$$\mathbf{y}_l(n) = [y_{l,0}, y_{l,1}, ..., y_{l,M-1}] \tag{10}$$

$$\mathbf{n}_l = [n_{0,l}\, n_{1,l}\, \ldots, n_{S-1,l}]^T. \tag{11}$$

Here $l = 0, 1, ..., N-1$, $^H$ denotes the Hermitian of a matrix, $c$ is the speed of light, $d$ is the spacing between antenna elements, $\mathbf{S}_l(\phi)$ is an $S \times M$ steering vector matrix for the $l$-th transmitted sample, $\mathbf{y}_l(n)$ is the $l$-th transmitted data sample for $M$-users and $\mathbf{n}_l$ is the AWGN at each receive antenna element for the $l$-th received data sample.

The received signal is processed at the adaptive array where a beamforming technique is implemented to determine optimal weights at the center frequency. These weights are determined using any classical adaptive weight technique such as minimum mean square error (MMSE), minimum output energy (MOE) etc.

The FSB algorithm is implemented at this stage to determine the weights for specific sub-carriers using the frequency information given in Eqn. (6). The first step of the FSB algorithm is to construct a polynomial using the optimal weights of the center sub-carrier as coefficients. The polynomial is given by

$$p(z) = w_0 + w_1 z + ... + w_{S-1} z^{S-1}. \tag{12}$$

The zeros of this polynomial represent the directions of interference (DOI) determined by the adaptive process. These zeros are found by taking the roots of the polynomial. Mathematically this is given as

$$p(z) = (z - z_{0,o})(z - z_{1,o})...(z - z_{S-2,o}). \tag{13}$$

Here $o$ represents the values of the zeros for the center frequency. The DOI can now be determined, and are given by

$$\hat{\cos(\phi_p)} = \frac{c}{j2\pi f_o d} \ln(z_{p,o}), \tag{14}$$

where $p = 0, 1, \ldots, S-2$. Physically the interference directions are independent of frequency. A polynomial similar to Eqn. (13) can therefore be determined for individual sub-carriers using new zeros. These new zeros are given by

$$z_{p,l} = e^{j2\pi\frac{f_l}{c}d\hat{cos(\phi_p)}}, \tag{15}$$

where $z_{p,l}$ represents the zeros for the $l$-th sub-carrier and $f_l$ is the frequency bin values given in Eqn. (6). The zeros for specific sub-carriers are represented in each column of the zeros matrix given by

$$\mathbf{Z}(n) = \begin{bmatrix} z_0^0(n) & z_1^0(n) & \cdots & z_{N-1}^0(n) \\ z_0^1(n) & z_1^1(n) & \cdots & z_{N-1}^1(n) \\ \vdots & \vdots & \ddots & \vdots \\ z_0^{S-2}(n) & z_1^{S-2}(n) & \cdots & z_{N-1}^{S-2}(n) \end{bmatrix} \tag{16}$$

The polynomial coefficients are determined from the roots. The coefficients of the new polynomials are the weights at each frequency for individual sub-carriers. The weight matrix is given by

$$\mathbf{W}(n) = \begin{bmatrix} w_0^0(n) & w_1^0(n) & \cdots & w_{N-1}^0(n) \\ w_0^1(n) & w_1^1(n) & \cdots & w_{N-1}^1(n) \\ \vdots & \vdots & \ddots & \vdots \\ w_0^{S-1}(n) & w_1^{S-1}(n) & \cdots & w_{N-1}^{S-1}(n) \end{bmatrix} \tag{17}$$

Here, the $l$-th column represents the weights for the $l$-th sub-carrier of the $n$-th block of data. Finally the hermitian of the weights is multiplied by the received signal, resulting in an $N \times N$ matrix. The diagonal of this matrix represents the corrupted version of the desired signal without interference. This is given by

$$\tilde{\mathbf{r}}(n) = \texttt{diag}(\mathbf{W}^H(n)\mathbf{V}(n)) \tag{18}$$

where, `diag` represents the diagonal entries of a matrix.

$$\tilde{\mathbf{r}}(n) = [\tilde{r}_0(n), \tilde{r}_1(n), \ldots, \tilde{r}_{N-1}(n)]^T. \tag{19}$$

and,

$$\mathbf{V}(n) = \begin{bmatrix} V_0^0(n) & V_1^0(n) & \cdots & V_{N-1}^0(n) \\ V_0^1(n) & V_1^1(n) & \cdots & V_{N-1}^1(n) \\ \vdots & \vdots & \ddots & \vdots \\ V_0^{S-1}(n) & V_1^{S-1}(n) & \cdots & V_{N-1}^{S-1}(n) \end{bmatrix} \tag{20}$$

Here, $\mathbf{V}(\mathbf{n})$ is the matrix representation of the signal received at the array and $\tilde{\mathbf{r}}(n)$ is the interference free but still distorted version of the received signal. The frequency-domain representation of the estimated signal after it is down-converted to the baseband, modulated by the FFT and fed through a decision device, can be written as

$$\tilde{\mathbf{x}}(n) = \mathbf{F}(n)\tilde{\mathbf{r}}(n) \tag{21}$$

where,

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} \tilde{x}_0(n) \\ \tilde{x}_1(n) \\ \vdots \\ \tilde{x}_{N-1}(n) \end{bmatrix} \tag{22}$$

It should be noted that a similar idea was introduced for uplink-downlink beamforming in [6].

## III. Simulation Results

This section compares the performance of minimum mean squared error (MMSE) using weights at only the center frequency with MMSE with the FSB algorithm in strong interference environments. The performance is measured in terms of BER verse SNR. The example uses a bandwidth of 150MHz with $N = 256$ sub-carriers centered around a center frequency of 5GHz. The $M = 5$ users use BPSK modulation.

Figure 1 shows the BER versus SNR for different interference powers in an OFDM system with an $S = 5$ element receive antenna. In the first case, four interfering users with signal powers 20 dB higher then the desired user are chosen. In the second case, the four interfering users signal powers are 12dB higher then the desired user. The larger the interference power the better the performance of the MMSE-FSB algorithm.

Figure 2 shows the BER versus SNR for different OFDM bandwidth sizes. The same parameters were used as in the first simulation, fixing the four interfering signals at 20dB. Notice that the MMSE-FSB algorithm performs better for larger bandwidths.

Figure 3 shows the reason for the improved performance of the MMSE-FSB algorithm. Directions of arrival for the five users are $(50^0, 70^0, 90^0, 110^0, 130^0)$. Without loss of generality the signal arriving at $90^0$ is chosen as the desired user. The beam-patterns for a classical MMSE algorithm are plotted for the first and last frequency of the 150MHz bandwidth OFDM signal. Even though the deviation appears to be small, Fig. 1 shows that in an interference limited environment it leads to significant performance loss. This divergence is not present in the nulls when the FSB algorithm is applied. Finally the larger the bandwidth used in the OFDM system the larger the null divergence becomes.

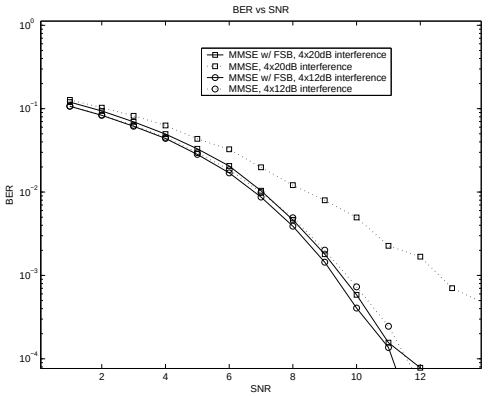


Fig. 1. BER vs SNR comaparison of MMSE and MMSE with FSB.

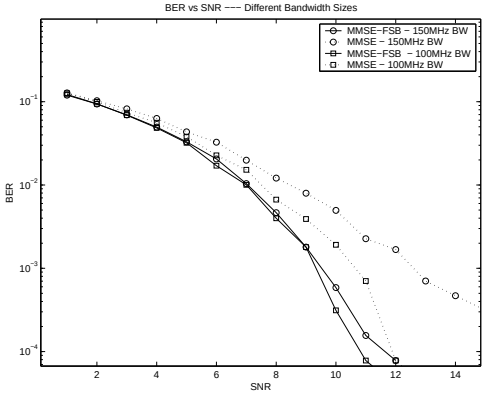


Fig. 2. BER vs SNR for different OFDM bandwidth sizes

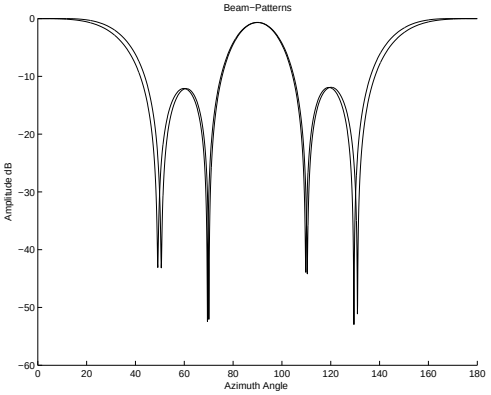


Fig. 3. Beam-patterns showing slight null deviation

## References


[1] A. L. Intini, "Orthogonal frequency division multiplexing for wireless networks," tech. rep., University of California, UCSB Center for Research in Electric Art Techinology, December 2002. http://www.create.ucsb.edu/ATON/01.01/OFDM.pdf.

[2] C. K. Kim, K. Lee, and Y. S. Cho, "Adaptive beamforming algorithm for ofdm systems with antenna arrays," *IEEE Transactions on Consumer Electronics*, vol. 46, pp. 1052–1058, Nov. 2000.

[3] Y. Lee and N. R. Sollenburger, "Adaptive antenna arrays for ofdm systems with cochannel interference," *IEEE Transactions on Communications*, vol. 47, pp. 217–229, Feb 1999.

[4] Y. Sun and H. Matsouka, "A novel adaptive antenna architecture - sub-carrier clustering for high speed ofdm systems in presence of rich interference," *VTC*, vol. 3, pp. 1564–1568, Spring 2002.

[5] B.-L. Cheung, F. Alam, J. H. Reed, and W. D. Woerner, "A new beamforming algorithm for ofdm systems," *In the Proceedings of the 14th Annual International Conference on Wireless Communications*, July 2002.

[6] Y.-C. Liang and F. Chin, "Downlink channel covariance matrix (dccm) estimation and its applications in wireless ds-cdma systems," *IEEE Journal on selected areas in Communications.*, vol. 19, pp. 222–232, Feb 2001.